CHAPTER-2

Polynomial

Revision Exercise :-

1. Sol:- Given $(25)^3+(-17)^3+(-8)^3$

We know the ~~cd~~ corollary : If $a+b+c=0$ then $a^3+b^3+c^3=3abc$

Then taking $a=25$, $b=-17$, $c=-8$.

$a+b+c = 25-17-8 = 25-25 = 0$.

Then $(a)^3+b^3+c^3 = 3abc$

$(25)^3+(-17)^3+(-8)^3 = 3\times25\times(-17)\times(-8) = 3\times3,400$

$= \cancel{7157\times1336}\ 10,200$ //

2. Sol:- Given, $y^3+ky+2k-2$ is exactly divisible by $(y+1)$

let $y+1=0 \Rightarrow y=-1$

Then $y^3+ky+2k-2 = 0$

$(-1)^3+k(-1)+2k-2 = 0$

$-1-k+2k-2 = 0$

$k-3=0$

$k=3$ //

3. sol: i) $\left[\frac{3}{2}x+1\right]^3$

$\Rightarrow (a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$\Rightarrow \left(\frac{3}{2}x\right)^3+3\left(\frac{3}{2}x\right)^2(1)+3\left(\frac{3}{2}x\right)(1)+(1)^3$

$\Rightarrow \cancel{9}\ \frac{27}{8}x^3+3\left(\frac{9}{4}x^2\right)(1)+\frac{9}{2}x+1$

$\Rightarrow \frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$

ii) Given, $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz.$

$\Rightarrow (\sqrt{2}x)^2+(y)^2+(-2\sqrt{2}z)^2+2\times(-\sqrt{2}x)\times y+2\times(\sqrt{2}z)y+2\times(\sqrt{2}x)\times\sqrt{2}z$

$\Rightarrow (-\sqrt{2}x+y+2\sqrt{2}z)^2$ $\quad [\because a^2+b^2+c^2+2ab+2bc+2ca = (a+b+c)^2]$

iii) Given, $27-125a^3-135a+225a^2$ $\quad [\because (a+b)^3 \Rightarrow a^3+b^3-3a^2b+3ab^2$

$\Rightarrow (3)^3+(5a)^3-3\times5a\times9+3\times3\times25a^2$

$\Rightarrow (3)^3+(-5a)^3-3\times3^2\times5a+3\times3\times(-5a)^2$

$\Rightarrow (3-5a)^3$

iv) Given $6\sqrt{3}a^2-47a+5\sqrt{3}$

$\Rightarrow 6\sqrt{3}a^2-2a-45a+5\sqrt{3}$

$\Rightarrow 2a(3\sqrt{3}a-1)-5\sqrt{3}(3\sqrt{3}-1)$

$\Rightarrow (3\sqrt{3}a-1)(2a-5\sqrt{3})$

v, Sol: Given $(2a+1)^2 - 9b^4$

$\Rightarrow (2a+1)^2 - (3b^2)^2$ [

$a^2 - b^2 = (a+b)(a-b)$

$\Rightarrow (2a+1+3b)(2a+1-3b)$ //

vi, $24\sqrt{3}x^3 - 125y^3$

$\Rightarrow (2\sqrt{3}x)^3 - (5y)^3$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= (2\sqrt{3}x - 5y)((2\sqrt{3}x)^2 + 2\sqrt{3}x(5y) + (5y)^2]$

$\Rightarrow (2\sqrt{3}x - 5y)(12x^2 + 10\sqrt{3}xy + 25y^2]$ //

vii, Sol: Given, $x(x-y)^3 + 3x^2y(x-y)$

$\Rightarrow x(x-y)[(x-y)^2 + 3xy]$

$\Rightarrow x(x-y)[x^2+y^2-2xy+3xy]$

$\Rightarrow x(x-1)[x^2+y^2+xy]$ //

viii, Sol:- Given $\left(a-\frac{2}{a}\right)\left(a+\frac{2}{a}\right)\left(a^2+\frac{2}{a^2}\right)\left(a^4+\frac{2}{a^4}\right]$

$\Rightarrow \left(a^2-\frac{1}{a^2}\right]\left(a^2+\frac{2}{a^2}\right]\left[a^4+\frac{2}{a^4}\right]$ $[\because (a+b)(a-b) \Rightarrow a^2-b^2]$

$\Rightarrow \left[(a^2)^2 - \left(\frac{1}{a^2}\right)^2\right]\left[a^4+\frac{2}{a^4}\right]$

$\Rightarrow \left[a^4 - \frac{2}{a^4}\right]\left[a^4+\frac{2}{a^4}\right]$

$\Rightarrow (a^4)^2 - \left(\frac{1}{a^4}\right)^2$

$\Rightarrow \left(a^8 - \frac{1}{a^8}\right)$

iv, Soli- Given $7x^2 + 2\sqrt{14}x + 2$

$\Rightarrow (\sqrt{7}x)^2 + 2\sqrt{7}x\sqrt{2} + 2$

$\Rightarrow (\sqrt{7}x)^2 + 2 \times \sqrt{7}x \times \sqrt{2} + (\sqrt{2})^2$

$(a+b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (\sqrt{7}x + \sqrt{2})^2$

x, Soli- Given. $x^3 + 13x^2 + 32x + 20$

let $x = -1$, Then

$$\begin{array}{r|rrrr} x=-1 & 1 & 13 & 32 & 20 \\ & 0 & -1 & -12 & -20 \\ \hline & 1 & 12 & 20 & 0 \end{array}$$

$\Rightarrow (x+1)(x^2 + 12x + 20)$

$\Rightarrow (x+1)(x^2 + 2x + 10x + 20)$

$\Rightarrow (x+1)(x(x+2) + 10(x+2))$

$\Rightarrow (x+1)((x+2)(x+10))$

$\Rightarrow (x+1)(x+2)(x+10)$

xi, Soli- Given $x^3 + 13x^2 - 22x - 8$, $x = -15$

$\Rightarrow (-15)^3 + 13(-15)^2 - 22(-15) - 8$

$\Rightarrow -3,375 + 13(225) + 330 - 8$

$\Rightarrow -3,375 + 2925 + 330 - 8$

$\Rightarrow -3383+3255$

$\Rightarrow -128$ //

4 Sol:- i, Given $x+\frac{2}{x}=7$,

$x^3+\frac{2}{x^3}=?$

$(a^3+b^3)=(a+b)(a^2+$ $(a+b)^3-3ab(a+b)$

$\left(x^3+\frac{2}{x^3}\right)=\left(x+\frac{2}{x}\right)^3-3\times \cancel{x}\times\frac{2}{\cancel{x}}\left(x+\frac{2}{x}\right)$

$=(7)^3-3(7)$

$\Rightarrow 343-21$

$\Rightarrow 322$

ii, Sol:- Given $x^2+\frac{1}{x^2}=14$.

$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times \cancel{x}\times\frac{1}{\cancel{x}}$

$\Rightarrow 14+2$

$=16$

$\left(x+\frac{1}{x}\right)=\sqrt{16}=4$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\times \cancel{x}\times\frac{1}{\cancel{x}}\left(x+\frac{1}{x}\right)$ $[\because (a^3+b^3)=(a+b)^3-3ab(a+b)]$

$=(4)^3-3(4)$

$=64-12$

$=52$ //

5 Sol: i, Given $3x^5-2x^4+x^2-2$

We divide $3x^5-2x^4+x^2-2$ by x^2+x+1 as shown in below.

$$x^2+x+1\,)\ \cancel{3x^5}-2x^4+x^2-2\ (\,3x^3-5x^2+2x+4$$

$$\underline{\cancel{3x^5}+3x^4+3x^3}$$
$$\underline{-\quad\ -\quad\ -}$$

$$-\cancel{5x^4}-3x^3+x^2-2$$

$$-\cancel{5x^4}-5x^3-5x^2$$
$$\underline{+\quad\ +\quad\ +}$$

$$\cancel{2x^3}+6x^2-2$$

$$\cancel{2x^3}+2x^2+2x$$
$$\underline{-\quad\ -\quad\ -}$$

$$4x^2-2x-2$$

$$4x^2+4x+4$$
$$\underline{-\quad\ -\quad\ -}$$

$$-6x-6$$

From the division, we observe that the quotient is $3x^3-5x^2+2x+4$

remainder is $-6x-6$

Hence, $-6x-6$ should be subtracted from $3x^5-2x^4-x^2-2$

So the result is exactly divisible by x^2+x+1

(ii) Sol:- Given. $x^4+2x^3-2x^2-2x-1$

We divide $x^4+2x^3-2x^2-2x-1$ by x^2+2x-3 as shown in below

$$
\begin{array}{r}
x^2+2x-3\,\big)\; x^4+2x^3-2x^2-2x-1\;(x^2+1 \\
\underline{\cancel{x^4}+\cancel{2x^3}-3x^2} \\
x^2-2x-1 \\
\underline{\cancel{x^2}+2x-3} \\
-4x+2 \\
(or)\; 4x-2
\end{array}
$$

$\therefore$ From the division we observe that the quotient x^2+1

remainder is $4x-2$.

Hence, $4x-2$ is should be added from $x^4+2x^3-2x^2-2x-1$

So the result is exactly divisible by x^2+2x-3

6. Sol:- (i) Given $x+y+z=1$, $xy+yz+zx=-1$, $xyz=-1$

$$(x^3+y^3+z^3) = (x+y+z)^3+3(xy+yz+zx)-3xyz$$

$$= (1)^3+3(-1)-3(-1)$$

$$\Rightarrow 1-\cancel{3}+\cancel{3} = 1$$

(ii) Sol: Given $a+b+c=9$, $ab+bc+ca=26$.

$$a^2+b^2+c^2 = (a+b+c)^2-2ab-2bc-2ca$$
$$= (a+b+c)^2-2(ab+bc+ca)$$
$$= (9)^2-2(26)$$
$$= 81-52$$
$$= 29$$

7 Sol: Given $4a^2+4a+3$

$$\Rightarrow 4a^2-2a+6a+3$$
$$\Rightarrow 2a(2a-1)+3(2a-1)$$
$$\Rightarrow (2a-1)(2a+3)$$

$\therefore$ The length and breath of rectangle are $(2a-1)$ and $(2a+3)$

8 Sol: (a) Given $p(x)=x^9-5x^4+1$, $q(x)=x-1$.

By using remainder theorem.

$$q(x)=0$$
$$x-1=0$$
$$x=1$$

Substitute in $p(x)$

$$p(x)=x^9-5x^4+1$$
$$p(1)=(1)^9-5(1)+1$$
$$\Rightarrow 1-5+1 = 2-5 = -3$$

ii) Sol:- Given, $p(x) = 4x^3 - 12x^2 + 11x - 5$, $g(x) = \cancel{4}\, x - \frac{1}{2}$

In remainder theorem.

$$g(x) = 0$$
$$x - \frac{1}{2} = 0$$
$$x = 1/2$$

$$p(x) = 4x^3 - 12x^2 + 11x - 5$$
$$p(\tfrac{1}{2}) = 4(\tfrac{1}{2})^3 - 12(\tfrac{1}{2})^2 + 11(\tfrac{1}{2}) - 5$$
$$\Rightarrow \cancel{4}(\tfrac{1}{\cancel{8}_2}) - \cancel{12}^{3}(\tfrac{1}{\cancel{4}}) + 11(\tfrac{1}{2}) - 5$$

10
11
6
27

$$\Rightarrow \frac{1}{2} - 3 + \frac{11}{2} - 5$$
$$\Rightarrow \frac{1 - 6 + 11 - 10}{2} \Rightarrow$$
$$= \frac{12 - 16}{2} \Rightarrow \frac{-4}{2} = -2$$

iii) Sol:- Given $p(x) = x^3 - 6x^2 - 2x - 4$, $g(x) = 1 - 3x$

In remainder theorem.

$$g(x) = 0$$
$$1 - 3x = 0$$
$$1 = 3x \Rightarrow x = 1/3$$

$$p(x) = x^3 - 6x^2 - 2x - 4$$
$$p(1/3) = (\tfrac{1}{3})^3 - 6(\tfrac{1}{3})^2 - 2(\tfrac{1}{3}) - 4$$
$$= \frac{1}{27} - \frac{6}{9} - \frac{2}{3} - 4$$
$$\Rightarrow \frac{1}{27} - \frac{2}{3} - \frac{2}{3} - 4$$
$$\Rightarrow \frac{1}{27} - \frac{4}{3} - 4$$

$\Rightarrow \frac{1-36-108}{27}$

$\Rightarrow \frac{-144+1}{27} = \frac{-143}{27}$

d, Sol:- Given $p(x) = x^3+3x^2+3x+1$, $g(x) = [x-\frac{1}{2}]$

In remainder theorm

$g(x) = 0$

$x - \frac{1}{2} = 0$

$x = \frac{1}{2}$

$p(x) = x^3+3x^2+3x+1$

$p(\frac{1}{2}) = (\frac{1}{2})^3+3(\frac{1}{2})^2+3(\frac{1}{2})+1$

$= \frac{1}{8}+3(\frac{1}{4})+\frac{3}{2}+1$

$\Rightarrow \frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$

$\Rightarrow \frac{1+6+12+8}{8}$

$\Rightarrow \frac{27}{8}$ //

9 Sol:- a, Given $\sqrt{2a^2+2\sqrt{6}ab+3b^2}$

$\Rightarrow 2a^2+2\sqrt{6}\,ab+3b^2$

$\Rightarrow 2a^2+\sqrt{6}ab+\sqrt{6}ab+3b^2$

$\Rightarrow \sqrt{2}a(\sqrt{2}a+\sqrt{3}b)+\sqrt{3}b(\sqrt{2}a+\sqrt{3}b)$

$(\cancel{2}\sqrt{a}+\sqrt{3}b)(\cancel{\sqrt{2}}\sqrt{2}\sqrt{a}+\sqrt{3}b)$

$\Rightarrow (\sqrt{2}a+\sqrt{3}b)^2$

$\sqrt{2a^2+2\sqrt{6}ab+3b^2} \Rightarrow \sqrt{(2a+\sqrt{3}b)^2}$

$\Rightarrow \sqrt{2}a+\sqrt{3}b$ //

b) Sol:- Given, $3(x+2y)^2+5(x+2y)+2$

let $x = x+2y$.

Then

$\Rightarrow 3x^2+5x+2$

$\Rightarrow 3x^2+3x+2x+2$

$\Rightarrow 3x(x+1)+2(x+1)$

$\Rightarrow (x+1)(3x+2)$

Substitute 'x' Value.

