Contents

Contents

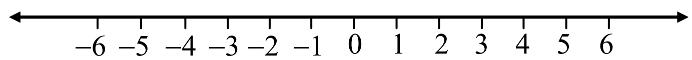
CONTENTS

CONTENTS

CONTENTS

1 Number System

INTRODUCTION



Natural Numbers (N)

Counting numbers are known as natural numbers. N = {1, 2, 3, 4, ……….}

Whole Numbers (W)

All natural numbers together with 0 form whole numbers. W = {0, 1, 2, 3, 4, ……….}

Integers (I or Z)

All natural numbers, 0 and the negatives of natural numbers form the collection of integers.

$$Z = \{-\infty, \ldots\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\ldots\infty\}$$

Rational Numbers (Q)

The number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ and $\frac{p}{q}$ is in the lowest form i.e., p and q have no common factor, is called a **rational number.**

i.e., All integers and fractions are rational numbers. The set of rational numbers is denoted by **Q**. A rational numbers is said to be positive if its numerator and denominator are either both positive or both negative.

E.g. 2/3, 0, –1, –3/4 etc.

IRRATIONAL NUMBERS, REAL NUMBERS AND THEIR DECIMAL EXPANSIONS

Irrational Numbers

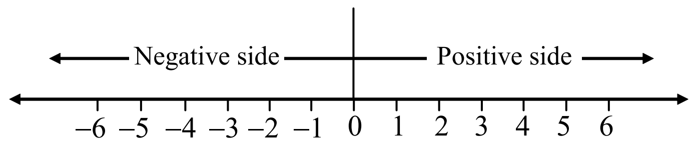
The number which cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ and $\frac{p}{q}$ is in the lowest form i.e., p and q have no common factor, is called an **Irrational Number.**

E.g. $\sqrt{2}, \sqrt{3}, \pi, \sqrt{5}, 1.02002000200002......$ etc.

Note: Decimal representation is non-terminating and non-repeating.

Real Numbers

The collection of rational and irrational numbers are called real numbers and is denoted by R.



A real number can be represented as a point on a line. This line is called real line or number line.

Note: (i) Terminating Decimal : Every fraction p/q can be expressed as a decimal. If the decimal expression of p/q terminates i.e. comes to an end then the decimal so obtained is called ***terminating decimal***.

(ii) A decimal number in which a digit or a set of digits repeats regularly, over a constant period is called a recurring decimal or ***periodic decimal***.

Example: 2.33….. = $2.\overline{3}$; 8.6232323…….. = $8.6\overline{23}$

(iii) A decimal fraction in which all the figure or digits occur repeatedly is called ***pure recurring decimal***.

Example: 2.3333... , 17.444... etc.

(iv) A decimal number in which some of the digits do not recur is called a ***mixed recurring***.

(v) A decimal number in which there is no any regular pattern of repetition of digits after decimal point is called a ***non-recurring decimal.***

Example: 3.143678..., 2.767834... etc.

(vi) A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an ***irrational number.***

Example: $\sqrt{2}$ = 1.4142135 ... , $\sqrt{3}$ = 1.7320508 ... .

(vii) If a + $\sqrt{b}$ is an irrational number then a - $\sqrt{b}$ is called its conjugate irrational number or ***conjugate.***

OPERATION ON IRRATIONAL NUMBERS

(i) $\sqrt[n]{x}.\sqrt[n]{y} = \sqrt[n]{xy} \text{ or } (xy)^{1/n}$

(ii) $\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}} \text{ or } \left(\frac{x}{y}\right)^{1/n}$

(iii) $\sqrt[n]{x^m} = x^{m/n}$

(iv) $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} \Rightarrow \sqrt[n]{\sqrt[m]{x}} \text{ or } \left(x^{1/mn}\right)$

(v) $\left(x^a\right)^{1/m} = x^{a/m} \Rightarrow x^{an/mn} \Rightarrow \left(x^{an}\right)^{1/mn}$

LAWS OF EXPONENTS FOR REAL NUMBERS

(i) $a^m \cdot a^n = a^{m+n}$	(v) $a^0 = 1$
(ii) $(a^m)^n = a^{mn}$	(vi) $\frac{1}{a^n} = a^{-n}$
(iii) $\frac{a^m}{a^n} = a^{m-n}, m > n$	(vii) $\sqrt[n]{a} = a^{1/n}$
(iv) $a^m.b^m = (ab)^m$	

EUCLID'S DIVISION LEMMA AND ALGORITHM

Euclid's Division Lemma

For any two given positive integers 'a' and 'b', there exist unique integers 'q' and 'r' such that $a = bq + r$, $0 \leq r < b$. 'a' and 'b' are called dividend and divisor respectively, 'q' and 'r' are called quotient and remainder respectively. Thus, dividend = (divisor × quotient) + remainder.

Remark: (i) The above lemma is a restatement of long division process

(ii) 'q' or 'r' can also be zero.

