CLASS-8 : Rational Numbers - Level-1

(1) $4 - \dfrac{5}{1+\dfrac{1}{3+\dfrac{4}{9}}} = 4 - \dfrac{5}{1+\dfrac{9}{31}} = 4 - \dfrac{5}{\dfrac{40}{31}}$

$= 4 - \dfrac{5 \cdot 31}{40} = \dfrac{32-31}{8} = \dfrac{1}{8}$

(2) $-2 + \dfrac{1}{2} = \dfrac{-4+1}{2} = \dfrac{-3}{4}$

(3) $\dfrac{3\sqrt{2}}{2\sqrt{3}} \cdot \dfrac{2\sqrt{3}}{2\sqrt{3}} = \dfrac{6\sqrt{6}}{4 \cdot 3} = \dfrac{\sqrt{6}}{2} = \dfrac{2.449}{2} = 1.224$

(4) $0.75 = \dfrac{75}{100} = \dfrac{3}{4}$

(5) $\dfrac{3}{7} + x + \dfrac{-8}{21} + \dfrac{5}{22} = \dfrac{-125}{462}$

$\dfrac{198 + 462x - 176 + 105}{462} = \dfrac{-125}{462}$

$\Rightarrow 462x = -252 \Rightarrow \boxed{x = \dfrac{-252}{462}}$

(6) (a) is true.

$\dfrac{5}{7} < \dfrac{7}{9} < \dfrac{9}{11} < \dfrac{11}{13} = 0.714 < 0.777 < 0.818 < 0.846$

(7) $\frac{3}{7} = \frac{x}{42} \Rightarrow 7x = 3 \times 42^{6} \Rightarrow \boxed{x = 18}$

(8) $|x+y| = \left|-\frac{1}{3} + \frac{2}{7}\right| = \left|\frac{-7+6}{21}\right| = \left|\frac{-1}{21}\right| = \frac{1}{21}$

[$\because |x|$ is always positive]

(9) $1 + \frac{2}{1+\frac{3}{1+4}} = 1 + \frac{2}{1+\frac{3}{5}} = 1 + \frac{2}{\frac{8}{5}} = 1 + 2 \cdot \frac{5}{8} = 1 + \frac{5}{4}$

$= \frac{9}{4} = 2.25$

(10) $15\frac{2}{3} \times 3\frac{1}{6} + 6.\frac{1}{3} = 11\frac{7}{18} + x$

$\frac{47}{3} \cdot \frac{19}{6} + \frac{19}{3} = \frac{205}{18} + x$

$\frac{893}{18} + \frac{19}{3} = \frac{205}{18} + x$

$x = \frac{893}{18} + \frac{19}{3} - \frac{205}{18}$

$x = \frac{26 + 9 + 342 - 615}{54}$

401

$x = \frac{2406}{5449} = \frac{401}{9}$

LEVEL-II Rational Numbers

(1) Let $x = 0.353535\ldots$ — (1)

Multiply with 100 on both sides

$100x = 35.3535\ldots$ — (2)

(2) − (1)

$100x = 35.3535\ldots$

$x = 0.3535\ldots$

$99x = 35.000$

$$x = \frac{35}{99}$$

(2) Let $x = 0.358358358\ldots$ — (1)

$1000x = 358.358358\ldots$ — (2)

(2) − (1)

$1000x = 358.358358\ldots$

$x = 0.358358\ldots$

$999x = 358$

$$\Rightarrow x = \frac{358}{999}$$

(3) Let $x = 1.3\bar{4} = 1.343434\ldots$ — (1)

(1) × 100

$100x = 134.343434\ldots$ — (2)

(2) − (1) [subtracting (1) from (2)]

$99x = 133 \Rightarrow x = \frac{99}{133}$

let $y = 4.12 = 4.12121212\ldots$ — ③

multiply the above with 100

$100y = 412.12121212\ldots$ — ④

④ − ③ [subtract ③ from ④]

$99y = 408$

$$\boxed{y = \frac{408}{99}}$$

$$x + y = \frac{133}{99} + \frac{408}{99} = \frac{541}{99}.$$

$$\boxed{\therefore 1.\overline{34} + 4.\overline{12} = \frac{541}{99}.}$$

(ii) 1 let $x = 0.\overline{356}$

$x = 0.356356356\ldots$ — ①

multiply by 1000 the above equation.