$\Rightarrow (x+2y+1)(3(x+2y)+1)$

$\Rightarrow (x+2y+1)(3x+6y+1)$ //

c) Sol:- Given, $125(x-y)^3+(5y-3z)^3+(3z-5x)^3$

$\Rightarrow [5(x-y)]^3+(5y-3z)^3+(3z-5x)^3$

In If $a+b+c=0$ Then $a^3+b^3+c^3=3abc$

a+b+c = z Taking $a = 5(x-y) = 5x-5y$

$b = 5y-3z$

$c = 3z-5x$

Then $a+b+c = 5x - 5y + 5y - 3z + 3z - 5x$

$= 0.$

$\therefore a^3+b^3+c^3 = 3abc$

$= 3(5(x-y)][5y-3z)(3z-5x)$

$= 15(x-y)(5y-3z)(3z-5x)$ //

d, Sol:- Given, $x^3 - 3x^2 - 9x - 5.$

Then let $(x+1)$ is a factor of x^3-3x^2-9x-5

Then

$$x=-1 \;\begin{array}{|cccc} 1 & -3 & -9 & -5 \\ 0 & -1 & 4 & 5 \\ \hline 1 & -4 & -5 & \underline{0} \end{array}$$

$\Rightarrow (x+1)(x^2-4x-5)$

$\Rightarrow (x+1)(x^2+x+5x-5)$

$\Rightarrow (x+1)(x(x+1)-5(x+1)$

$\Rightarrow (x+1)(x+1)(x-5)$

$\Rightarrow (x+1)^2(x-5)$ //

10 a Sol: Given $(104)^2$

$\Rightarrow 104 \times 104$

$\Rightarrow 10816$

Level - II

1 Sol:- Given $(y-2)$ and $(y-\frac{1}{2})$ are factors of my^2+5y+n.

(i) $y-2=0 \Rightarrow y=2$

Then $my^2+5y+n=0$

$m(2)^2+5(2)+n=0$

$4m+10+n=0$

$4m+n=-10 \longrightarrow (1)$

(ii) $y-\frac{1}{2}=0 \Rightarrow y=\frac{1}{2}$

Then $my^2+5y+n=0$

$m(\frac{1}{2})^2+5(\frac{1}{2})+n=0$

$\frac{m}{4}+\frac{5}{2}+n=0$

$m+10+4n=0$

$m+4n=-10 \longrightarrow (2)$

$(1)\times 2-(2) \Rightarrow$

$$\begin{array}{r} 16m+4n=-40 \\ \underset{-}{m}+\underset{-}{4n}=\underset{+}{-10} \\ \hline 15m=-30 \end{array}$$

$m=-\frac{30}{15}=-2$

Subtitute 'm' in eq's (2)

$m+4n=-10$

$-2+4n=-10$

$4n=-10+2=-8$

$4n = -8$

$n = -8/4 = -2$

$\therefore m = n$ //

2 Sol:- (i) Given $x+\sqrt{a}$ is factor of $2x^4 - 2a^2x^2 - 3x + 2a^3 - 2a^2 + 3$

$\Rightarrow 2x^4 - 2a^2x^2 - 3x + 3 + 2a^3 - 2a^2 + 3 = 0$

let $x - \sqrt{a} = 0$

$x = \sqrt{a}$

$\Rightarrow 2(\sqrt{a})^4 - 2a^2(\sqrt{a})^2 - 3\sqrt{a} + 3 + 2a^3 - 2a^2 = 0$

$2a^2 - 2a^2a - 3\sqrt{a} + 3 + 2a^3 - 2a^2 = 0$

$\cancel{2a^2} - \cancel{2a^3} - 3\sqrt{a} + 3 + \cancel{2a^3} - \cancel{2a^2} = 0$

$-3\sqrt{a} = -3$

$\sqrt{a} = \frac{+3}{+3} = 1$

$a = 1$ //

(ii) Sol: Given $(x+\sqrt{a})$ is factor of $5x^4 + 5\sqrt{a}x^3 + 2x^2 - 3a + 5$

let $x + \sqrt{a} = 0$

$x = -\sqrt{a}$

Then substitute in eq's

$5x^4 + 5\sqrt{a}x^3 + 2x^2 - 3a + 5 = 0$

$5(-\sqrt{a})^4 + 5\sqrt{a}(-\sqrt{a})^3 + 2(-\sqrt{a})^2 - 3a + 5 = 0$

$5a^2 + 5\sqrt{a}\,a\sqrt{a} + 2a - 3a + 5 = 0$

b) Given, $x + \frac{1}{x} = \sqrt{3}$,

Then $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \cdot \cancel{x} \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)$ $\left[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)\right]$

$= (\sqrt{3})^3 - 3(\sqrt{3}) \Rightarrow \sqrt{27} - 3\sqrt{3}$

$\Rightarrow 3\sqrt{3} - 3\sqrt{3}$

$\Rightarrow 0$ //

c) Sol:- Given $x - \sqrt{3}$ is a factor of the polynomial $ax^2 + bx - 3$

and $a + b = 2 - \sqrt{3}$.

Then let $x - \sqrt{3} = 0$

$x = \sqrt{3}$.

Then $ax^2 + bx - 3 = 0$

$a(\sqrt{3})^2 + b(\sqrt{3}) = 3$

$3a + \sqrt{3}b = 3 \rightarrow ①$

$a + b = 2 - \sqrt{3} \rightarrow ②$

① - ② × 3 $\Rightarrow$ $\cancel{3a} + \sqrt{3}b = 3$

$\cancel{3a} + 3b = 6 - 3\sqrt{3}$

$\sqrt{3}b - 3b = 3 - (6 - 3\sqrt{3})$

$\sqrt{3}b - \sqrt{3}\sqrt{3}b = 3 - 6 + 3\sqrt{3}$

$\sqrt{3}b(1 - \sqrt{3}) = -3 + 3\sqrt{3}$

$\sqrt{3}b(\cancel{1 - \sqrt{3}}) = -3(1 - \cancel{\sqrt{3}})$ ~~$-3(1+$~~

$\sqrt{3}b = -3$

$b = \frac{-3}{\sqrt{3}} = \frac{-\sqrt{3}\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$

$b = -\sqrt{3}.$

from ② $\Rightarrow$ $a + b = 2 - \sqrt{3}$

$a - \sqrt{3} = 2 - \sqrt{3}$

$a = 2 - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} = 2$

$\therefore a = 2, \; b = -\sqrt{3}$ //

d Soli– Given, polynomial $3x^4 - 4x^3 - 3x - 1$ by $x - 1$

Then,

$$
\begin{array}{l}
x - 1 \,)\, 3x^4 - 4x^3 - 3x - 1 \,(\, 3x^3 - x^2 - x - 4 \\
\quad\quad 3x^4 - 3x^3 \\
\quad\quad - \quad + \\
\hline
\quad\quad\quad - x^3 - 3x - 1 \\
\quad\quad\quad - x^3 + x^2 \\
\quad\quad\quad + \quad - \\
\hline
\quad\quad\quad\quad - x^2 - 3x - 1 \\
\quad\quad\quad\quad - x^2 + x \\
\quad\quad\quad\quad + \quad - \\
\hline
\quad\quad\quad\quad\quad - 4x - 1 \\
\quad\quad\quad\quad\quad - 4x + 4 \\
\quad\quad\quad\quad\quad + \quad - \\
\hline
\quad\quad\quad\quad\quad\quad -5
\end{array}
$$

//

$\therefore$ The Quotient $= 3x^3 - x^2 - x - 4$

Remainder $= -5$ //

$5a^2 - 5a^2 + 2a - 3a + 5 = 0$

$-a + 5 = 0$

$-a = -5$

$a = 5$

$a^2 = 25$ //

3 Sol:- Given $x = \frac{\sqrt{p+2q} + \sqrt{p-2q}}{\sqrt{p+2q} - \sqrt{p-2q}}$

$\Rightarrow x = \frac{\sqrt{p+2q} + \sqrt{p-2q}}{\sqrt{p+2q} - \sqrt{p-2q}} \times \frac{\sqrt{p+2q} + \sqrt{p-2q}}{\sqrt{p+2q} + \sqrt{p-2q}}$

$\Rightarrow x = \frac{(\sqrt{p+2q} + \sqrt{p-2q})^2}{(\sqrt{p+2q})^2 - (\sqrt{p-2q})^2}$

$\Rightarrow x = \frac{(\sqrt{p+2q})^2 + (\sqrt{p-2q})^2 + 2\sqrt{(p+2q)(p-2q)}}{p+2q-(p-2q)}$

$\Rightarrow x = \frac{p+\cancel{2q} + p - \cancel{2q} + 2\sqrt{p^2-(2q)^2}}{\cancel{p}+2q-\cancel{p}+2q}$

$\Rightarrow x = \frac{2p + 2\sqrt{p^2-4q^2}}{4q}$

$\Rightarrow x = \frac{\cancel{2}(p+\sqrt{p^2-4q^2})}{\cancel{4}q}$

$\Rightarrow x = \frac{p+\sqrt{p^2-4q}}{2q}$

$\Rightarrow 2qx = p + \sqrt{p^2 - 4q^2}$

$2qx - p = \sqrt{p^2 - 4q^2}$

Squaring on Both sides

$(2qx - p)^2 = (\sqrt{p^2 - 4q^2})^2$

$4q^2x^2 + p^2 - 2 \times 2qx \times p = p^2 - 4q^2$

$4q^2x^2 - 4pqx = -4q^2$

$\cancel{4}(q^2x^2 - pqx) = -\cancel{4}q^2$

$q^2x^2 - pqx = -q^2$

$\cancel{q}(qx^2 - px) = -\cancel{q}q$

$qx^2 - px = -q$

$\therefore qx^2 - px + q = 0$

4 Sol: a, Given $a+b+c = 9$, $a^2+b^2+c^2 = 35$.

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow 2ab+2bc+2ca = (a+b+c)^2 - (a^2+b^2+c^2)$

$2(ab+bc+ca) = (9)^2 - 35$

$2(ab+bc+ca) = 81 - 35 = 46$

$ab+bc+ca = 46/2 = 23$

$(a+b+c)^3 - 3abc = (a^2+b^2+c^2 - ab - bc - ca)(a+b+c)$

$(a+b+c)^3 - 3abc = [(a^2+b^2+c^2) - (ab+bc+ca)](a+b+c)$

$= [35-23] \times 9)$

$= 12 \times 9$

$= 108$

ii. Sol:- Given $p+q+r = 1$, $pq+qr+pr = -1$, and $pqr = -1$

$p^3+q^3+r^3 = (p+q+r)(p^2+q^2+r^2 - pq - qr - pr) + \cancel{3abc}\ 3pqr$

$= (p+q+r)((p+q+r)^2 - 2pq - 2qr - 2pr - pq - qr - pr)$
$+ \cancel{3abc}\ 3pqr$

$= (p+q+r)[(p+q+r)^2 - 3(pq+qr+pr)] + 3pqr$

$\Rightarrow (1)[(1)^2 - 3(-1)] + 3(-1)$

$\Rightarrow 1[1+3] - 3$

$= 4 - 3 = 1$

5 Sol:- Given $2x^3 - 9x^2 + 15x + p$ when divided by $x-2$ leaves $(-p)$ as remainder

$\Rightarrow$ let $x-2 = 0$

$x = 2$

Then

$2x^3 - 9x^2 + 15x + p = -p$

$2(2)^3 - 9(2)^2 + 15(2) + p = -p$

$2(8)-9(4)+15(2)+p=-p$

$16-36+30+p+p=0$

$10+2p=0$

$2p=-10$

$p=-5$ //

6. Sol: i) Given polynomial $x^3-3x^2+6x-18$, factor $x-3$

let $x-3=0$

$x=3$

Then

$p(x)=x^3-3x^2+6x-18=0$

$(3)^3-3(3)^2+6(3)-18=0$

$27-3(9)+18-18=0$

$p(3)=27-27+18-18=0$

$p(3)\Rightarrow 0$

$\therefore$ $(x-3)$ is factor of $x^3-3x^2+6x-18$

ii) Sol: Given $x+5$ is factor of $3x^3-x^2+6x+200$

let $x+5=0$

$x=-5$

Then

$p(x)=3x^3-x^2+6x+200$

$p(-5)=3(-5)^3-(-5)^2+6(-5)+200$

$=3(-125)-25-30+200$

$=-375-55+200=-43$

$\Rightarrow -430+200$

$p(-15) = -230$

$\therefore$ $(x+15)$ is not a factor of $3x^3 - x^2 + 6x + 200$

iii, Sol: Given, polynomial $x^3 - 5x^2 + 3x + 1$

Let $x - 1 = 0$

$x = 1$

Then

$p(x) = x^3 - 5x^2 + 3x + 1$

$p(1) = (1)^3 - 5(1)^2 + 3(1) + 1$

$= 1 - 5 + 3 + 1$

$= 5 - 5 = 0$

$\therefore$ $(x-1)$ is a factor of polynomial $x^3 - 5x^2 + 3x + 1$

iv, Sol:- Given, $x+b$ is factor of $x^3 - b^2x + x + 2$

Let $x + b = 0$

$x = -b$

Then

$p(x) = x^3 - b^2x + x + 2 = 0$

$\Rightarrow (-b)^3 - b^2(-b) + (-b) + 2 = 0$

$-b^3 + b^3 - b + 2 = 0$

$-b + 2 = 0$

$+b = +2$

$b = 2$ //

vi, Soln Given, $2x^3+3x^2+7x-2p$ is divisible by $x+1$.

Let $x+1=0$

$x=-1$

Then $2x^3+3x^2+7x-2p=0$

$2(-1)^3+3(-1)^2+7(-1)-2p=0$

$2(-1)+3(1)+7-2p=0$

$-2+3+7-2p=0$

$-\cancel{8}-2p=0$

$-2p=-\cancel{8}\ 6$

$p=+\cancel{6}/-2=\cancel{4}\ 6/-2=-3$

$p=-3$ //

vii, Given, $x-2$ is factor of x^3-2bx^2+bx-1.

Let $x-2=0$

$x=2$

Then $x^3-2bx^2+bx-1=0$

$(2)^3-2b(2)^2+b(2)+1=0$

$8-2b(4)+2b+1=0$

$8-8b+2b+1=0$

$-6b+7=0$

$-6b=-7$

$b=\cancel{-}7/\cancel{-}6=7/6$ //

7,(i) Given, px^4-3x^2+20 and $4x^2+7x-p$ when divided by $x-2$ leaves the same remainder.

$f(x) = px^4-3x^2+20$

$g(x) = 4x^2+7x-p$

Let $x-2=0$

$x=2$

Then

$f(2) = p(2)^4-3(2)^2+20 = 16p-3(4)+20$

$= 16p-24+20 = 16p-4$

$g(2) = 4(2)^2+7(2)-p$

$= 4(4)+14-p = 16+14-p$

$= 30-p$

$\therefore f(x) = g(x)$

$16p-4 = 30-p$

$16p+p = 30+4$

$17p = 34$

$p = 34/17 = 2$

(ii) Sol:- Given $f(x)$ is divided by $(x-2)$ and $(x-5)$, the remainders are 17 and 11.