H.C.F. of two numbers a and b by Euclid's Division Algorithm

Step 1: Apply the Algorithm to a and b.
$a = bq + r_1 \quad (0 \leq r_1 < b)$

Step 2: If $r_1 = 0$, then b is the HCF of a and b.

Step 3: If $r_1 \neq 0$, then apply the Algorithm to divisor b and remainder r_1.
$b = r_1 s + r_2$

Step 4: Continue the process till the remainder become zero.
Here the last divisor will be the HCF of a and b.

THE FUNDAMENTAL THEOREM OF ARITHMETIC

Theorem : Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

REVISITING RATIONAL, IRRATIONAL NUMBERS AND THEIR DECIMAL EXPANSION

Theorem 1: Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

Theorem 2 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

SOLVED EXAMPLES

Example 1: Express $0.4\overline{7}$ is the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

Solution: Let x = $0.4\overline{7}$

$\therefore$ x = 0.4777 ... (1)

Multiplied (1) by 10

$\therefore$ 10x = 4.777... (2)

Now (2) multiplied by 10 again

$\therefore$ 100x = 47.777 .. (3)

Subtract (2) from (3)

90x = 43

$\therefore x = \frac{43}{90} \therefore \quad 0.4\overline{7} = \frac{43}{90}$.

Example 2: Prove that $\sqrt{2}$ is not a rational numbers.

Solution: We shall prove this by the method of contradiction.

If possible, let us assume that $\sqrt{2}$ is a rational number, then

$\sqrt{2} = \frac{p}{q}$, where p and q are integers having no common factor other than 1 and $q \neq 0$.

$\Rightarrow 2 = \frac{p^2}{q^2}$ (squaring both the side)$\Rightarrow 2q^2 = p^2$ ... (1)

$\Rightarrow p^2$ is even integer

$\therefore$ p is an even integer

(If p is not even. Then $p = 2m + 1$, $m \in Z$, $p^2 = (2m + 1)^2 = 4m^2 + 4m + 1$, which is odd, thus is a contradiction of the fact that p^2 is even)

$\therefore$ p = 2m where m is an integer $\Rightarrow p^2 = 4m^2$ .... (2)

from (1) and (2), $2q^2 = 4m^2$

$\Rightarrow q^2 = 2m^2$

$\therefore q^2$ is an even integer $\Rightarrow$ q is an even integer

So both p and q are even integers and therefore have a common factor 2. But this contradicts the fact that p and q have no common factors other than 1.

Hence, $\sqrt{2}$ is not a rational number, it is irrational.

Example 3: Simplify: $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$.

Solution:

$$\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}} = \frac{2^4 \times 2^{n+1} - 2^2 \times 2^n}{2^4 \times 2^{n+2} - 2 \times 2^{n+2}} = \frac{2^{n+5} - 2^{n+2}}{2^{n+6} - 2^{n+3}}$$

$$= \frac{2^{n+5} - 2^{n+2}}{2.2^{n+5} - 2.2^{n+2}} = \frac{1}{2}.$$

Example 4: Write the rationalizing factor of the following and then rationalize:

(a) $\sqrt{27}$ (b) $\sqrt[3]{16}$

Solution (a) $\sqrt{27} = 3\sqrt{3}$

$\sqrt{3} = 3^{1/2}$

Rationalizing factor $= 3^{1-\frac{1}{2}} = 3^{1/2} = \sqrt{3}$

$\therefore \quad 3\sqrt{3} \times \sqrt{3} = 3\sqrt{3^2} = 3 \times 3 = 9,$ which is a rational number.

(b) $\sqrt[3]{16} = \sqrt[3]{2 \times 2 \times 2 \times 2} = 2\sqrt[3]{2} = 2.2^{1/3}$

$\sqrt[3]{2} = 2^{1/3}$

Rationalizing factor of $\sqrt[3]{2} = 2^{1-\frac{1}{3}} = 2^{2/3}$

Now, $2.2^{1/3} \times 2^{2/3} = 2 \times 2 = 4$, which is a rational number.

Hence, $2^{2/3} = \left(2^2\right)^{1/3} = \sqrt[3]{4}$ which is the rationalizing factor.

Example 5: Prove that : $\dfrac{1}{3-\sqrt{8}} - \dfrac{1}{\sqrt{8}-\sqrt{7}} + \dfrac{1}{\sqrt{7}-\sqrt{6}} - \dfrac{1}{\sqrt{6}-\sqrt{5}} + \dfrac{1}{\sqrt{5}-2} = 5$