$\Rightarrow 1000x = 356.356356\ldots$ — ②

② − ① [Subtract ① from ②]

$999x = 356$

$$\boxed{x = \frac{356}{999}}$$

(5) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ = - - -

rationalise the denominator.

$= \frac{\sqrt{2}-1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1}$ [$\because \sqrt{2}-1$ is a rationalising factor]

$= \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2}$ [$\because (a-b)(a+b) = a^2-b^2$]

$= \frac{2+1-2\sqrt{2}}{2-1} = \frac{3-2\sqrt{2}}{1}$

$= 3-2\sqrt{2}$

(6) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Rationlising Denominator.

$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \cdot \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{5+3-2\sqrt{15}}{5-3}$

$= \frac{8-2\sqrt{15}}{2} = 4-\sqrt{15}$

If $\frac{x}{y} = \frac{3}{5}$

$$\frac{x-y}{x+y} = \frac{y\left(\frac{x}{y}-1\right)}{y\left(\frac{x}{y}+1\right)} \quad [\because y \text{ take common}]$$

$$= \frac{\frac{3}{5}-1}{\frac{3}{5}+1}$$

$$= \frac{3-5}{3+5}$$

$$= \frac{-2}{8} = \frac{-1}{4}$$

(8) Let the required number be x.

$$\frac{27}{13} - x = \frac{-3}{7}$$

$$\Rightarrow x = \frac{27}{13} + \frac{3}{7}$$

$$= \frac{189+39}{91}$$

$$= \frac{228}{91}$$

(9) $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{120}\right)$

$= \frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdots\frac{121}{120}$

$= \frac{1}{2}\cdot\frac{121}{1} = \frac{121}{2}$

(10) $\sqrt{0.04 \times 0.4 \times x} = 0.4 \times 0.004 \times \sqrt{y}$

Squaring on both sides.

$0.04 \times 0.4 \times x = (0.4)^2 \cdot (0.004)^2 \cdot y$

$0.04 \times 0.4 \times x = (0.4)^2 \cdot \left(\frac{0.04}{10}\right)^2 \cdot y$

$0.04 \times 0.4 \times x = (0.4)^2 \cdot (0.04)^2 \cdot \frac{1}{10^2} \cdot y$

$\Rightarrow \frac{x}{y} = 0.4 \times 0.04 \cdot \frac{1}{10^2}$

$\Rightarrow \frac{x}{y} = \frac{4}{10} \times \frac{4}{100} \cdot \frac{1}{10^2}$

$$\boxed{\frac{x}{y} = 16 \times 10^{-5}}$$

Rational Numbers: MCQs

① $\frac{1}{4}$ and $\frac{1}{3}$

0.25 and 0.33

You can take any number between the above two numbers

0.25 0.29 0.33

Ans: B.

② As per definition

a non terminating and non repeating Decimal.

Ans: B

③ Ans: C

$\frac{22}{7}$ is a rational number.

④ Here irrational number is multiplying by 2

so it is also irrational number.

Ans: B

⑤ Ans: A

It is irrational number.

⑥ The product of two irrational numbers

e.g: $\sqrt{2}$ is an irrational

$\sqrt{2} \times \sqrt{2} = 2$

so. either rational numbers or irrational number.

Ans: C.

⑦ $\pi = 3.142857142857\ldots$

It's an irrational number.

Ans: B

⑧ Irrational numbers are ~~Added~~ closed under addition.

ex: $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$2\sqrt{2}$ is the irrational number.