Using Division Algorithm:

Dividend = Divisor x Quotient + Remainder

Divided = Divisor × Quotient + Remainder.

let $g(x)$, $k(x)$ be quotient when $f(x)$ is divided by $(x-2)$ and $(x-5)$

Then $f(x) = (x-2)\,g(x) + 17$ $\left[\because \text{Divisor} = x-2, \text{Quotient} = g(x), \text{Remainder} = 17\right]$

$f(2) = (2-2)\,g(2) + 17$

$f(2) = 0 \cdot g(2) + 17 = 17$.

$f(2) = 17 \longrightarrow (1)$

Also $f(x) = (x-5)\,k(x) + 11$

$f(5) = (5-5)\,k(5) + 11$

$= (0)(k(5)) + 11$

$f(5) = 11 \longrightarrow (2)$

Now, let $ax+b$ be remainder when $f(x)$ is divided by $(x-2)(x-5)$ and $g(x)$ be Quotient

$f(x) = (x-2)(x-5)\,g(x) + (ax+b)$.

using (1) & (2).

$f(2) = (2-2)(x-5)\,g(2) + (a(2)+b)$

$17 = (0)(x-5)\,g(2) + (2a+b)$

$17 = 2a+b \longrightarrow (3)$

$f(5) = (5-2)(5-5)\, g(5) + (a(5)+b)$

$11 = (3)(0)\, g(5) + (5a+b)$

$11 = 5a+b \rightarrow (4)$

Solving (3) and (4), subtract (3)-(4).

$$\begin{aligned} 2a+b &= 17 \\ 5a+b &= 11 \\ \hline -3a &= 6 \end{aligned}$$

$a = 6/-3 = -2$

Substitute a in eq (3)

$2a+b = 17$

$2(-2)+b = 17$

$-4+b = 17$

$b = 17+4 = 21.$

$\therefore$ The remainder $ax+b = -2x+21$ //

$\therefore$ $-2x+21$ is a remainder when $f(x)$ is divided by $(x-2)(x-5)$

iii) Given $f(x)$ is divided by $x-3$ and $x+6$

The remainder are 7 and 22.

Using Division Algorithm:

Divided = Divisor x Quotient + Remainder

Let $g(x)$, $k(x)$ be Quotient when $f(x)$ is divided by $x-3$ and $x+6$.

$f(x) = (x-3)\,g(x) + 7$

$f(3) = (3-3)\,g(3) + 7$

$f(3) = (0)\,g(3) + 7$

$f(3) = 7 \longrightarrow (1)$

$f(x) = (x+6)\,k(x) + 22$

$f(-6) = (-6+6)\,k(-6) + 22$

$f(-6) = (0)\,k(-6) + 22$

$f(-6) = 22 \longrightarrow (2)$.

Now, let $ax+b$ be remainder when $f(x)$ is divided by $(x-3)(x+6)$ and $g(x)$ be Quotient

$f(x) = (x-3)(x+6)\,g(x) + (ax+b)$

using (1) & (2).

$f(3) = (3-3)(3+6)\,g(3) + (a(3)+b)$

$f(3) = (0)(9)\,g(3) + (3a+b)$

$7 = \cancel{f(3)} = 3a+b \longrightarrow (3)$.

$f(-6) = (-6-3)(-6+6)\,g(-6) + (a(-6)+b)$

$22 = (-9)(0)\,g(-6) + (-6a+b)$

$22 = -6a + b \rightarrow$ (4)

Subtracting (3) - (4).

$\Rightarrow \quad 3a + \cancel{b} = 7$

$\quad\; -6a + \cancel{b} = 22$

$\quad\;\; + \quad\; - \quad\;\; -$

$9a = -15$

$a = -15/9 = -5/3$

Substitute a in eq (3)

$3a + b = 7$

$\cancel{3}\left(\frac{-5}{\cancel{3}}\right) + b = 7$

$-5 + b = 7$

$b = 7 + 5 = 12$

$\therefore$ a remainder $ax + b = \frac{-5}{3}x + 12$

$\therefore \frac{-5}{3}x + 12$ is remainder when $f(x)$ is divided by $(x-3)(x+6)$

8 Soli (i) Given $a^3(b-c) + b^3(c-a) + c^3(a-b)$

$\Rightarrow a^3(b-c) - b^3(a-c) - c^3(b-a)$

$\Rightarrow a^3(b-c) - b^3(a-c+b-b) - c^3(b-a+c-c)$

$\Rightarrow a^3(b-c) - b^3(b-c+a-b) - c^3(b-c+c-a)$

$\Rightarrow a^3(b-c) - b^3(b-c) + b^3(a-b) - c^3(b-c) + c^3(c-a)$

$\Rightarrow a^3(b-c) - b^3(b-c) + b^3(b-a) - c^3(b-c) - c^3(c-a)$

$\Rightarrow a^3(b-c) - b^3(b-c) + b^3(b-a+c-c) - c^3(b-c) - c^3(c-a)$

$\Rightarrow a^3(b-c) - b^3(b-c) + b^3(b-c+c-a) - c^3(b-c) - c^3(c-a)$

$\Rightarrow a^3(b-c) - b^3(\cancel{b-c}) + b^3(b-c) + b^3(c-a) - c^3(b-c) - c^3(c-a)$

$\Rightarrow (b-c)(a^3-c^3) + (c-a)(b^3-c^3)$

$\Rightarrow (b-c)(a^3-c^3) - (a-c)(b^3-c^3)$ $\quad [\because a^3-b^3 = (a-b)(a^2+ab+b^2)]$

$\Rightarrow (b-c)(a-c)[a^2+ac+c^2) - (a-c)(b-c)(b^2+bc+c^2)$

$\Rightarrow (b-c)(a-c)[a^2+ac+\cancel{c^2} - b^2 - bc - \cancel{c^2}]$

$\Rightarrow (b-c)(a-c)(a^2+ac-bc-b^2)$

$\Rightarrow (b-c)(a-c)[a^2-b^2+ac-bc]$

$\Rightarrow (b-c)(a-c)((a+b)(a-b)+c(a-b)]$

$\Rightarrow (b-c)(a-c)((a-b)(a+b+c)$

$\Rightarrow -(a-b)(b-c)(c-a)(a+b+c)$

(ii) Sol: Given $(a+b+c)^3 - (b+c-a)^3 + (c+a-b)^3 - (a+b-c)^3$

$\Rightarrow (a+b+c)^3 - (-a+b-c)^3 - (a-b+c)^3 - (a+b-c)^3$

$(a+b+c)^3 = a^3+b^3+c^3+3a^2b+3a^2c+3ab^2+3b^2c+3ac^2+3bc^2+6abc$

$(a+b+c)^3 = -a^3+b^3+c^3+3a^2b+3a^2c-3ab^2+3b^2c-3ac^2+3bc^2+6abc$

$(a-b+c)^3 = a^3 - b^3 + c^3 - 3a^2b + 3a^2c + 3ab^2 + 3b^2c + 3ac^2 - 3bc^2 - 6abc$

$(a+b-c)^3 = a^3 + b^3 - c^3 + 3a^2b - 3a^2c + 3ab^2 - 3b^2c + 3ac^2 + 3bc^2 - 6abc$

$\therefore (a+b+c)^3 - (-a+b+c)^3 - (a-b+c)^3 - (a+b-c)^3$

$\Rightarrow \cancel{a^3} + \cancel{b^3} + \cancel{c^3} + 3\cancel{a^2b} + 3\cancel{a^2c} + 3\cancel{ab^2} + 3\cancel{b^2c} + 3\cancel{ac^2} + 3\cancel{bc^2} + 6abc$

$+ \cancel{a^3} - \cancel{b^3} - \cancel{c^3} - 3\cancel{a^2b} - 3\cancel{a^2c} + 3\cancel{ab^2} - 3\cancel{b^2c} + 3\cancel{ac^2} - 3\cancel{bc^2} + 6abc$

$- \cancel{a^3} + \cancel{b^3} - \cancel{c^3} + 3\cancel{a^2b} + 3\cancel{a^2c} - 3\cancel{ab^2} - 3\cancel{b^2c} - 3\cancel{ac^2} + 3\cancel{bc^2} + 6abc$

$- \cancel{a^3} - \cancel{b^3} + \cancel{c^3} - 3\cancel{a^2b} + 3\cancel{a^2c} - 3\cancel{ab^2} + 3\cancel{b^2c} - 3\cancel{ac^2} + 3\cancel{bc^2} + 6abc$

$\Rightarrow 6abc + 6abc + 6abc + 6abc$

$\Rightarrow 24abc$ //

(iii) Given $a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc$

$\Rightarrow a(b^2+c^2-2bc) + b(c^2+a^2-2ac) + c(a^2+b^2-2ab) + 8abc$

$\Rightarrow ab^2 + ac^2 - 2abc + bc^2 + a^2b - 2abc + ca^2 + cb^2 - 2abc + 8abc$

$\Rightarrow ab^2 + ac^2 + bc^2 + ba^2 + ca^2 + cb^2 - 6abc + 8abc$

$\Rightarrow ab^2 + ac^2 + bc^2 + ba^2 + ca^2 + cb^2 + 2abc \quad \text{—} (1)$

$\Rightarrow ab^2 + ac^2 + bc^2 + ba^2 + ca^2 + cb^2$

$(a+b)(b+c)(c+a) = (ab + ac + b^2 + bc)(c+a)$

$= abc + a^2b + ac^2 + a^2c + b^2c + ab^2 + bc^2 + abc$

$\Rightarrow a^2b + ac^2 + b^2c + bc^2 + ac^2 + a^2c + 2abc \quad \text{—} (2)$

from (1) and (2)

$a(b-c)^2 + b(c-a)^2 + c(a-b) + 8abc$

$= (a+b)(b+c)(c+a)$ //

9 Sol:- Given $a^4+2a^3-2a^2+2a-3$.

$f(x) = a^4+2a^3-2a^2+2a-3$

$g(x) = a^2+2a-3$

$= a^2-a+3a-3$

$= a(a-1)+3(a-1)$

$g(x) = (a-1)(a+3)$

If $a-1=0 \Rightarrow a=1$

$f(1) = (1)^4+2(1)^3-2(1)^2+2(1)-3$

$= 1+2-\cancel{2}+\cancel{2}-3$

$= 3-3 = 0.$

If $a+3=0$

$a=-3$

$f(-3) = (-3)^4+2(-3)^3-2(-3)^2+2(-3)-3$

$= 81+2(-27)-2(9)-6-3$

$= 81-54-18-9$

$= 81-54-27$

$= 81-81 = 0$

$\therefore$ $(a-1)$ and $(a+3)$ are factors of $a^4+2a^3-2a^2+2a-3$

$\therefore$ a^2+2a-3 is exactly divisible by $a^4+2a^3-2a^2+2a-3$

10 Ans: Given, $(x+1)$ and $(x-1)$ are factors of mx^3+x^2-2x+n

If $x+1=0 \Rightarrow x=-1$

Then $mx^3+x^2-2x+n=0$

$m(-1)^3+(-1)^2-2(-1)+n=0$

$m(-1)+1+2+n=0$

$-m+n=-3 \longrightarrow (1)$

If $x-1=0 \Rightarrow x=1$

Then $mx^3+x^2-2x+n=0$

$m(1)^3+(1)^2-2(1)+n=0$

$m+1-2+n=0$

$m+n-1=0$

$m+n=1 \longrightarrow (2)$

(1)+(2).

$$\begin{array}{r} -\cancel{m}+n=-3 \\ \cancel{m}+n=1 \\ \hline 2n=-2 \\ n=-1 \end{array}$$

Substitute 'n' in (2)

$m+n=1$

$m-1=1$

$m=1+1=2$

$\therefore m=2, n=-1$

11 Sol: Given, $(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$

$(a^3+b^3+c^3) = (a+b+c)^3 - 3(ab+bc+ca)(a+b+c) + 3abc.$

$(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3 = (\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}) - 3((a^2-b^2)(b^2-c^2)$
$+(b^2-c^2)(c^2-a^2)+(c^2-a^2)(a^2-b^2))(a^2-b^2+b^2-c^2+c^2-a^2)$

$+3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

$\Rightarrow 0-3((a^2-b^2)(b^2-c^2)+(b^2-c^2)(c^2-a^2)+(c^2-a^2)(a^2-b^2))(0)$
$+3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

$\Rightarrow 0-3(0)+3(a+b)(a-b)(b+c)(b-c)(c+a)(c-a)$

$\Rightarrow 0-0+3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

$\Rightarrow 3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

$\therefore (a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3 = 3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$

12 Sol: Given $x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)((x-y)^2+(y-z)^2+(z-x)^2)$

$\Rightarrow \frac{1}{2}(x+y+z)[x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx]$

$\Rightarrow \frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$

$\Rightarrow \frac{1}{\cancel{2}}(x+y+z)\cancel{2}(x^2+y^2+z^2-xy-yz-zx)$

$\Rightarrow (x+y+z)((x+y+z)^2-2xy-2yz-2zx-xy-yz-zx)$

$\Rightarrow (x+y+z)\left[(x+y+z)^2 - 3(xy+yz+zx)\right]$

$x^3+y^3+z^3 \Rightarrow (x+y+z)\left[(x+y+z)^2 - 3(xy+yz+zx)\right] + 3xyz$

$\Rightarrow (x+y+z)^3 - 3(xy+yz+zx)(x+y+z) + 3xyz$

Then $\Rightarrow$

$(27)^3+(-14)^3+(-13)^3 \Rightarrow (27-14-13)^3 - 3(27\times(-14)+(-14)(-13)+(-13)(27))[27-14-13]$

$+3\times 27\times(-14)\times(-13)$

$\Rightarrow (0)^3 - 3(27\times(-14)+(-14)\times(-13)+(-13)(27))(0)$

$+3(27)(-14)(-13)$

$\Rightarrow 0 - 0 + 3\times 27\times(-14)\times(-13)$

$\Rightarrow 14742$ //

13. Sol: Given $(3x-1)^4 = a_4x^4+a_3x^3+a_2x^2+a_1x+a_0 \longrightarrow (1)$

In L.H.S. $(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

$(3x-1)^4 = (3x)^4 - 4(3x)^3(1) + 6(3x)^2(1)^2 - 4(3x)(1)^3 + (1)^4$

$\Rightarrow 81x^4 - 4(27)x^3 + 6(9x^2) - 12x + 1$

$\Rightarrow 81x^4 - 108x^3 + 54x^2 - 12 + 1 \longrightarrow (2)$

Comparing (1) & (2).