Solution: Rationalizing the denominator of each term on the left hand side, we have

$$\text{L.H.S.} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$

$$- \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{3+\sqrt{8}}{3^2-\left(\sqrt{8}\right)^2} - \frac{\sqrt{8}+\sqrt{7}}{\left(\sqrt{8}\right)^2-\left(\sqrt{7}\right)^2} + \frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2} - \frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2} + \frac{\sqrt{5}+2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$

$$= \left(3+\sqrt{8}\right) - \left(\sqrt{8}+\sqrt{7}\right) + \left(\sqrt{7}+\sqrt{6}\right) - \left(\sqrt{6}+\sqrt{5}\right) + \left(\sqrt{5}+2\right)$$

$$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2 = 3+2 = 5$$

IMPORTANT DEFINITIONS

- A number is called a rational number, if it can be written in the form p/q, where p and q are integers and $q \neq 0$.
- A number which cannot be expressed in the form p/q (where, p and q are integers and $q \neq 0$) is called an irrational number.
- All rational numbers and all irrational numbers together make the collection of real numbers.
- Decimal expansion of a rational number is either terminating or non-terminating recurring, while the decimal expansion of an irrational number is non-terminating non-recurring.
- If r is a rational number and s is an irrational number, then $r + s$ and $r - s$ are irrationals. Further, if r is a non-zero rational, then rs and $\dfrac{r}{s}$ are irrationals.

REVISION EXERCISE

1. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(a) 1.5 (b) $\frac{\sqrt{2}-\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}\sqrt{3}}{2}$ (d) 1.8

2. Which of the following is irrational?

(a) 0.14 (b) $0.14\overline{16}$

(c) $0.\overline{1416}$ (d) 0.4014001400014…..

3. If $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7\sqrt{5}}{11}$, then $a =$

(a) 0 (b) 1 (c) 2 (d) 3

4. What is the value of $\left(\frac{\sqrt{2}}{5}\right)^{8} \div \left(\frac{\sqrt{2}}{5}\right)^{13}$

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) $\left(\frac{5}{\sqrt{2}}\right)^{5}$

5. Which of the following will change into a terminating decimal?

(a) $\frac{7}{12}$ (b) $\frac{5}{44}$ (c) $\frac{13}{125}$ (d) $\frac{2}{9}$

6. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find the value of $x^2 + y^2$

(a) 96 (b) 98 (c) 10 (d) $2\sqrt{2}$

7. Which of the following is equal to x ?

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{1/3}}$ (c) $(\sqrt{x^3})^{\frac{2}{3}}$ (d) $x^{\frac{12}{7}} x^{\frac{7}{12}}$

8. If $a = 5 + 2\sqrt{6}$ and $b = \frac{1}{a}$, then $a^2 + b^2 =$

(a) 49 (b) 98 (c) 100 (d) 102

9. $9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} =$

(a) 15 (b) 0 (c) 12 (d) 1

10. If $2\sqrt[3]{189} + 3\sqrt[3]{875} - 7\sqrt[3]{56}$ is simplified, then the resultant answer is

(a) $8\sqrt[3]{7}$ (b) $6\sqrt[3]{7}$ (c) $7\sqrt[3]{7}$ (d) $9\sqrt[3]{7}$

EXERCISE (Single Correct Type)

LEVEL - I

1. The product of any two irrational numbers is:
   (a) always an irrational number (b) always a rational number
   (c) always an integer (d) sometimes rational, sometimes irrational

2. The decimal expansion of the number $\sqrt{2}$ is:
   (a) a finite decimal (b) 1.41421
   (c) non-terminating recurring (d) non-terminating recurring

3. Which of the following is irrational?
   (a) $\sqrt{\frac{4}{9}}$ (b) $\frac{\sqrt{12}}{\sqrt{3}}$ (c) $\sqrt{7}$ (d) $\sqrt{81}$

4. The value of $23.\overline{43}$ is in the form of $\frac{p}{q}$, is
   (a) $\frac{232}{99}$ (b) $\frac{2320}{99}$ (c) $\frac{232}{9}$ (d) $\frac{2320}{9}$

5. The value of 1.999... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is
   (a) $\frac{19}{10}$ (b) $\frac{1999}{1000}$ (c) 2 (d) $\frac{1}{9}$

6. The number obtained on rationalising the denominator $\frac{1}{\sqrt{7}-2}$ is
   (a) $\frac{\sqrt{7}+2}{3}$ (b) $\frac{\sqrt{7}-2}{3}$ (c) $\frac{\sqrt{7}+2}{5}$ (d) $\frac{\sqrt{7}+2}{45}$

7. $\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to
   (a) $\frac{1}{2}\left(3-2\sqrt{2}\right)$ (b) $\frac{1}{3+2\sqrt{2}}$ (c) $3-2\sqrt{2}$ (d) $3+2\sqrt{2}$

8. The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to
   (a) $\sqrt{2}$ (b) 2 (c) 4 (d) 8

9. If $\sqrt{2}=1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to
   (a) 2.4142 (b) 5.8282 (c) 0.4142 (d) 0.1718

10. $\sqrt[4]{\sqrt[2]{2^2}}$ equals
   (a) $2^{-\frac{1}{6}}$ (b) 2^{-6} (c) $2^{\frac{1}{4}}$ (d) 2^6

11. The product $\sqrt[3]{2} \cdot \sqrt[4]{2} \cdot \sqrt[12]{32}$ equals

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt[12]{2}$ (d) $\sqrt[12]{32}$