Ans: A

⑨ multiplicative inverse of '0' does not exist.

Ans: C

⑩ Ans: D

11. Any rational number with its denominator is in the form of $2^m \times 5^n$ where m, n are positive integers and are terminating decimals.

Ans: A

12. ∵ '0' does not have multiplicative inverse.

So

Ans: A

13. Step-4: which is violating axioms.

Ans: A

14. $\sqrt{3} = 1.732$

Ans: C

15. Already this question in level 2 is there

$1.\overline{34} + 4.\overline{12} = \frac{541}{99}$

Ans: D

16 Let $x = \sqrt{5\sqrt{5\sqrt{5\sqrt{5}\ldots}}}$

squaring on both sides

$x^2 = 5\sqrt{5\sqrt{5\sqrt{5}\ldots}}$

$x^2 = 5x$.

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ & $x = 5$

$\therefore x = 5 \quad (\because x \neq 0)$

Ans: B

17. I. $\sqrt[3]{3.728} = 1.8884$ [connection $\left(\frac{1}{2}\right)^2$

II. $\frac{\sqrt{3}-1}{\sqrt{3}+1} < 1$

III. $\left(\frac{1}{2}\right)^{+2} = \frac{1}{4} = 0.25$ [$\left(\frac{1}{2}\right)^2$ should be taken]

(iv) $= \frac{17}{8} = 2.125$

Ans: B

18. Additive inverse of $-\frac{a}{b}$ is $-\left(-\frac{a}{b}\right) = \frac{a}{b}$

Ans: A.

19. $\frac{1}{1\times4} + \frac{1}{4\times7} + \frac{1}{7\times10} + \frac{1}{10\times13} + \frac{1}{13\times16}$

$= \frac{1}{3}\left[\left(1-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{7}\right) + \left(\frac{1}{7}-\frac{1}{10}\right) + \left(\frac{1}{10}-\frac{1}{13}\right) + \left(\frac{1}{13}-\frac{1}{16}\right)\right]$

$= \frac{1}{3}\left(1-\frac{1}{16}\right) = \frac{1}{3}\cdot\frac{15}{16} = \frac{5}{16}$

$= \frac{5}{16}$

Ans: B

20. $\frac{a+b}{a} = \frac{b}{a+b}$

$\Rightarrow a^2 + b^2 + 2ab = ab$

$\Rightarrow a^2 + ab + b^2 = 0$

$\therefore a = \frac{-b \pm \sqrt{b^2-4b^2}}{2} = \frac{-b \pm ib\sqrt{3}}{2}$

If 'b' is real, a is not real

If b is not real a may be real.

Ans: D

21. Ans: A (conceptual)

22. $\frac{1+\sqrt{2}}{1-\sqrt{2}} + \frac{1-\sqrt{2}}{1+\sqrt{2}}$

Applying concept of rationalisation.

$$\frac{1+\sqrt{2}}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} + \frac{1-\sqrt{2}}{1+\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1-\sqrt{2}}$$

$$= \frac{3+2\sqrt{2}}{-1} + \frac{3-2\sqrt{2}}{-1}$$

$$= -3-2\sqrt{2}-3+2\sqrt{2}$$

$$= -6$$

Ans: A

23. $2.\overline{47} + 3.\overline{53} + 0.\overline{05}$

$$= \frac{245}{99} + \frac{351}{99} + \frac{5}{99}$$

$$= \frac{601}{99} = 6.\overline{06}$$

Ans: D

24. B : additive inverse. [∵ all are positive real numbers]

25. $I = \frac{1}{1+\sqrt{3}} + \frac{1}{1-\sqrt{3}}$

$= \frac{1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} + \frac{1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \frac{1-\sqrt{3}}{-2} + \frac{1+\sqrt{3}}{-2}$

$= -\frac{1}{2}[1-\sqrt{3}+1+\sqrt{3}]$

$= -1$

Ans: A

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