$a_4 = 81,\ a_3 = -108,\ a_2 = 54,\ a_1 = -12,\ a_0 = 1$

Then

$a_4 + 3a_1 + 9a_2 + 27a_1 + 81a_0 = 81 + 3(-108) + 9(54) + 27(-12) + 81(1)$

$\Rightarrow 81 - 324 + 486 - 324 + 81$

$\Rightarrow 648 - 648$

$\Rightarrow 0$ //

14 Sol: i, Given $p(x) = (x-2)^2 - (x+2)^2$

$(a^2 - b^2) = (a+b)(a-b)$

$p(x) = (x - \cancel{2} + x + \cancel{2})[x - 2 - (x+2)]$

$= (2x)(\cancel{x} - 2 - \cancel{x} - 2)$

$= (2x)(-4) = -8x$

Then $p(x) = 0$

$-8x = 0 \Rightarrow x = 0/-8 = 0$

$x = 0$

Hence $x = 0$ is the zero of the polynomial $p(x)$

ii, Sol: Given, $p-1$ is factor of $p^{10} - 1$

let $p - 1 = 0$

$p = 1$

Then $p^{10} - 1 = (1)^{10} - 1 = 1 - 1 = 0$

$p^{11} - 1 = (1)^{11} - 1 = 1 - 1 = 0$

$\therefore$ p-1 is a factor of $p^{10}-1$ and $p^{11}-1$

iii, Sol: Given, $x^5 - 2mx^2 + 16$ is divisible by $x+2$

let $x+2=0$

$x=-2$

Then $x^5 - 2mx^2 + 16 = 0$

$(-2)^5 - 2m(-2)^2 + 16 = 0$

$-32 - 2m(4) + 16 = 0$

$-8m - 16 = 0$

$-8m = 16 \Rightarrow m = \frac{16}{-8} = -2$

$\therefore m = -2$ //

iv, Sol: Given $x+2a$ is factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$

let $x+2a=0$

$x = -2a$

Then $x^5 - 4a^2x^3 + 2x + 2a + 3 = 0$

$(-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 = 0$

$-32a^5 - 4a^2(-8a^3) + 4a + 2a + 3 = 0$

$\cancel{-32a^5} + \cancel{32a^5} + 2a + 3 = 0$

$-2a = -3$

$a = \frac{-3}{-2} = \frac{+3}{2}$ // $\Rightarrow a = 3/2$ //

V. Sol:- Given $2x-1$ is factor of $8x^4+4x^3-16x^2+10x+m$

let $2x-1=0$

$\Rightarrow 2x=1 \Rightarrow x=1/2$

Then

$8x^4+4x^3-16x^2+10x+m=0$

$8(\frac{1}{2})^4+4(\frac{1}{2})^3-16(\frac{1}{2})^2+10(\frac{1}{2})+m=0$

$\cancel{8}(\frac{1}{\cancel{16}_2})+\cancel{4}(\frac{1}{\cancel{8}_2})\cancel{+}\cancel{16}^4(\frac{1}{\cancel{4}})+\cancel{10}^5(\frac{1}{\cancel{2}})+m=0$

$\frac{1}{2}+\frac{1}{2}+4+5+m=0$

$\frac{1+1+8+10+2m}{2}=0$

$\Rightarrow 12+18+2m=0$

$2m=-20 \Rightarrow m=+10$

$\Rightarrow 4+2m=0$

$\Rightarrow 2m=-4 \Rightarrow m=-2$ //

Vi, Sol:- Given $(x-2y)^3+(2y-3z)^3+(3z-x)^3$

We know If $a+b+c=0$.

Then $a^3+b^3+c^3=3abc$.

$\Rightarrow (x-2y+2y-3z+3z-x)$

$(a+b+c)=(\cancel{x}-\cancel{2y}+\cancel{2y}-\cancel{3z}+\cancel{3z}-\cancel{x})$

$=0$.

Then $(x-2y)^3+(2y-3z)^3+(3z-x)^3 = 3(x-2y)(2y-3z)(3z-x)$

15 Sol:- Given, az^3+4z^2+3z-4 and z^3-4z+a leave the same remainder when divided by $z-3$

$f(x) = az^3+4z^2+3z-4$

$g(x) = z^3-4z+a$

let $z-3=0$

$z=3$

Then $f(3) = a(3)^3+4(3)^2+3(3)-4$

$\Rightarrow 9a+4(9)+9-4 \Rightarrow a(27)+4(9)+9-4$

$\Rightarrow 9a+48+9-4 \Rightarrow 27a+36+9-4$

$\Rightarrow 9a+53 \Rightarrow 27a+32+9 \Rightarrow 27a+41$

$g(3) = (3)^3-4(3)+a$

$= 27-12+a$

$\Rightarrow 15+a$

$f(x) = g(x)$

$f(3) = g(3)$

$27a+41 = 15+a$

$27a-a = 15-41$

$26a = -26 \Rightarrow a = -1$ //

(ii) Sol: Given $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$

let $x+1=0 \Rightarrow x=-1$

$p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7 = 19$

$\Rightarrow 1 - 2(-1) + 3(1) + a + 3a - 7 = 19$

$1 + 2 + 3 + 4a - 7 = 19$

$6 + 4a - 7 = 19$

$4a - 1 = 19$

$4a = 19 + 1 = 20$

$a = 20/4 = 5.$

$\therefore p(x) = x^4 - 2x^3 + 3x^2 - 5x + 3(5) - 7$

$= x^4 - 2x^3 + 3x^2 - 5x + 15 - 7$

$p(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

Also $p(x)$ is divided by $x+2$

let $x+2=0 \Rightarrow x=-2$

Then $p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8$

$= 16 - 2(-8) + 3(4) + 10 + 8$

$= 16 + 16 + 12 + 18$

$= 32 + 30$

$= 62$ //

iii, Sol: Given $a+b+c=0$

$$\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab} \Rightarrow \frac{a^3+b^3+c^3}{abc}$$

$$\Rightarrow \frac{(a+b+c)^3-3(ab+bc+ca)(a+b+c)+3abc}{abc}$$

$$\Rightarrow \frac{(0)^3-3(ab+bc+ca)(0)+3abc}{abc}$$

$$\Rightarrow \frac{3abc}{abc}$$

$$\Rightarrow 3$$

$$\therefore \frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=3 \;//$$

iv, Sol: Given $a+b+c=5$, $ab+bc+ca=10$

$$a^3+b^3+c^3-3abc=(a+b+c)^3-3(ab+bc+ca)(a+b+c)$$

$$=(5)^3-3(10)(5)$$

$$=125-150$$

$$=-25 \;//$$

v, Given, $(a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$

let $(a+b+c)^3 = ((a+b)+c)^3$ $\quad$ [$\because (a+b)^3 = a^3+3a^2b+3ab^2+b^3$]

$\Rightarrow (a+b+c)^3 = (a+b)^3 + 3(a+b)^2c + 3(a+b)c^2 + c^3$

$= (a^3+3a^2b+3ab^2+b^3) + 3(a^2+2ab+b^2)c + 3(a+b)c^2 + c^3$

$= a^3+b^3+c^3+3a^2b+3a^2c+3ab^2+3ac^2+3bc^2+6abc$

$= a^3+b^3+c^3+3a^2b+3a^2c+3ab^2+3ac^2+3bc^2+3abc+3abc$

$= a^3+b^3+c^3+3a(ab+ac+b^2+bc)+3c(ab+ac+b^2+bc)$

$= a^3+b^3+c^3+(3a+3c)(ab+ac+b^2+bc)$

$\Rightarrow a^3+b^3+c^3+3(a+c)(a(b+c)+b(b+c))$

$\Rightarrow a^3+b^3+c^3+3(a+c)[(b+c)(a+b)]$

$(a+b+c)^3 \Rightarrow a^3+b^3+c^3+3(a+b)(b+c)(c+a)$

$\therefore (a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$ //

5 Sol:- $-\sqrt{3}$

Explanation: Given polynomial $p(x) = 6x^4 - \sqrt{3}x^2 - \frac{5}{3}$

The coefficient of $x^3 = -\sqrt{3}$.

6 Sol: (b) 0.

Explanation:

The degree of a polynomial is the highest degree of its variable term.

Degree of a constant zero.

Here. $p(x) = 4$

$\Rightarrow p(x) = 4x^0$.

$p(x)$ · Degree of $x = 0$

7 Sol:- (d) 5.

Explanation: Given $(y^3-2)(y^2+11)$

The degree of a polynomial is the highest degree its variable term.

Here $p(y) = (y^3-2)(y^2+11)$

$= y^5 + 11y^3 - 2y^2 - 22$

$\therefore$ Degree of $y = 5$

8. Sol: (a) 3.

Explanation:- Given polynomial $2x^3 - x\sqrt{3} + 3x - 4$

The degree of polynomial is the highest degree its variable term.

let $p(x) = 2x^3 - x\sqrt{5} + 3x - 4$.

The degree of x is 3.

9. Sol:- (a) 6.

Explanation: Given $p(t) = t^4 - t^2 + 6$.

Then $p(-1) = (-1)^4 - (-1)^2 + 6$.

$= 1 - 1 + 6$

$= 6$

10. Sol:- Given, zero of the polynomial $p(x)$

Then, $p(x) = ax + 1$

$0 = ax + 1$

$ax = -1$

$x = -1/a$

Ans:- (d) $-1/a$,,

11 Sol:- (a) 3.

Explanation: Given x^2-5x+6

To get the zero of p(x)

$p(x) = 0$

$p(x) = x^2-5x+6$

$x^2-5x+6 = 0$

Splitting the middle term

$x^2-2x-3x+6 = 0$

$x(x-2)-3(x-2) = 0$

$(x-2)(x-3) = 0$

So.

$x-2 = 0$

$x = 2$

$x-3 = 0$

$x = 3$.

$\therefore$ The value of $x = 2$ and 3.

So. 2 and 3 are the zero's of the polynomial x^2-5x+6,

12 Sol: (a) i,

Explanation: Given.

i, x $f(x) = x^3+x^2+x+1$.

let $x+1 = 0$

$x = -1$.

$f(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= -2 + 2 = 0$

$f(x) = 0.$

$\therefore (x+1)$ is factor of $x^3 + x^2 + x + 1.$

ii, Given polynomial $f(x) = x^4 + x^3 + x^2 + x + 1$

let $x + 1 = 0$

$x = -1$

$f(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1$

$= 3 - 2 = 1$

$\therefore f(x) \neq 0.$

$\therefore (x+1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1.$

iii, Given polynomial $f(x) = x^4 + 3x^3 + 3x^2 + x + 1$

let $x + 1 = 0, \Rightarrow x = -1$

$f(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1 = 1 + 3(-1) + 3 - 1 + 1$

$\Rightarrow 1 - \cancel{3} + \cancel{3} - 1 + 1 \Rightarrow 2 - 1 = 1$

$f(x) \neq 0.$

$\therefore (x+1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1.$

iii) Given polynomial $x^3-x^2-(2+\sqrt{2})x+\sqrt{2} = f(x)$

let $x+1=0 \Rightarrow x=-1$

$f(-1) = (-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$= -1+1+2+\sqrt{2}+\sqrt{2}$

$= -\cancel{2}+\cancel{2}+2\sqrt{2}$

$= 2\sqrt{2}$

$f(x) \neq 0$

$\therefore (x+1)$ is not a factor of $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

13 Sol:- c) $\sqrt{2}-1$

Explanation: Given $(x-1)$ is factor of ~~kx^2~~ $kx^2-\sqrt{2}x+1$.

let $x-1=0 \Rightarrow x=1$

Then $kx^2-\sqrt{2}x+1=0$

$k(1)^2-\sqrt{2}(1)+1=0$

$k-\sqrt{2}+1=0$

$k=\sqrt{2}-1$ //

14 Sol: d) $(x+2)(x+9)$

Explanation: Given $x^2+11x+18$

splitting the middle term

$x^2+2x+9x+18$

$x(x+2)+9(x+2)$

$(x+2)(x+9)$ //

15 Sol:- Given: $(x-y)^3$ (c) $x^3-y^3-3x^2y+3xy^2$

Explanation: Given $(x-y)^3$

$(x-y)^3 = x^3-y^3-3x^2y+3xy^2$ $[\because (a-b)^3 = a^3-b^3-3a^2b+3ab^2]$

16 Sol:- d, $a^2-b^2 = (a-b)(a+b)$

Explanation: Given $x^2-\frac{y^2}{100}$

$x^2-\frac{y^2}{100} \Rightarrow x^2-\left(\frac{y}{10}\right)^2$

$x^2-\frac{y^2}{100} \Rightarrow \left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$

$a^2-b^2 = (a+b)(a-b)$ //

17 Sol:- b, 3, $(x+3)(x-3)$

Explanation:- Given Volume of cuboid is $3x^2-27$.

Volume of cuboid is given by $= l \times b \times h$.

Given $3x^2-27$

$\Rightarrow 3(x^2-9)$ $[\because a^2-b^2 = (a+b)(a-b)]$

$\Rightarrow 3(x^2-3^2)$

$\Rightarrow 3(x+3)(x-3)$

$\therefore$ The possible dimensions are 3, $(x+3)$, $(x-3)$

18 Sol: (c) $\frac{1}{4}$

Explanation:- Given $49x^2 - b = (7x + \frac{1}{2})(7x - \frac{1}{2})$

$$49x^2 - b = (7x)^2 - (\tfrac{1}{2})^2 \qquad [a^2 - b^2 = (a+b)(a-b)]$$

$$49x^2 - b = 49x^2 - \frac{1}{4}$$

$$-b = -\frac{1}{4}$$

$$b = \frac{1}{4}$$

19 Sol:- (c) 3abc.

Explanation: Given $a+b+c=0$

$$a^3+b^3+c^3 = (a+b+c)^3 + 3(ab+bc+ca)(a+b+c) + 3abc$$

$$= (0) + 3(ab+bc+ca)(0) + 3abc$$

$$= 3abc$$

20 Sol: (5x+1), 10x, (c) $5x+1$

Explanation: Given $(25x^2-1) + (1+5x)^2$

$$\Rightarrow [(5x)^2 - (1)^2] + (1+5x)^2$$

$$\Rightarrow (5x+1)(5x-1) + (1+5x)^2$$

$$\Rightarrow (5x+1)[5x \cancel{-1} + 5x \cancel{+1}]$$

$$\Rightarrow (5x+1)[10x]$$

$\therefore$ $5x+1$ is one of the factor $(25x^2-1) + (1+5x)^2$

21 Sol:- (b) $(2x+1)(2x+3)$

Explanation: Given $4x^2+8x+3$

splitting the middle term

$\Rightarrow 4x^2+6x+2x+3$

$\Rightarrow 2x(2x+3)+1(2x+3)$

$\Rightarrow (2x+3)(2x+1)$

22 Sol:- (c) 0.

Explanation: Given $\frac{x}{y}+\frac{y}{x}=-1$

$$\Rightarrow \frac{x^2+y^2}{xy}=-1$$

$\Rightarrow x^2+y^2=-xy$

$\Rightarrow x^2+y^2+xy=0$

then

$x^3-y^3=(x+y)(x^2+xy+y^2)$

$x^3-y^3=(x-y)(0)$

$\therefore x^3-y^3=0$

23 Sol:- (d) Not defined

Explanation: Zero of a polynomial is the value of a variable for which the polynomial becomes 0.
zero polynomial is a constant polynomial

Whose coefficent are equal to 0.