12. If $xy^2 = a^3$, $yz^2 = b^3$ and $zx^2 = c^3$ then z^3 equals :

(a) $\frac{bc^4}{a^2}$ (b) $\frac{b^4c}{a^2}$ (c) $\frac{b^4c^4}{a^2}$ (d) $\frac{ab^4}{c^2}$

13. If $3^{2x^2-9x} = 81^{-1}$, then x is:

(a) $4, \frac{1}{2}$ (b) $2, \frac{1}{4}$ (c) 2, 1 (d) Both (a) and (b)

14. If $A = \sqrt{7} - \sqrt{6}$ and $B = \sqrt{6} - \sqrt{5}$, then

(a) $A > B$ (b) $A = B$ (c) $A < B$ (d) $A \geq B$

15. The value of $\frac{1}{\sqrt{5-2\sqrt{6}}}$ is

(a) 2 (b) –1 (c) $\sqrt{3}+\sqrt{2}$ (d) 1

16. The value of expression $\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1} \cdot \left(\frac{3}{2}\right)^3 + \left(\frac{-1}{3}\right)^{-1}}$ is

(a) $\frac{-3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{9}{4}$

17. The decimal expansion of the rational number $\frac{33}{2^2.5}$ will terminate after

(a) one decimal place (b) two decimal place
(c) three decimal place (d) more than 3 decimal places

18. For some integer *m*, every even integer is of the form

(a) m (b) $m + 1$ (c) $2m$ (d) $2m + 1$

19. For some integer *q*, every odd integer is of the form

(a) q (b) $q + 1$ (c) $2q$ (d) $2q + 1$

20. $n^2 - 1$ is divisible by 8, if n is

(a) an integer (b) a natural number (c) an odd integer (d) an even integer

21. The product of a non-zero rational and an irrational number is

(a) always irrational (b) always rational (c) rational or irrational (d) none

22 If $\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$ then a is

(a) –2 (b) +2 (c) 3 (d) –3

23. Value of $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^{3}\right]^{\frac{1}{4}}$ is

(a) 2 (b) 5 (c) 3 (d) 4

24. Value of $(256)^{-\left(4^{\frac{-3}{2}}\right)}$ is

(a) $\frac{1}{2}$ (b) 2 (c) 3 (d) $\frac{1}{3}$

25. Value of $(1^3+2^3+3^3)^{\frac{1}{2}}$ is

(a) 2 (b) 3 (c) 5 (d) 6

LEVEL - II

26. The value of $\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}$ is

(a) $\frac{b^2}{b^2-a^2}$ (b) $\frac{a^2}{2b^2-a^2}$ (c) $\frac{2a^2}{b^2-a^2}$ (d) $\frac{2b^2}{b^2-a^2}$

27. If $x=\frac{1}{2-\sqrt{3}}$, then the value of x^3-2x^2-7x+5 is

(a) 1 (b) 2 (c) 3 (d) 4

28. If $a=\frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $b=\frac{2+\sqrt{5}}{2-\sqrt{5}}$, then the value of a^2-b^2 is

(a) $144\sqrt{5}$ (b) $-144\sqrt{5}$ (c) $\sqrt{5}$ (d) 144

29. The value of $\frac{7^{2n+3}-(49)^{n+2}}{((343)^{n+1})^{2/3}}$ is equal to

(a) –39 (b) –42 (c) –44 (d) –45

30. Value of $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$

(a) 2 (b) 3 (c) 4 (d) 5

31. Find the value of

$\frac{160}{2\times 7}+\frac{160}{7\times 12}+\frac{160}{17\times 12}+\frac{160}{17\times 22}+\frac{160}{22\times 27}+\frac{160}{27\times 32}$

(a) 17 (b) 15 (c) 13 (d) 11

32. Radhica has two ribbons which are 80 cm and 96 cm long. She has to cut them into equal pieces to make bows. What is the length of the longest piece of ribbon that she can cut, so that no ribbon is wasted?

(a) 16 (b) 15 (c) 14 (d) 13

33. In a problem involving division, the divisor is eight times the quotient and four times the remainder. If the remainder be 12, then the dividend is

(a) 400 (b) 342 (c) 300 (d) 450

34. If $1^3 + 2^3 + 3^3 + ... + 10^3 = 3025$ then 4 + 32 + 108 + … + 4000 is equal to

(a) 1200 (b) 12100 (c) 12200 (d) 12400

35. If $x = \sqrt[3]{4} + \sqrt[3]{2} + 1$, then the value of $\frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}$ is

(a) 1 (b) 0 (c) −1 (d) $3\sqrt[3]{3}$

36. If $60^a = 3$ and $60^b = 5$, then the value of $12^{\frac{1-a-b}{2(1-b)}}$ is

(a) $\sqrt{60}$ (b) $\sqrt{3}$ (c) 2 (d) 5

37. If $x = \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$, then the value of $\left(x^2 - 12x + 1\right)^{2017}$ is

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{8}$

38. If N = 1 + 11 + 111 + ….. + 1111111111

The sum of digits of N is in the form of ab. Average of a and b is

(a) 3 (b) 4 (c) 6 (d) 5

39. The value of $\sqrt[3]{20+14\sqrt{2}} + \sqrt[3]{20-14\sqrt{2}}$ is

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) 4 (d) −4

40. $(\text{Odd}^{\text{even}} + \text{even}^{\text{odd}})$ + even is always:

(a) either even or odd (b) even (c) odd (d) None of these

41. X, Y, Z are distinct integers, X and Y are even and ‘Z’ is odd. Also ‘Y’ is a negative number and X and Z are positive: Which of these is not true?

(a) $(X - Y)^3Z^2$ is even (b) $(X + Z)^2Y$ is negative and even

(c) $(X + Y + Z)^3Y^3$ is odd (d) $(X - Z)^2Y^2$ is even

42. If $a^x = b$, $b^y = c$, $c^z = a$, then find the value of xyz

(a) 0 (b) 1 (c) 2 (d) None of these

43. If N is a natural number, then how many values of ‘N’ are possible such that $\frac{(15N^2 + 13N + 36)}{N}$ is also a natural number

(a) 8 (b) 9 (c) 10 (d) 6

44. Find the values of x and y such that the number 56129137X51Y is divisible by 88

(a) 2, 2 (b) 4, 2 (c) 4, 4 (d) none of these

45. Find the value of $(a^{\frac{1}{2}}+b^{\frac{1}{2}})(a^{\frac{1}{2}}-b^{\frac{1}{2}})+(a^{\frac{1}{3}}+b^{\frac{1}{3}})(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}})$

(a) 1 (b) a (c) 2a (d) b

46. Find the value of $\frac{4^{-\frac{2}{3}}\times\sqrt[3]{16}\times x^{-\frac{9}{2}}\times x^{3}}{9\times16^{-1}\times\sqrt{x^{-3}}\times2^{4}\times3^{-2}}$

(a) 1 (b) 2 (c) 3 (d) None of these

47. What is the remainder when 7^{84} is divided by 2402?

(a) 1 (b) 6 (c) 2401 (d) None of these

48. What is the remainder in $\frac{2^{643}}{96}$?

(a) 32 (b) 95 (c) 1 (d) None of these

49. What are the last two digits of x,
if $x=(11^3+12^3+13^3+14^3)(57^3-55^3)$

(a) 00 (b) 30 (c) 40 (d) 55

50. Find the remainder if 30^{40} is divided by 17

(a) 16 (b) 1 (c) 4 (d) 17

51. x = remainder when (1! + 2! + 3! + … 100!) is divided by 15. Find x

(a) 3 (b) 4 (c) 5 (d) 9

52. What is the remainder when $74^{13}-41^{13}+75^{13}-42^{13}$ is divided by 66?

(a) 2 (b) 0 (c) 1 (d) 65

53. What is the remainder when $82^{361}+83^{361}+84^{361}+85^{361}+86^{361}$ is divided by 7?

(a) 2 (b) 0 (c) 5 (d) None of these

54. $33^{24}-17^{24}$ is atleast divisible by

(a) 450 (b) 800 (c) 40,000 (d) 250

55. Find the unit digit of $2^{69}\times3^{68}\times7^{67}$

(a) 2 (b) 3 (c) 6 (d) None of these

56. Find the unit digit of $7^{23^{12^{98}}}$

(a) 7 (b) 9 (c) 1 (d) 3

57. What is the right most non-zero digit of the number N where N = $(6!+7!+8!+9!+10!)^{131!}$?

(a) 2 (b) 3 (c) 4 (d) 6

58. What is the unit's digit of
$1^{781}+2^{781}+3^{781}+\ldots\ldots+9^{781}$

(a) 1 (b) 3 (c) 5 (d) 7

59. How many zeroes are there at the end of 200!

(a) 48 (b) 49 (c) 24 (d) None of these

60. $(3x - 2y) : (2x + 3y) = 5 : 6$ then one of the values of $\left(\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right)^2$ is:

(a) $\frac{1}{25}$ (b) 5 (c) $\frac{1}{5}$ (d) 25

MULTIPLE CORRECT ANSWER TYPE

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which ***ONE or MORE*** *is correct. Choose the correct options.*

61. Which is not the rationalizing factor of $1 + \sqrt{2}+\sqrt{3}$?

(a) $1 + \sqrt{2}-\sqrt{3}$ (b) 2 (c) 4 (d) $1+\sqrt{2}+\sqrt{3}$

62. Evaluate $\sqrt[3]{\left(\frac{1}{64}\right)^{-2}}$

(a) 4 (b) 16 (c) 32 (d) $\frac{1}{2^{-4}}$

63. $\frac{2^{n+3}-2(2^n)}{2^{(2n+2)}}$ when simplified is

(a) $1 - 2(2^n)$ (b) $2^{n+3} - \frac{1}{4}$ (c) $\frac{3}{2^{n+1}}$ (d) $\frac{6}{2^{n+2}}$