Here', there is no variable

. Hence, zero of zero polynomial is not defined.,

24 Sol:- c) 2.

Explanation:- Given $(x+1)$ is a factor of 2k polynomial $2x^2+kx$

let $x+1=0 \Rightarrow x=-1$

Then $2x^2+kx=0$

$2(-1)^2+k(-1)=0$

$2-k=0$

$k=2$,,

25 Sol:- d) 50.

Explanation: Given $x^{51}+51$ is divided by $x+1$.

let $p(x) = x^{51}+51$

We divide $p(x)$ by $x+1$, We get the remainder $p(-1)$.

let $x+1=0 \Rightarrow x=-1$

$p(-1) = (-1)^{51}+51 = -1+50 = 50.$

Hence, the remainder is 50 ,,

26 Sol: (b) $\frac{1}{2}$

Explanation: Given polynomial $2x^2+7x-4$.

$p(x)=0$.

$2x^2+7x-4=0$

$2x^2-x+8x-4=0$

$x(2x-1)+4(2x-1)=0$

$(2x-1)(x+4)=0$

$2x-1=0$

$2x=1 \Rightarrow x=1/2$

$x+4=0$

$x=-4$.

$\therefore$ $1/2$ is one of zero's of the polynomial $2x^2+7x-4$.

27 Sol: d) 6.

Explanation:- Given $p(x)=x+3$.

$p(-x)=(-x)+3$

Then $p(x)+p(-x)=\cancel{x}+3-\cancel{x}+3=6$ //

28 Sol:- d) 27

Explanation: Given $(x+3)^3$

$(x+3)^3 = x^3+(3)^3+3x^2(3)+3x(3)^2$

$(x+3)^3 = x^3 + 27 + 9x^2 + 27x.$

$= x^3 + 27 + 27x + 9x^2$

$(x+3)^3 = x^3 + 9x^2 + 27x + 27.$

$\therefore$ x Coefficient of $x = 27$,,

29. Sol: (a) 3.

Explanation: Given $x^2 + 9y^2 = 369.$, $xy = 60.$

$\Rightarrow x^2 + (3y)^2 = 369.$

$(x - 3y)^2 = x^2 + (3y)^2 - 2 \times x \times 3y$

$= x^2 + 9y^2 - 6 \times xy$

$= 369 - 6 \times 60$

$= 369 - 360$

$(x-3y)^2$

$(x-3y) = 9$,,

30. Sol: (a) $-\frac{5}{2}, \frac{7}{2}$

Explanation: Given $lx^4 + mx^3 + 2x^2 + 4$

Let $x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x+1)(x-2) = 0$

$x + 1 = 0 \Rightarrow x = -1$

$x-2=0 \Rightarrow x=2$

If $x=-1$

Then $\cdot\ lx^4+mx^3+2x^2+4=0$

$l(-1)^4+m(-1)^3+2(-1)^2+4=0$

$l(1)+m(-1)+2(1)+4=0$

$l-m+6=0$

$l-m=-6 \longrightarrow (1)$

If $x=2$

Then $lx^4+mx^3+2x^2+4=0$

$l(2)^4+m(2)^3+2(2)^2+4=0$

$16l+8m+8+4=0$

$16l+8m=-12 \longrightarrow (21)$

$4(4l+2m=-3)$.

$4l+2m=-3 \longrightarrow (2)$

$(1)\times 2+(2) \Rightarrow$

$2l-\cancel{2m}=-12$

$4l+\cancel{2m}=-3$

$\overline{6l=-15}$

$l=-15/6=\frac{-5}{2}$

~~fro~~ Substitute l in (1).

$l - m = -6$

$-\frac{5}{2} - m = -6$

$-m = -6 + \frac{5}{2} = \frac{-12+5}{2} = \frac{-7}{2}$

$m = \frac{7}{2}$

$\therefore l = -\frac{5}{2}, m = 7/2$,,

Level - II

31) Sol: c)

Explanation: Given $f(x) = x^3 + ax^2 + bx + 3$.

$f(x)$ is divided by $(x+2)$, remainder $= 7$.

let $x+2 = 0 \Rightarrow x = -2$

Then $f(-2) = (-2)^3 + a(-2)^2 + b(-2) + 3 = 7$

$\Rightarrow -8 + a(4) - 2b + 3 = 7$

$\Rightarrow -8 + 4a - 2b + 3 = 7$

$\Rightarrow 4a - 2b + 3 - 5 = 7$

$\Rightarrow 4a - 2b = 7 + 5 = 12$

$\Rightarrow 4a - 2b = 12 \longrightarrow (1)$

$\Rightarrow 2a - b = 6 \longrightarrow (1)$

$f(x)$ is also divided by $(x-1)$, the remainder $= 4$.

let $x - 1 = 0 \Rightarrow x = 1$

Then $f(x) = (1)^3 + a(1)^2 + b(1) + 3 = 4$

$\Rightarrow 1 + a + b + 3 = 4$

$\Rightarrow a+b+4=4$

$\Rightarrow a+b=4-4=0$

$\Rightarrow a+b=0 \longrightarrow (2)$.

(1) + (2) $\Rightarrow$ $2a-b=6$

$a+b=0$

$3a=6$

$a=6/3=2$

Substitute 'a' in eq's (2)

$a+b=0$

$2+b=0$

$b=-2$

$\therefore 3a-b=3(2)-(-2)=6+2$

$=8$ //

32. Sol:- (b) 1, -3, 2

Explanation: Given ax^2+bx+c is exactly divisible by $(x-1)(x-2)$

Then $f(x)=ax^2+bx+c$.

let $x-1=0 \Rightarrow x=1$

$f(1) \Rightarrow a(1)^2+b(1)+c=0$

$\Rightarrow a+b+c=0 \longrightarrow (1)$

let $x-2=0 \Rightarrow x=2$

$f(2) \Rightarrow a(2)^2+b(2)+c=0$

$4a+2b+c=0 \longrightarrow (2)$

$f(x)$ is also divided by $(x+1)$, the remainder 6.

Let $x+1=0$

$x=-1$

Then $f(-1) = a(-1)^2+b(-1)+c=6$

$\Rightarrow a-b+c=6 \longrightarrow (3)$

$(1)-(3) \Rightarrow$

$$\begin{array}{r} \cancel{a}+b+\cancel{c}=0 \\ \cancel{a}-b+\cancel{c}=6 \\ -\ \ +\ \ -\ \ \ - \\ \hline 2b=-6 \end{array}$$

$b=-6/2=-3$

Substitute b in eq (1) & eq (2)

from eq (1) $\Rightarrow$ $a+b+c=0$

$a-3+c=0$

$a+c=3 \longrightarrow (4)$

from eq (2) $\Rightarrow$ $4a+2b+c=0$

$4a+2(-3)+c=0$

$4a+c-6=0$

$4a+c=6 \longrightarrow (5)$

$(4)-(5) \Rightarrow$

$$\begin{array}{r} a+\cancel{c}=3 \\ 4a+\cancel{c}=6 \\ - \ \ \ - \ \ \ - \\ \hline -3a=-3 \end{array}$$

$a=-3/-3=1$

Substitute 'a' in (4).

④ $\Rightarrow$ $a+c=3$

$1+c=3$

$c=3-1=2$

$\therefore a=1,\ b=-3,\ c=2$ //

33 Sol: (b) $(2a+3b+8)(4a-5b-6)$

Explanation: Given $8(a+1)^2+2(a+1)(b+2)-15(b+2)^2$

$\Rightarrow 8(a+1)^2+12(a+1)(b+2)-10(a+1)(b+2)-15(b+2)^2$

$\Rightarrow 4(a+1)[2(a+1)+3(b+2)]-5(b+2)[2(a+1)+3(b+2)]$

$\Rightarrow [2(a+1)+3(b+2)][4(a+1)-5(b+2)]$

$\Rightarrow [2a+2+3b+6][4a+4-5b-10]$

$\Rightarrow [2a+3b+8][4a-5b-6]$ //

34 Sol: Given $(a+1)x^2+(2a+3)x+(3a+4)=0$ is 2.

The product of roots $=\dfrac{c}{a}=\dfrac{3a+4}{a+1}=2$

$\Rightarrow \dfrac{3a+4}{a+1}=2$

$\Rightarrow 3a+4=2(a+1)$

$3a+4=2a+2$

$3a-2a=2-4$

$a=-2$

The Sum of the roots ca $= \frac{-b}{a}$

$$= \frac{-(2a+3)}{a+1}$$

$$= \frac{-[2(-2)+3]}{-2+1}$$

$$= \frac{-[-4+3]}{-1}$$

$$= \frac{+[-1]}{+1}$$

$$= -1 \quad //$$

Ans: $b = (-1)$ //

35 Sol: $\alpha, \frac{-2}{a}$

Explanation: Given, α, β are the roots of the eq's $ax^2+bx+c=0$.

In eq's the sum of the roots. $\alpha+\beta = \frac{-b}{a}$

The product of the roots $\alpha\beta = \frac{c}{a}$

Then $\frac{\alpha}{a\beta+b} + \frac{\beta}{a\alpha+b} = \frac{\alpha(a\alpha+b)+\beta(a\beta+b)}{(a\beta+b)(a\alpha+b)}$

$$= \frac{a\alpha^2+b\alpha+a\beta^2+b\beta}{a^2\alpha\beta+ab\beta+ab\alpha+b^2}$$

$$= \frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$= \frac{a((\alpha+\beta)^2-2\alpha\beta)+b(\alpha+\beta)}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$\Rightarrow \frac{a\left[\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)\right] + b\left(\frac{-b}{a}\right)}{a^2\left(\frac{c}{a}\right) + ab\left(\frac{-b}{a}\right) + b^2}$$ $[\because \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta]$

$$\Rightarrow \frac{a\left[\frac{b^2}{a^2} - \frac{2c}{a}\right] - \frac{b^2}{a}}{ac - \cancel{b^2} + \cancel{b^2}}$$

$$\Rightarrow \frac{\cancel{a}\left[\frac{b^2 - 2ac}{a^{\cancel{2}}}\right] - \frac{b^2}{a}}{ac}$$

$$\Rightarrow \frac{\frac{\cancel{b^2} - 2ac - \cancel{b^2}}{a}}{ac}$$

$$\Rightarrow \frac{-2ac}{ac} \quad \frac{\frac{-2\cancel{a}c}{\cancel{a}}}{ac}$$

$$\Rightarrow \frac{-2\cancel{c}}{a\cancel{c}}$$

$$\Rightarrow \frac{-2}{a} //$$

36 Sol:- d) -2.

Explanation:- Given $ax^2+bx+c=0$.

The product of eq's $\alpha\beta = \frac{c}{a} = 3$

$\Rightarrow c = 3a$

The sum of eq's $\alpha+\beta = \frac{-b}{a}$

Also given a, b, c are in A.p then:

$2b = a+c$

$b = \frac{a+c}{2}$

$$b = \frac{a+3a}{2}$$

$$b = \frac{4a}{\cancel{4}} = 2a$$

$$\therefore \alpha+\beta = \frac{-b}{a}$$

$$= \frac{-2\cancel{a}}{\cancel{a}} = -2$$

$$\alpha+\beta = -2 \text{ //}$$

37 Sol: (a) 3

Explanation: Given $p(x+2) = 2x^3 - 2x^2 + 2x + 3$.

Then $p(x)$ is divided by $x-3$.

let $x-3=0 \Rightarrow x=3$

$p(x+2) = 2x^3 - 4x^2 + 2x + 3$

x is replaced by $x-2$

$p(x-\cancel{2}+2)^2 = 2(x-2)^3 - 4(x-2)^2 + 2(x-2) + 3$

$p(x) = 2(x-2)^3 - 4(x-2)^2 + 2(x-2) + 3$

Then $p(x)$ is divided by $x-3$

let $x-3=0 \Rightarrow x=3$

$p(3) = 2(3-2)^3 - 4(3-2)^2 + 2(3-2) + 3$

$= 2(1)^3 - 4(1)^2 + 2(1) + 3$

$\Rightarrow 2 - 4 + 2 + 3$

$\Rightarrow 7 - 4$

$\Rightarrow 3$ //

38 Sol: d, 128.

Explanation: Given $a^3+4a-8=0$

$$a^3+4a=8$$

$$a^3=8-4a$$

Squaring on Both sides

$$(a^3)^2=(8-4a)^2$$

$$a^6=64+16a^2-2\times8\times4a$$

$$a^6=64+16a^2-64a$$

Multiply 'a' on Both sides

$$a.a^6=a(64+16a^2-64a)$$

$$a^7=64a+16a^3-64a^2$$

$$a^7+64a^2=16a(a^3+4a).$$

$$=16(8)=128 \;,,$$

39 Sol: (a) x.

Explanation: Given x^{2017} is divided by (x^2-1)

factor of $x^2-1=(x+1)(x-1)$

$$p(x)=x^{2017}$$

We know that

divided = divisor x Quotient + Remainder

Since $p(x)$ is a polynomial of degree 2017. So it will a linear remainder in form of $ax+b$.

where a and b are constants

$$x^{2017} = \text{divisor} * (x+1)(x-1) + ax+b$$

Put $x=1$

$$1^{2017} = \text{divisor} * (1+1)(1-1) + a(1)+b$$

$$1 = \text{divisor} * (2)(0) + a+b$$

$$1 = a+b \quad \text{—} (1)$$

put $x=-1$ in eq.

$$(-1)^{2017} = \text{divisor} * (-1+1)(-1-1) + a(-1)+b$$

$$-1 = \text{divisor} * (0)(-2) - a+b$$

$$-1 = -a+b \rightarrow (2)$$

(1) + (2) $\Rightarrow$

$$\begin{array}{r} \cancel{a}+b=1 \\ -\cancel{a}+b=-1 \\ \hline 2b=0 \\ b=0 \end{array}$$

from (1) $\Rightarrow$ $a+b=1$

$a+0=1 \Rightarrow a=1$

Then remainder $ax+b = 1(x)+0 = x$ //

40 Sol:- (a) 31

Explanation:- Given $x = \dfrac{(2017\ 2016)^2}{(2017\ 2015)^2 + (2017\ 2017)^2 - 2}$

let 2017 2016 = y

$x \Rightarrow \dfrac{y^2}{(y-1)^2+(y+1)^2-2}$

$\Rightarrow \dfrac{y^2}{y^2+1-\cancel{2y}+y^2+\cancel{2y}+1-2}$

$\Rightarrow \dfrac{y^2}{2y^2+\cancel{2}-\cancel{2}}$

$x \Rightarrow \dfrac{\cancel{y^2}}{2\cancel{y^2}} \Rightarrow \dfrac{1}{2}$

$x^{-11} - 2017 = \left(\frac{1}{2}\right)^{-11} - 2017$

$= 2^{11} - 2017$

$= 2048 - 2017$

$= 31$ ✓

41 Sol: (c) 2x+3.

Explanation: Given f(x) is divided by (x-1) and (x-2), it leaves remainder 5 and 7.