64. Which of the following is not the value of b in $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

(a) 2 (b) –1 (c) –3 (d) 4

65. Which of the following is equal to $\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}$

(a) $8\sqrt{5}$ (b) $-8\sqrt{5}$ (c) $5\sqrt{2}$ (d) $-\sqrt{320}$

MATRIX MATCH TYPE

66. Match the column

Column I	**Column II**
(A) Integer	(p) positive integer
(B) Zero	(q) Irrational number
(C) natural number	(r) whole number
(D) sum of a rational and irrational number	(s) rational number

(a) (A- p), (B - r), (C - q), (D – s) (b) (A- s), (B - r), (C - p), (D – q)

(c) (A- r), (B - s), (C - p), (D – q) (d) (A- s), (B - q), (C - p), (D – r)

67. Match the column

Column I	**Column II**
(A) $\frac{1}{2+\sqrt{3}}$	(p) 2
(B) $64^{1/2}$	(q) $56^{1/2}$
(C) $16^{1/4}$	(r) 8
(D) $7^{1/2}\ 8^{1/2}$	(s) 2 - $\sqrt{3}$

(a) (A- p), (B - r), (C - q), (D – s)
(b) (A- s), (B - q), (C - p), (D – r)
(c) (A- r), (B - s), (C - p), (D – q)
(d) (A- s), (B - r), (C - p), (D – q)

68. Match the column

Column - I	**Column - II**
(A) If $x=\frac{\sqrt{3}}{2}$, the value of $\frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$	(p) $\frac{9}{4}$
(B) If $x^{x\sqrt{x}}=\left(x\sqrt{x}\right)^{x}$, then the value of x is	(q) $\frac{4}{3}$
(C) If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$, $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$ then the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$	(r) $\frac{1}{8}$
(D) For what value of 'a' is $x+\frac{1}{4}\sqrt{x}+a^2$ a perfect square	(s) $\frac{7}{\sqrt{3}}$

(a) (A- s), (B - p), (C - q), (D – r)
(b) (A- p), (B - q), (C - r), (D – s)
(c) (A- r), (B - s), (C - p), (D – q)
(d) (A- r), (B - s), (C - p), (D – q)

69. Match the column

Column - I	**Column - II**
(A) If $x=3+2\sqrt{2}$, then the value of $\frac{x^6+x^4+x^2+1}{x^3}$ is	(p) 2
(B) Value of $\sqrt{x}-\frac{1}{\sqrt{x}}$ when $x=3+2\sqrt{2}$	(q) $1.\overline{4}$
(C) Value of $\frac{(6.25)^{\frac{1}{2}}\times(0.0144)^{\frac{1}{2}}+1}{(0.027)^{\frac{1}{3}}\times(81)^{\frac{1}{4}}}$ is	(r) 4
(D) Value of $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$ is	(s) 204

(a) (A- s), (B - p), (C - q), (D – r)
(b) (A- p), (B - q), (C - r), (D – s)
(c) (A- r), (B - s), (C - p), (D – q)
(d) (A- r), (B - s), (C - p), (D – q)

70. Match the column

Column - I	**Column - II**
(A) Value of $\left(3+2\sqrt{2}\right)^{-3}+\left(3-2\sqrt{2}\right)^{-3}$	(p) 330
(B) Value of $\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$ is	(q) 7
(C) Value of $\frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt{3}-\sqrt{4+2\sqrt{3}}}}$ is	(r) $\frac{1}{\sqrt{2}}$
(D) $11\sqrt{n}=\sqrt{112}+\sqrt{343}$, then the value of n is	(s) 198

(a) (A- s), (B - p), (C - q), (D – r)
(b) (A- s), (B - r), (C - p), (D – q)
(c) (A- p), (B - q), (C - r), (D – s)
(d) (A- r), (B - s), (C - p), (D – q)

INTEGER ANSWER TYPE

The answer to each of the questions is a single-digit integer, ranging from 0 to 9.

71. Express 0.99999 ….. in the form $\frac{p}{q}$,

72. Evaluate 2^{55} 2^{60} - 2^{97} ′ 2^{18}.

73. If $25^{x-1} = 5^{2x-1} - 100$, find the value of x.

74. If $\frac{9^n \times 3^2 \times \left(3^{-n/2}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$ then the value of m - n.

75. Find the value of x, if $5^{x-3}.3^{2x-8} = 225$.

76. If $27^{2n-1} = (243)^3$, then find the value of n

77. Find the value of $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

78. Find the value of $\left(\sqrt[3]{3.5}+\sqrt[3]{2.5}\right)\left\{\left(\sqrt[3]{3.5}\right)^2-\sqrt[3]{8.75}+\left(\sqrt[3]{2.5}\right)^2\right\}$

79. Evaluate: $\left[8-\left(\frac{4^{9/4}\sqrt{2.2^2}}{2\sqrt{2^{-2}}}\right)^{1/2}\right]$

80. Find the value of ‘a’ of $5\sqrt{5}\times 5^3 \div 5^{-3/2} = 5^{a+2}$

2 Polynomial

Polynomials

An expression of the form of $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n = f(x)$, where $a_0, a_1, a_2, ..., a_n$ are real numbers, n is non-negative integer and $a_0 \neq 0$, is called polynomial of degree n.