Using Division Algorithm here:-

Dividend = Divisor × Quotient + Remainder

Let g(x), k(x) be Quotient when f(x) is divided by x-1 and x-2

$f(x) = (x-1)\,g(x) + 5$

$f(1) = (1-1)\,g(x) + 5$

$f(1) = (0) q(1) + 5$

$f(1) = 5 \longrightarrow ①$

$\therefore$ Also $f(x) = (x-2) K(x) + 7$

$f(2) = (2-2) K(2) + 7$

$f(2) = 0 \cdot K(2) + 7$

$f(2) = 7 \longrightarrow ②$

Now, let $ax+b$ be remainder when $f(x)$ is divided by $(x-1)$ $(x-2)$ and $g(x)$ be Quotient

$f(x) = (x-1)(x-2) \cdot g(x) + (ax+b)$

from using ① & ②.

$f(1) = (1-1)(1-2) g(1) + (a(1)+b)$

$5 = 0(-1) g(1) + a+b$

$5 = a+b \longrightarrow ③$

$f(2) = (2-1)(2-2) \cdot g(2) + (a(2)+b)$

$7 = (1)(0) g(2) + (2a+b)$

$7 = 2a+b \longrightarrow ④$

③ − ④ $\Rightarrow$

$$\begin{array}{r} a + \cancel{b} = 5 \\ 2a + \cancel{b} = 7 \\ \hline -a = -2 \\ a = 2 \end{array}$$

substitute 'a' in eq(3)

$a+b=5$

$2+b=5$

$b=3.$

The Remainder $\cancel{2x+3} = ax+b = 2x+3.$

$\therefore 2x+3$ is remainder when $f(x)$ is divided by $(x-1)(x-2)$

42 Sol:- (c) 10.

Explanation:- Given $p(x) = ax^9 + bx^5 + cx - 11$, $P(1024) = -32$

$$P(1042) = a(1042)^9 + b(1024)^5 + c(1024)\cancel{x} - 11$$

$$P(-1042) = a(-1042)^9 + b(-1024)^5 + c(-1024) - 11$$

$$= -a(1024)^9 - b(1024)^5 - c(1024) - 11$$

$$P(1024) + P(-1042) = a(\cancel{1042})^9 + b(\cancel{1024})^5 + c(\cancel{1024}) - 11 - a(\cancel{1042})^9$$
$$- b(\cancel{1024})^5 - c(\cancel{1024}) - 11$$

$$= -11 - 11.$$

$$P(1024) + P(-1042) = -22$$

$$P(-1042) = -22 - P(1024)$$

$$= -22 - (-32)$$

$$= -22 + 32$$

$$= 10$$

43 Soln (C) $\frac{1}{8}$

Explanation: Given $a+b=1$.

$AM \geq GM$.

$\frac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow \frac{1}{2} \geq \sqrt{ab}$

$\Rightarrow ab \leq \frac{1}{4}$

$a^4+b^4 \geq (a+b)^4 - 4a^3b - 6a^2b^2 - 4ab^3$

$\geq (a+b)^4 - 4ab(a^2+b^2) - 6a^2b^2$

$\geq (a+b)^4 - 4ab((a+b)^2 - 2ab) - 6a^2b^2$

$\geq (1)^4 - 4(\frac{1}{4})[(1)^2 - 2(\frac{1}{4})] - 6(\frac{1}{4})^2$

$a^4+b^4 \geq 1 - (1-\frac{1}{2}) - \frac{6}{16}$

$\geq 1 - (\frac{2-1}{2}) - \frac{3}{8}$

$\geq 1 - \frac{1}{2} - \frac{3}{8}$

$\geq \frac{8-4-3}{8}$

$a^4+b^4 \geq \frac{1}{8}$ //

44 Sol: (a) 45

Explanation: Given $f(10) - f(5) = 15$,

$f(x)$ is be a polynomial of degree 1.

Let $f(x) = ax$.

Then $f(10) = a(10) = 10a$

$f(5) = a(5) = 5a$

$f(10) - f(5) = 10a - 5a$

$15 = 5a$

$\Rightarrow 5a = 15$

$a = 15/5 = 3.$

$f(20) - f(5) = 20a - 5a$

$= 20 \times 3 - 5 \times 3 = 60 - 15$

$= 45$ //

45 Sol:- d) $x = -P/2$

Explanation:- Given $f(x) = x^2 + px + 2$

$p > 0$.

$ax^2 + bx + c$ has its minimum value at $x = \frac{-b}{2a}$

Here $a = 1, b = p, c = 2$

$\therefore f(x)$ has its minimum value at $x = \frac{-p}{2(1)} = \frac{-p}{2}$

$= -P/2$ //

46 Sol:- (b) 63

Explanation: Given $f(x) = x^3 - 3x^2 - 7x + 21$

In the form $ax^3 + bx^2 + cx + d$.

Here $a = 1$, $b = -3$, $c = -7$, $d = 21$

α, β, γ are zero's of given polynomial.

$$\therefore \alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-3)}{1} = 3$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-21}{1} = -21$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-7}{1} = -7$$

Therefore

$$\frac{(14 - 7\alpha)(22 - 11\beta)(114 - 57\gamma)}{209} = \frac{7(2-\alpha) \cdot 11(2-\beta) \cdot 57(2-\gamma)}{209}$$

$$\Rightarrow \frac{\overset{21}{4389}(2-\alpha)(2-\beta)(2-\gamma)}{209}$$

$$\Rightarrow 21(2-\alpha)(2-\beta)(2-\gamma)$$

$$\Rightarrow 21[4 - 2\beta - 2\alpha + \alpha\beta](2-\gamma)$$

$$\Rightarrow 21[8 - 4\gamma - 4\beta + 2\beta\gamma - 4\alpha + 2\alpha\gamma + 2\alpha\beta - \alpha\beta\gamma]$$

$$\Rightarrow 21[8 - 4(\alpha + \beta + \gamma) + 2(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma]$$

$$\Rightarrow 21[8 - 4(3) + 2(-7) - (-21)]$$

$$\Rightarrow 21[8 - 12 - 14 + 21]$$

$\Rightarrow 21[29-26]$

$\Rightarrow 21 \times 3$

$= 63$ //

47.Sol: (a) $(a^4+3b^4)[a-\sqrt{6}b)(a^2+6b^2)(a+\sqrt{6}b)$

Explanation: Given $a^8-33a^4b^4-108b^4$

$\Rightarrow a^8+3a^4b^4-36a^4b^4-108b^4$

$\Rightarrow a^4[a^4+3b^4]-36b^4[a^4+3b^4]$

$\Rightarrow [a^4+3b^4][a^4-36b^4]$

$\Rightarrow [a^4+3b^4][(a^2)^2-(6b)^2]$ $[\because a^2-b^2=(a+b)(a-b)]$

$\Rightarrow [a^4+3b^4][a^2+6b^2][a^2-6b^2]$

$\Rightarrow [a^4+3b^4][a^2+6b^2][(a)^2-(\sqrt{6}b)^2]$

$\Rightarrow [a^4+3b^4][a^2+6b^2][a+\sqrt{6}b][a-\sqrt{6}b]$ //

$\Rightarrow [a^4+3b^4][a-\sqrt{6}b][a^2+6b^2][a+\sqrt{6}b]$ //

48.Sol: (a) $\frac{1}{2^{128}}$

Explanation: $((\sqrt{x})^{\sqrt{x}})^{\sqrt{x}\cdots\infty} = \frac{1}{16}$

$\therefore (\sqrt{x})^{\frac{1}{16}} = \frac{1}{16}$ [$\because$ from the given problem]

$(x^{\frac{1}{2}})^{\frac{1}{16}} = \frac{1}{16}$

$x^{\frac{1}{32}} = \frac{1}{16}$

$x = \left(\frac{1}{16}\right)^{32}$

$x = \left(\frac{1}{2^4}\right)^{32}$

$x = \frac{1}{2^{128}}$ //

49. Sol: (d) $\left[(a+b)^2+\sqrt{2}ab-2ab\right]\left[(a-b)^2-\sqrt{2}ab+2ab\right]$

Explanation: Given a^4+b^4

$a^4+b^4 = (a^2)^2+(b^2)^2$

$= (a^2+b^2)^2 - 2a^2b^2$

$= (a^2+b^2)^2 - (\sqrt{2}ab)^2$

$= (a^2+b^2+\sqrt{2}ab)(a^2+b^2-\sqrt{2}ab)$

$= ((a+b)^2-2ab+\sqrt{2}ab)((a-b)^2+2ab-\sqrt{2}ab)$

$= \left[(a+b)^2+\sqrt{2}ab-2ab\right]\left[(a-b)^2-\sqrt{2}ab+2ab\right]$ //

50. Sol: (c) 50.

Explanation: Given $x = 7^{1/3} + \frac{2}{7^{1/3}}$

$7x^3 - 21x \Rightarrow 7\left[7^{1/3}+\frac{2}{7^{1/3}}\right]^3 - 21\left[7^{1/3}+\frac{2}{7^{1/3}}\right]$

$\Rightarrow 7\left[(7^{1/3})^3+\left(\frac{2}{7^{1/3}}\right)^3 + 3\,7^{1/3}\frac{2}{7^{1/3}}\left(7^{1/3}+\frac{2}{7^{1/3}}\right)\right] - 21\left[7^{1/3}+\frac{2}{7^{1/3}}\right]$

$\Rightarrow 7\left[7+\frac{2}{7}+3\left[7^{1/3}+\frac{2}{7^{1/3}}\right]\right] - 21\left[7^{1/3}+\frac{2}{7^{1/3}}\right]$

$\Rightarrow 7\left[\frac{49+1}{7} + 3\left[7^{1/3} + \frac{1}{7^{1/3}}\right]\right] - 21\left[7^{1/3} + \frac{1}{7^{1/3}}\right]$

$\Rightarrow 7\left[\frac{50}{7} + 3\left[7^{1/3} + \frac{1}{7^{1/3}}\right]\right] - 21\left[7^{1/3} + \frac{1}{7^{1/3}}\right]$

$\Rightarrow \cancel{7} \times \frac{50}{\cancel{7}} + 21\left[7^{1/3} + \frac{1}{7^{1/3}}\right] - 21\left[7^{1/3} + \frac{1}{7^{1/3}}\right]$

$\Rightarrow 50$

Multiple Correct Answer Type

51 Sol:- (c) $a^2 + 4a + 18$

Explanation: Given $x = \frac{a}{2}$

Then $4x^2 + 8x + 18$

$\Rightarrow 4\left[\frac{a}{2}\right]^2 + \cancel{8}^{4}\left[\frac{a}{\cancel{2}}\right] + 18$

$\Rightarrow \cancel{4}\left[\frac{a^2}{\cancel{4}}\right] + 4a + 18$

$\Rightarrow a^2 + 4a + 18$ c

52 Sol:- b, $a = -3$, c, $b = -1$

Explanation: Given $x^3 + ax^2 + bx + 6$. has $x-2$ as factor

$f(x) = x^3 + ax^2 + bx + 6 = 0$

let $x - 2 = 0 \Rightarrow x = 2$

$f(2) = (2)^3 + a(2)^2 + b(2) + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$4a + 2b = -14$

$\Rightarrow$ ~~4~~ $2a+b=-7 \longrightarrow (1)$

Also.
~~f(x)~~ The remainder 3 when divided by $x-3$.

Then $x-3=0 \Rightarrow x=3$

$f(-3) = (3)^3+a(3)^2+b(3)+6 = 3$

$\Rightarrow 27+9a+3b+6=3$

$9a+3b = 3-33 = -30$

$3a+b = -10 \longrightarrow (2)$

$(1)-(2) \Rightarrow$

$2a+\cancel{b} = -7$

$\underline{3a+\cancel{b} = -10}$

$-a = 3$

$a=-3$

Substitute 'a' in eq(1)

$2a+b = -7$

$2(-3)+b = -7$

$-6+b=-7$

$b=-7+6=-1$

$\therefore a=-3, b=-1$

53 ~~Sol~~: c) $x+y+z$, d) $a+b$.

Explanation: ~~a~~ Given $a(x+y+z)+bx+by+bz$

$\Rightarrow a(x+y+z)+b(x+y+z)$

$\Rightarrow (x+y+z)(a+b)$

$\therefore (x+y+z)(a+b)$ are factors of $a(x+y+z)+bx+by+bz$.

54 Sol: (a) 318, (b) 325, (c) 343. d) $\sqrt{1296}$

Explanation: Given $x+\frac{1}{x}=7$

$x^3+\frac{1}{x^3} = (x+\frac{1}{x})^3 - 3\cancel{x}\times\frac{1}{\cancel{x}}(x+\frac{1}{x})$ $[\because a^3+b^3=(a+b)^3-3ab(a+b)]$

$= (7)^3 - 3(7)$

$= 343 - 21$

$= 322$

$\therefore$ The values of a, b, c, d is not the values of $x^3+\frac{1}{x^3}$ //

Match Matrix Type

55 Sol: A-s, B-r, C-q, D-p

Explanation:

Column - 1	Column II
A) $2-y^2-y^3+2y^8$	(P) 2
B) 2	q) 1
C) $5x-\sqrt{7}$	r) 0
D) $4-x^2$	s) 8.

A Sol: The degree of polynomial $2-y^2-y^3+2y^8 = 8$ (s)

B Sol: The degree of 2 is 0. (r)

C Sol: The degree of $5x-\sqrt{7}$ is 1 (q)

D Sol: The degree of $4-x^2$ is 2 (P)

56 Sol: (b) A - q, B - p, C - s, D - r

Explanation: Given

	Column - I	Column - II
	A. $x-1y^4-y^3+2$	p
	A. kx^2-3x+k	(P) -2
	B. x^2+x+k	q) $3/2$
	C. $2x^2+kx+\sqrt{2}$	(r) $\sqrt{2}-1$
	D. $kx^2-\sqrt{2}x+1$	(s) $-(2+\sqrt{2})$

A Sol: Given polynomial kx^2-3x+k when divided by $x-1$

let $x-1=0 \Rightarrow x=1$

$f(x)=0$

$kx^2-3x+k=0$

$k(1)-3(1)+k=0$

$2k-3=0 \Rightarrow 2k=3 \Rightarrow k=3/2$ (q)

B Sol: Given polynomial x^2+x+k when divided by $x-1$

let $x-1=0 \Rightarrow x=1$

$f(x)=x^2+x+k=0$

$f(1)=(1)^2+1+k=0 \Rightarrow 2+k=0 \Rightarrow k=-2$ (p)

C Sol: Given polynomial $2x^2+kx+\sqrt{2}$ when divided by $x-1$

let $x-1=0 \Rightarrow x=1$

$f(x)=2x^2+kx+\sqrt{2}=0$

$f(1)=2(1)^2+k(1)+\sqrt{2}=0 \Rightarrow k+2+\sqrt{2}=0 \Rightarrow k=-(2+\sqrt{2})$ (s)

D Sol) Given polynomial $kx^2-\sqrt{2}x+k$ when divided by $x-1$

let $x-1=0 \Rightarrow x=1$

$f(x)=kx^2-\sqrt{2}x+k=0$

$f(1)=k(1)-\sqrt{2}(1)+1k=0 \Rightarrow 2k-\sqrt{2}+1=0 \Rightarrow k=\sqrt{2}-1$ (r)

5 Sol: (c) A-s, B-r, C-p, D-q

Explanation: Given

Column-I	Column-II
A, $x+1$	p. $27/8$
B, x	q, $-27/8$
C, $x-\frac{1}{2}$	r, 1
D, $5+2x$	s, 0.

A Sol: Given polynomial x^3+3x^2+3x+1 when divided $x+1$

let $x+1=0 \Rightarrow x=-1$

$p(x)=x^3+3x^2+3x+1$

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1 \Rightarrow \cancel{-1}+3(1)+\cancel{3}+\cancel{1}=0$ (s)

B Sol:- Given polynomial x^3+3x^2+3x+1 when divided by x

$$x)\ x^3+3x^2+3x+1\ (x^2+3x+3$$
$$\underline{-x^3}$$
$$3x^2+3x+1$$
$$\underline{-3x^2}$$
$$3x+1$$
$$\underline{-3x}$$
$$1$$

remainder = 1 (r)

C, Sol: Given polynomial x^3+3x^2+3x+1 when divided by $x-\frac{1}{2}$

let $x-\frac{1}{2}=0 \Rightarrow x=\frac{1}{2}$

$p(x)=x^3+3x^2+3x+1$

$P(\frac{1}{2})=(\frac{1}{2})^3+3(\frac{1}{2})^2+3(\frac{1}{2})+1 \Rightarrow \frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$

$\Rightarrow \frac{1+6+12+8}{8}$

$\Rightarrow 27/8 \quad (p)$

D Sol: Given polynomial x^3+3x^2+3x+1 when divided by $5+2a$

let $5+2x=0 \Rightarrow 2x=-5 \Rightarrow x=-5/2$

$p(x)=x^3+3x^2+3x+1$

$P(-\frac{5}{2})=(\frac{-5}{2})^3+3(\frac{-5}{2})^2+3(\frac{-5}{2})+1$

$=-\frac{125}{8}+\frac{3\times 25}{4}-\frac{15}{2}+1$

$=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1 \Rightarrow \frac{-125+150-60+8}{8}$

$=\frac{-185+158}{8}$

$=-27/8 \ (q)$

58 Sol: (c) A-q, B-p, C-s, D-r

Explanation: Given

	Column-I	Column-II
	A. $\frac{(0.337+0.126)^2-(0.337-0.126)^2}{0.337\times 0.126}$	P, 2.60

B. $\dfrac{(2.3)^3 + (0.027)}{(2.3)^2 - 0.69 + 0.09}$ 9, 4

C. $\dfrac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3\times1.5\times4.7\times3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5\times4.7 - 4.7\times3.8 - 3.8\times1.5}$ 8, 10

D. If $\dfrac{2p}{p^2+2p+1} = \dfrac{1}{4}$, then value of $(p+\frac{1}{p})$ is 5, 10.

Asol: Given $\dfrac{(0.337+0.126)^2 - (0.337-0.126)^2}{0.337\times0.126}$

$\Rightarrow \dfrac{(0.337)^2 + (0.126)^2 + 2\times0.337 + 0.126 - (0.337)^2 + (0.126)^2 + 2\times0.337\times0.126}{0.337\times0.126}$

$\Rightarrow \dfrac{4\times(0.337\times0.126)}{(0.337\times0.126)}$ $[\because (a+b)^2 = a^2+b^2+2ab$
$(a-b)^2 = a^2+b^2-2ab$

$\Rightarrow 4$ (9)

B sol: Given $\dfrac{(2.3)^3 + (0.027)^3}{(2.3)^2 - 0.69 + 0.09}$

$\Rightarrow \dfrac{(2.3)^3 + (0.3)^3}{(2.3)^2 - 2.3\times0.3 + (0.3)^2}$ $[\because a^3+b^3 = (a+b)^3 - 3ab(a+b)$
$a^2+b^2-ab = (a+b)^2 - 3ab$

$= \dfrac{(2.3+0.3)^3 - 3\times2.3\times0.3(2.3+0.3)}{(2.3+0.3)^2 - 3\times2.3\times0.3}$

$= \dfrac{(2.3+0.3)\,((2.3+0.3)^2 - \cancel{3\times2.3\times0.3})}{(2.3+0.3)^2 - 3\times2.3\times0.3}$

$\Rightarrow (0.3 + 2.3)$

$= \cancel{2.60}\ 2.60$ (p)

C Sol: Given $\dfrac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5 \times 4.7 - 4.7 \times 3.8 - 3.8 \times 1.5}$

$$\Rightarrow \frac{(1.5 + 4.7 + 3.8)^3 - 3(1.5 \times 4.7 + \cancel{4.7} \times 3.8 + 3.8 \times 1.5)(1.5 + 4.7 + 3.8)}{(1.5 + 4.7 + 3.8)^2 - 3 \cdot (1.5 \times 4.7 + 4.7 \times 3.8 + 3.8 \times 1.5)}$$

$$\Rightarrow \frac{(1.5 + 4.7 + 3.8)[(1.5 + 4.7 + 3.8)^2 - 3(1.5 \times 4.7 + 4.7 \times 3.8 + 3.8 \times 1.5)]}{(1.5 + 4.7 + 3.8)^2 - 3(1.5 \times 4.7 + 4.7 \times 3.8 + 3.8 \times 1.5)]}$$

$\Rightarrow (1.5 + 4.7 + 3.8)$

$\Rightarrow 10$ (s)

D Sol: $\dfrac{2p}{p^2 + 2p + 1} = \dfrac{1}{4}$

$8p = p^2 + 2p + 1$

$p^2 + 1 = 8p + 2p = 10p$

Now, $p + \dfrac{1}{p} = \dfrac{p^2 + 1}{p}$.

$= \dfrac{10\cancel{p}}{\cancel{p}}$

$= 10$ (r)

59 Sol: (a) A - q, B - P, C - S, D - r

Explanation: Given

Column - I	Column - II
A, $a^2+b^2+c^2=2(a-b-c)-3$ then the Value of $4a-3b+5c$	(P) $\frac{7}{8}$
B, If $2x+\frac{2}{x}=3$ then the Value of $x^3+\frac{1}{x^3}+2$ is	(q) 2
C, If $a^3-b^3=56$ and $a-b=2$ then the Value of (a^2+b^2) is	(r) $\frac{2}{3}$
D. If $(a^2+b^2)^3=(a^3+b^3)^2$, then the Value of $(\frac{a}{b}+\frac{b}{a})$.	(S) 20.

Ans: Given $a^2+b^2+c^2=2(a-b-c)-3$.

$$a^2+b^2+c^2=2a-2b-2c-3$$

$$a^2+b^2+c^2-2a+2b+2c+3=0$$

$$a^2-2a+1+b^2+2b+1+c^2+2c+1=0$$

$$(a-1)^2+(b+1)^2+(c+1)^2=0$$

$$(a-1)^2=0 \Rightarrow a-1=0$$
$$a=1$$

$$(b+1)^2=0 \Rightarrow b+1=0 \Rightarrow b=-1$$

$$(c+1)^2=0 \Rightarrow c+1=0 \Rightarrow c=-1$$

Then $4a-3b+5c=4(1)+3(-1)+5(-1)$

$$=4+3-5$$

$$=7-5=2\ (q)$$

B, Sol Given $2x+\frac{2}{x}=3$ Then $x^3+\frac{1}{x^3}+2$

$2(x+\frac{1}{x})=3$

$x+\frac{1}{x}=\frac{3}{2}$

$x^3+\frac{1}{x^3}+2 = (x+\frac{1}{x})^3-3x\times\frac{1}{x}(x+\frac{1}{x})+2$ $\quad [\because a^3+b^3 = (a+b)^3-3ab(a+b)]$

$=(\frac{3}{2})^3-3(\frac{3}{2})+2$

$=\frac{27}{8}-\frac{9}{2}+2$

$=\frac{27-36+16}{8}$

$=\frac{43-36}{8}=\frac{7}{8}$ (P)

C, Sol 2 Given $a^3-b^3=56$, $a-b=2$ Then a^2+b^2?

$(a-b)^3=a^3-b^3-3\times a\times b(a-b)$

$(2)^3 = 56-3ab(2)$

$8 = 56-6ab$

$6ab = 56-8=48$

$ab = 48/6 = 8$

$(a-b)^2 = a^2+b^2-2ab$

$a^2+b^2 = (a-b)^2+2ab$

$=(2)^2+2(8)$

$=4+16$

$=20$ (S)

D8 Sol: Given $(a^2+b^2)^3 = (a^3+b^3)^2$ then $\left(\frac{a}{b}+\frac{b}{a}\right)$

$(a^2)^3+(b^2)^3+3a^2\times b^2(a^2+b^2) = (a^3)^2+(b^3)^2+2a^3b^3$

$a^6+b^6+3a^2b^2(a^2+b^2) = a^6+b^6+2a^3b^3$

$3a^2b^2(a^2+b^2) = 2a^3b^3$

$\frac{\cancel{a^2}\cancel{b^2}(a^2+b^2)}{a^3b^3} = \frac{2}{3}$

$\frac{a^2+b^2}{ab} = \frac{2}{3}$

$\Rightarrow \frac{a^2}{ab}+\frac{b^2}{ab} = \frac{2}{3}$

$\Rightarrow \frac{a}{b}+\frac{b}{a} = \frac{2}{3}$ (r)

Integer Type

G0 Sol: Given kx^3+3x^2-3 and $2x^3-5x+k$ are divided by $x-4$.

Then $p(x) = kx^3+3x^2-3$

$g(x) = 2x^3-5x+k$

$p(x) = g(x)$

$kx^3+3x^2-3 = 2x^3-5x+k$

let $x-4=0 \Rightarrow x=4$

$k(4)^3+3(4)^2-3 = 2(4)^3-5(4)+k$

$64k+3(16)-3 = 2(64)-20+k$

$\Rightarrow$ $64k + 48 - 3 = 128 - 20 + k$

$64k + 45 = 108 + k$

$64k - k = 108 - 45$

$63k = 63$

$k = 63/63 = 1$

61 Sol: Given $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ is divided by $g(x) = x + \frac{2}{3}$

let $g(x) = 0$

$x + \frac{2}{3} = 0 \Rightarrow x = -\frac{2}{3}$

Then $f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^4 + 2\left(-\frac{2}{3}\right)^3 - \frac{\left(-\frac{2}{3}\right)^2}{3} - \frac{\left(-\frac{2}{3}\right)}{9} + \frac{2}{27}$

$= \cancel{3} \times \frac{16}{\cancel{81}_{27}} - 2\left(\frac{8}{27}\right) - \frac{\frac{4}{9}}{3} + \frac{\frac{2}{3}}{9} + \frac{2}{27}$

$= \frac{\cancel{16}}{\cancel{27}} - \frac{\cancel{16}}{\cancel{27}} - \frac{4}{27} + \frac{2}{27} + \frac{2}{27}$

$= -\frac{4}{27} + \frac{2+2}{27}$

$= -\frac{4}{27} + \frac{4}{27} = 0$ //

62 Sol: Given $A = -8x^2 - 6x + 10$, when $x = \frac{1}{2}$

$A = -8x^2 - 6x + 10$

$= -8\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right) + 10$

$= -8\left(\frac{1}{4}\right) - 3 + 10$

$= -2 - 3 + 10 = -5 + 10 = 5$ //

63 Sol:- Given $\frac{1}{2}x^5+3x^4+2x^3+3x^2$ is

The degree of polynomial of 'x' is '5'

The degree of a polynomial is the highest of degree of its ~~variba~~ variable term.

64 Sol:- Given $f(x)$ ax^3+4x^2+3x-4 and x^3-4x+a leave the same remainder when divided by $(x-3)$

$f(x) = ax^3+4x^2+3x-4$, $g(x) = x^3-4x+a$

$f(x) = g(x)$

$ax^3+4x^2+3x-4 = x^3-4x+a$

let $x-3=0 \Rightarrow x=3$.

Then $a(3)^3+4(3)^2+3(3)-4 = (3)^3-4(3)+a$.