For example: (i) $f(x) = x - 3$ is a polynomial of degree 1.

(ii) $g(x) = x^2 + 3x + 7$ is a polynomial of degree 2 and so on.

Note-1: In the above definition $a_0, a_1, a_2, ..., a_n$ are known as the co-efficients of the polynomial.

Note-2: If all a_i's are integers then $f(x)$ is called polynomial with integral co-efficients.

Note-3: Every non-zero real number is a polynomial of degree zero and it is called the **constant polynomial**.

Note-4: The constant polynomial 0 is called the **zero polynomial.** Its degree is not defined.

Note-5: A polynomial of degree 1 is called a **linear polynomial,** *e.g.,* $x + 2$, $4x + 3$, $y - 2$ *etc*.

Note-6: A polynomial of degree 2 is called a **quadratic polynomial,** *e.g.,* $x^2 + 2$, $4x^2 + 3x$, $y^2 - 2y + 4$ *etc*.

Note-7: A polynomial of degree 3 is called a **cubic polynomial,** *e.g.,* $x^3 + 2$, $4x^3 + 3x^2$, $y^3 - 2y + 3$, $x^3 + 3x^2 + 3x + 1$ *etc*.

Zero of a Polynomial

If $g(x)$ is a polynomial and if k is any constant then value obtained by replacing x by k in $g(x)$ is called the value of $g(x)$ at $x = k$. But when $g(k)$ becomes 0 *i.e.,* $g(k) = 0$, then k is called the zero of the polynomial $g(x)$ *e.g.,* $x = 2$ is the zero of the polynomial $g(x) = x^2 - 3x + 2$, because $g(2) = (2)^2 - 3 \times 2 + 2 = 4 - 6 + 2 = 0$

Geometrical meaning of zeroes of a polynomial

The zeroes of a polynomial are the x coordinates of the points where the graph of $y = p(x)$ intersects the x-axis, where $p(x)$ is the given polynomial.

Factor

A polynomial $g(x)$ is called a factor of a polynomial $f(x)$ if there is a polynomial $h(x)$ such that $f(x) = g(x) \cdot h(x)$.

Remainder Theorem: If a polynomial $P(x)$ is divided by $(x - a)$, then remainder is $P(a)$.

For example: If $f(x) = x^2 - 5x + 7$ and we want to find the remainder when $f(x)$ is divided by $(x - 1)$, then remainder is $f(1) = 1^2 - 5 \cdot 1 + 7 = 3$

Note: If remainder $P(a) = 0$ for any polynomial $P(x)$ then a is called zero of the polynomial $P(x)$.

Factor Theorem

If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then:

(a) $x - a$ is a factor of $p(x)$, if $p(a) = 0$.

(b) $p(a) = 0$, if $x - a$ is a factor of $p(x)$..

For example: Let $f(x) = x^2 - 5x + 6$ or $f(x) = (x - 2)(x - 3)$.

$\therefore$ $(x - 2)$ as well as $(x - 3)$ both are factors of $f(x)$.

Note: In the above definition $g(x)$ is also called divisor of $f(x)$. Generally we use term divisor.

Division Algorithm for Polynomials

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

Factorization of the polynomial $ax^2 + bx + c$ by splitting the middle term:

Let $x + p$ and $x + q$ be two linear factors of the polynomial $ax^2 + bx + c$

$\therefore$ $x^2 + bx + c = (x + p)(x + q)$

$\Rightarrow$ $x^2 + bx + c = x^2 + (p + q)x + pq$

Now, comparing the coefficients of like power (exponent) of x on both sides, we get

$b = p + q, c = pq$

To factorize the given quadratic polynomial, we have to find two numbers p and q such that $p + q = b$ and $pq = c$.

Algebraic Identities

Identity: An identity is a statement that two expressions are equal for all values of the letters involved.

Example: $7x = 5x + 2x$ is an identity

(i) Since the expression $7x$ and $5x + 2x$ are equal for all values of x.

(ii) The sides of this identity are $7x$ and $5x + 2x$, $7x$ being the left hand side and $5x + 2x$, the right hand side.