$a(27)+4(9)+9-4 = 27-12+a$

$27a+36+5 = 15+a$

$27a+41 = 15+a$

$27a-a = 15-41$

$26a = -26$

$a = -26/26 = -1$

$-a = 1$ //

65. Sol: Given α, β are zero's of $2x^2+3x+10$.

In the form $ax^2+bx+c=0$

Here $a=2, b=3, c=10$,

The product of polynomial $\alpha\beta = \frac{c}{a} = \frac{10}{2} = 5$

$\therefore \alpha\beta = 5$ //

66. Sol: Given $f(x) = x^4-2x^3+3x^2-ax+b$ is divisible by $(x-1)$ and $(x+1)$

If $x-1=0 \Rightarrow x=1$

$f(1) \Rightarrow (1)^4-2(1)^3+3(1)^2-a(1)+b=0$

$\Rightarrow 1-2(1)+3(1)-a+b=0$

$= 4-2-a+b=0$

$\Rightarrow -a+b=-4+2=-2 \longrightarrow$

$\Rightarrow -a+b=-2 \longrightarrow (1)$

If $x+1=0 \Rightarrow x=-1$

$f(-1) \Rightarrow (-1)^4-2(-1)^3+3(-1)^2-a(-1)+b=0$

$1-2(-1)+3(1)+a+b=0$

$\Rightarrow 1+2+3+a+b=0$

$a+b=-6 \longrightarrow (2)$

(1)+(2) $\Rightarrow$

$$\begin{aligned} -\cancel{a}+b &= -2 \\ \cancel{a}+b &= -6 \\ \hline 2b &= -8 \end{aligned} \Rightarrow b = -8/2 = -4$$

Substitute b in eq's (2)

$a+b=-6$

$a-4=-6$

$a=-6+4=-2$

$\therefore a-b=-2-(-4)=-2+4=2$

57 Sol: Given HCF of $(x^2-xy+2y^2)$ and $(2x^2-xy-y^2)$

(i) $x^2-xy+2y^2 = x^2-2xy+xy+2y^2$

$= x(x-2y)+y(x-2y)$

$= (x-2y)(x+y)$

(ii) $2x^2-xy+y^2 = 2x^2-2xy+xy+y^2$

$= 2x(x-y)+y(x-y)$

$= (x-y)(2x+y)$

There is no common factors

$\therefore$ HCF $= 1$

68 Sol: Given $2x^3+ax^2+3x-5$ and x^3+x^2-2x+a are divided by $(x-2)$, the same remainder are obtained

$f(x) = 2x^3+ax^2+3x-5$, $g(x)=x^3+x^2-2x+a$

$f(x) = g(x)$

$2x^3+ax^2+3x-5 = x^3+x^2-2x+a$

If $x-2=0 \Rightarrow x=2$

$2(2)^3+a(2)^2+3(2)-5 = (2)^3+(2)^2-2(2)+a$

$2(8)+a(4)+6-5 = 8+4-4+a$

$16+4a+1 = 8+a$

$4a-1 = 8-16-1$

$3a = -\cancel{8}9$

$a = -9/3 = -3$

$\therefore -a = 3$

69 Sol: Given $a+b+c=0$, then $\frac{a^2+b^2+c^2}{a^2-bc}$

$a+b+c=0$

$b+c=-a$

Squaring on both sides

$(b+c)^2 = (-a)^2$

$b^2+c^2+2bc = a^2$

$b^2+c^2 = a^2-2bc$

$\frac{a^2+b^2+c^2}{a^2-bc} = \frac{a^2+a^2-2bc}{a^2-bc}$

$= \frac{2a^2-2bc}{a^2-bc}$

$= \frac{2(a^2-bc)}{(a^2-bc)} = 2$ //

70 Sol: Given $x = 1-\sqrt{2}$ then $\left(x - \frac{1}{x}\right)^3$

$$\frac{1}{x} = \frac{1}{1-\sqrt{2}}$$

$$= \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{(1)^2-(\sqrt{2})^2}$$

$$\frac{1}{x} = \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1} = -(1+\sqrt{2}).$$

$$\left(x-\frac{1}{x}\right)^3 = (1-\sqrt{2}-(-(1+\sqrt{2}))]^3$$

$$= (1-\cancel{\sqrt{2}}+1+\cancel{\sqrt{2}})^3$$

$$= (2)^3 = 8$$

NTSE Questions (Previous Years)

1 Sol: (c) 42.

Explanation: Given α, β, γ are the zero's of ~~H~~ ~~as~~ of the polynomial

$$p(x) = x^3 - 64x - 14.$$

$p(x)$ is in the form $ax^3 + bx^2 + cx + d$.

Here $a = 1$, $b = 0$, $c = -64$, $d = -14$

Then $\alpha+\beta+\gamma = \frac{-b}{a} = \frac{-0}{1} = 0$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = \frac{-64}{1} = -64$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(-14)}{1} = \frac{14}{1} = 14$$

$$\alpha^3+\beta^3+\gamma^3 = (\alpha+\beta+\gamma)^3 - 3(\alpha\beta+\beta\gamma+\gamma\alpha)(\alpha+\beta+\gamma)+3\alpha\beta\gamma$$

$[\because a^3+b^3+c^3 = (a+b+c)^3 - 3(ab+bc+ca)(a+b+c)+3abc]$

$= (0)^3 - 3(-64)(0) + 3(14)$

$= 0+0+42$

$= 42$ //

2 Sol: Given $p(x) = x^{200} - 2x^{199} + x^{50} - 2x^{49} + x^2 + x + 1$ is divided by $(x-1)(x-2)$

Let $f(x) = ax+b$ be remainder when $p(x)$ is divided by $(x-1)(x-2)$ and $g(x)$ be Quotient

$p(x) = (x-1)(x-2)g(x) + (ax+b)$ [$\because$ using division algorith dividend = Divisor × Quotient + Remainder]

When $x = 1$

$x^{200} - 2x^{199} + x^{50} - 2x^{49} + x^2 + x + 1 = (x-1)(x-2)g(x) + (ax+b)$

$(1)^{200} - 2(1)^{199} + (1)^{50} - 2(1)^{49} + (1)^2 + 1 + 1 = (1-1)(1-2)g(1) + (a(1)+b)$

$1 - 2 + 1 - 2 + 1 + 1 + 1 = a+b$

$5 - 4 = a+b$

$1 = a+b \longrightarrow (1)$

When $x = 2$

$(2)^{200} - 2(2)^{199} + (2)^{50} - 2(2)^{49} + (2)^2 + 2 + 1 = (2-1)(2-2)g(2) + (a(2)+b)$

$2^{200} + 2^{200} + 2^{150} - 2^{70} + 2^2 + 2 + 1 = 2a + b$

$4 + 2 + 1 = 2a + b$

$7 = 2a + b \longrightarrow (2)$

$(1) - (2) \Rightarrow$ $a + \cancel{b} = 1.$

$$\begin{array}{r} 2a + b = 7 \\ \hline -a = -6 \end{array}$$

$a = 6.$

Substitute 'a' in eq's (1)

$a + b = 1$

$6 + b = 1$

$b = 1 - 6 = -5$

$\therefore$ The Remainder $ax + b = 6x - 5$,, (d) $6x - 5$

38 Sol:- Given, $81a$ (a) $x^2 - 13ax + 31a^2$

Given $81a^4 + (x-2a)(x-5a)(x-8a)(x-11a)$

$\Rightarrow (x-2a)(x-11a)(x-5a)(x-8a) + 81a^4$

$\Rightarrow (x^2 - 11ax - 2ax + 22a^2)(x^2 - 8ax - 5ax + 40a^2) + 81a^4$

$\Rightarrow (x^2 - 13ax + 22a^2)(x^2 - 13ax + 40a^2) + 81a^4$

let $x^2 - 13ax = t$ assume

$\Rightarrow (t + 22a^2)(t + 40a^2) + 81a^4$

$\Rightarrow (t+22a^2)(t+40a^2)+81a^4$

$\Rightarrow (t^2+40a^2t+22a^2t+880a^4)+81a^4$

$\Rightarrow (t^2+62a^2t+880a^4+81a^4$

$\Rightarrow t^2+62a^2t+961a^4$

$\Rightarrow (t+31a^2)^2$

$\Rightarrow (x^2-13ax+31a^2)^2$

One of the facto of polynomial is $(x^2-13ax+31a^2)$

40 Sol^n (b) 2015

Explanation: Given $\dfrac{(2014^2-2020)(2014^2+4028-3)(2015)}{(2011)(2013)(2016)(2017)}$

$$\Rightarrow \frac{(2014^2-2016-4)(2014^2+2\times2014\times1+1-4)(2015)}{(2011)(2013)(2016)(2017)}$$

$$\Rightarrow \frac{((2014^2-2^2)-2016)\left(\cdot(2014+1)^2-4\right)](2015)}{(2011)(2013)(2016)(2017)}$$

$$\Rightarrow \frac{((2014+2)(2014-2)-2016)\left((2015)^2-2^2\right)(2015)}{(2011)(2013)(2016)(2017)}$$

$$\Rightarrow \frac{[(2016)(2012)-2016]\left((2015+2)(2015-2)\right](2015)}{(2011)(2013)(2016)(2017)}$$

$$\Rightarrow \frac{\cancel{2016}(2012-1)\cdot\cancel{(2017)}\cancel{(2013)}(2015)}{(2011)\cancel{(2013)}\cancel{(2016)}\cancel{(2017)}}$$

$$\Rightarrow \frac{\cancel{(2011)}(2015)}{\cancel{(2011)}}$$

$\Rightarrow 2015$ //

5 Sol: (d) 4

Explanation: Given $p(x) = x^2 + bx + c$.

$p(x)$ is factor of both x^4+6x^2+25 and $3x^4+4x^2+28x+5$.

$p(x)$ is HCF of the both equations.

Then $3x^4+4x^2+28x+5 \div x^4+6x^2+25 \Rightarrow$ Remainder = HCF

$$x^4+6x^2+25 \,)\, 3x^4+4x^2+28x+5 \,(\, 3$$

$$3x^4+18x^2+75$$

$$-14x^2+28x-70$$

$$-14[x^2+2x+\cancel{10}] = 0$$

$$x^2-2x+\cancel{10}\,5$$

$\therefore p(x) = x^2-2x+\cancel{10}\,5$

$p(1) = 1^2-2(1)+\cancel{10}\,5$

$= 6-2 = 4.$ //

6 Sol: (a) −8.

Explanation: Given $(1+2x-x^2)^4$

General term given expansion

$T_{r+1} = \sum_{r=0}^{4} {}^4C_r (x(2-x))^r$

When we put r=4 We get term having x^7

$T_{3+1} = {}^4C_4 x^4 (2-x)^4$

We can written as

$T_4 = x^4 \sum_{r=0}^{4} {}^4C_r (2)^{4-r} x^r (-1)^r$

Here on putting r=3

$T_4 = {}^4C_3 (2)^1 x^3 (-1)^3 x^4$

$= -2\ {}^4C_3 x^7$

$x^7 = -2\ {}^4C_3 = -2 \cdot \frac{4!}{1!\,3!} = -2 \frac{4\times 3\times 2\times 1}{1\times 3\times 2\times 1} \Rightarrow -2\times 4 = -8$

7 Sol: (d) $(x+65)(x-50)$

Explanation:- Given $x^2+15x-3250$

$\Rightarrow x^2 - 50x + 65x - 3250$

$\Rightarrow x(x-50) + 65(x-50)$

$\Rightarrow (x-50)(x+65)$

8 Sol: Given $\frac{(0.44)^2+(0.06)^2+(0.024)^2}{(0.044)^2+(0.006)^2+(0.0024)^2}$

$\Rightarrow \frac{(0.44)^2+(0.06)^2+(0.0024)^2}{(0.1\times 0.44)^2+(0.1\times 0.06)^2+(0.1\times 0.024)^2}$

$$\Rightarrow \frac{(0.44)^2+(0.06)^2+(0.024)}{(0.1)^2\left[(0.44)^2+(0.06)^2+(0.024)^2\right]}$$

$$\Rightarrow \frac{1}{(0.1)^2} \Rightarrow \frac{1}{0.01}$$

$\Rightarrow 100$

Ans: c, 100

9. Sol: c, $1-a+b$.

Explanation: Given one of the zero's of the cubic polynomial

x^3+ax^2+bx+c is -1

Let α, β, γ are zero of the given polynomial $p(x)$

$\therefore \alpha = -1$, and $p(-1) = 0$

$p(-1) = (-1)^3+a(-1)^2+b(-1)+c = 0$

$\Rightarrow 1 + a(1) - b + c = 0$

$\Rightarrow c = 1 - a + b$.

We know the product of all zero's $= \frac{(-1)^3 \text{ Constant term}}{x^3 \text{ coefficient of } x^2}$

$\alpha\beta\gamma = \frac{(-1)c}{1}$

$(-1)\beta\gamma \Rightarrow -c$

$-\beta\gamma = -c$

$\therefore \alpha\beta\gamma = C$

$\therefore \alpha\beta\gamma = 1-a+b$

$\therefore$ The product of other two zero's $= 1-a+b$ //

10. Sol: (b) 2

Explanation: Given $a^2+b^2+2c^2-4a+2c-2bc+5$

$a^2+b^2+c^2+c^2-4a+2c-2bc+1+4=0$

$a^2-4a+4+b^2-2bc+c^2+c^2+2c+1=0$

$(a-2)^2+(b-c)^2+(c+1)^2=0$

$(a-2)^2=0$, $(b-c)^2=0$, $(c+1)^2=0$

$a-2=0$ $b-c=0$ $c+1=0$

$a=2$ $b=c$ $c=-1$

$b=-1$

$\therefore a+b-c = 2-1-(-1) = 2-1+1 = 3-1 = 2$ //

11. Sol: (c) 373

Explanation: Given $\dfrac{(10^4+324)(22^4+324)(34^4+324)(46^4+324)(58^4+324)}{(4^4+324)(16^4+324)(28^4+324)(40^4+324)(52^4+324)}$

let $x^4+324 = [(x-3)^2+9][(x+3)^2+9]$

$$\Rightarrow \frac{((10-3)^2+9)[(10+3)^2+9][(22-3)^2+9][(22+3)^2+9]((34-3)^2+9)[(34+3)^2+9][(46-3)^2+9][(46+3)^2+9][(58-3)^2+9][(58+3)^2+9]}{[(4-3)^2+9][(4+3)^2+9][(16-3)^2+9][(16+3)^2+9][(28-3)^2+9][(28+3)^2+9][(40-3)^2+9][(40+3)^2+9][(52-3)^2+9][(52+3)^2+9]}$$

$$\Rightarrow \frac{\cancel{(7^2+9)}\cancel{(13^2+9)}\cancel{(19^2+9)}\cancel{(25^2+9)}\cancel{(31^2+9)}\cancel{(37^2+9)}\cancel{(43^2+9)}\cancel{(49^2+9)}\cancel{(55^2+9)}(61^2+9)}{(1^2+9)\cancel{(7^2+9)}\cancel{(13^2+9)}\cancel{(19^2+9)}\cancel{(25^2+9)}\cancel{(31^2+9)}\cancel{(37^2+9)}\cancel{(43^2+9)}\cancel{(49^2+9)}\cancel{(55^2+9)}}$$

$$\Rightarrow \frac{61^2+9}{1^2+9} \Rightarrow \frac{3721+9}{10}$$

$$\Rightarrow \frac{3730}{10} = 373 \;//$$

12 Sol: (b) 0

Explanation: Given $xy+yz+zx=0$ — (1)

from (1) $\Rightarrow$ $xy = -(yz+zx)$

$yz = -(xy+zx)$

$zx = -(xy+yz)$

Then $\frac{1}{x^2-yz} + \frac{1}{y^2-zx} + \frac{1}{z^2-xy}$

$$\Rightarrow \frac{1}{x^2-(-(xy+zx))} + \frac{1}{y^2-(-(xy+yz))} + \frac{1}{z^2-(-(yz+zx))}$$

$$\Rightarrow \frac{1}{x^2+xy+zx} + \frac{1}{y^2+xy+yz} + \frac{1}{z^2+yz+zx}$$

$$\Rightarrow \frac{1}{x(x+y+z)} + \frac{1}{y(y+x+z)} + \frac{1}{z(z+y+x)}$$

$$\Rightarrow \frac{1}{x+y+z}\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right]$$

$$\Rightarrow \frac{1}{x+y+z}\left[\frac{yz+zx+xy}{xyz}\right]$$

$$\Rightarrow \frac{1}{x+y+z}\left[\frac{0}{xyz}\right]$$

$$\Rightarrow \frac{1}{x+y+z}(0)$$

$$= 0 \text{ //}$$