(i) $(a + b)^2 = a^2 + 2ab + b^2$
(ii) $(a - b)^2 = a^2 - 2ab + b^2$
(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(vi) $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

(vii) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + 3ab(a + b) + b^3$

(viii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - 3ab(a - b) - b^3$

(ix) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(x) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(xi) $a^3 + b^3 + c^3 - 3ab = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(xii) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Some more Identities **:**

(i) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(ii) $(a + b)^2 - (a - b)^2 = 4ab$

Graph of Polynomials

In algebraic or in set theoretic language the graph of a polynomial $f(x)$ is the collection (or set) of all points (x, y), where $y = f(x)$. In geometrical or in graphical language the graph of a polynomial $f(x)$ is a smooth free hand curve passing through points (x_1, y_1), (x_2, y_2), (x_3, y_3), ... etc. y_1, y_2, y_3, ... are the values of the polynomial $f(x)$ at x_1, x_2, x_3, ... respectively.

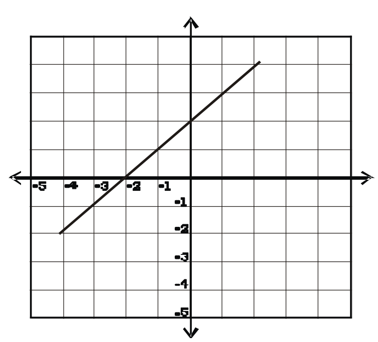
Graph of a Linear Polynomial

Consider a linear polynomial $f(x) = ax + b$, $a \neq 0$ Graph of $y = ax + b$ is a straight line. That is why $f(x) = (ax + b)$ is called a linear polynomial. Since two points determine a straight line, so only two points need to plotted to draw the line $y = ax + b$. The line represented by $y = ax + b$ crosses the x-axis at exactly one point, namely $\left(-\frac{b}{a}, 0\right)$.

ZEROES OF POLYNOMIAL AND ITS GEOMETRICAL REPRESENTATION

Geometrical Representation of the Zeroes of A Linear Polynomial

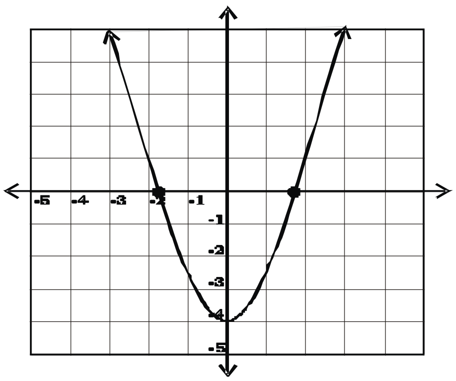
A linear polynomial $p(x) = ax + b$, $a \neq 0$, has exactly one zero and it is a straight line. In the graph it intersects x-coordinate at only one point. We observe the graph, the line intersects x-axis at $x = -2$.

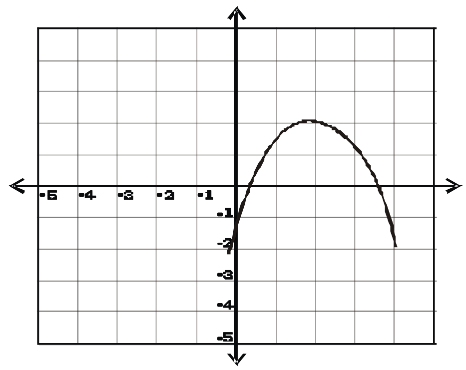


Geometrical Representation Of the Zeroes of A Quadratic Polynomial

In fact for any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, has two zeroes. The graph of the corresponding polynomial either opens upwards or downwards depends on $a > 0$, $a < 0$. The shape of the curve is called **parabola.**

Case – I:

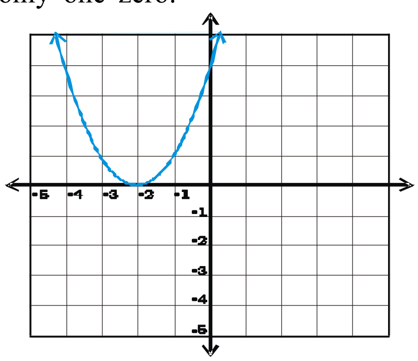


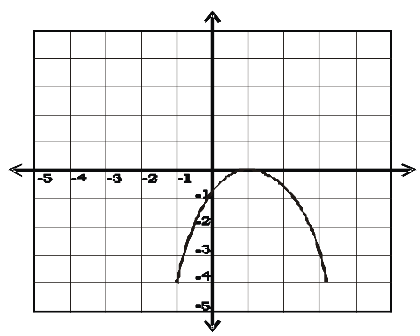


We observe that (2,–2) are the zeroes of the quadratic polynomial in the first graph and (0.5,3.5) in the second graph., where the graphs intersect the x-axis at these two points.

Case – II:

Here the graph touches the x-axis at only one point(two coincident points).Therefore the quadratic polynomial has only one zero.

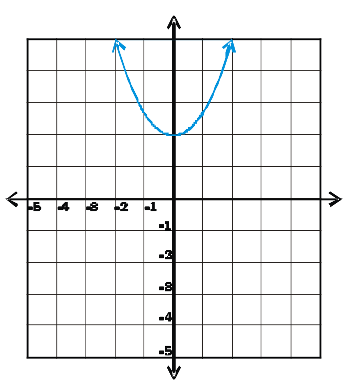


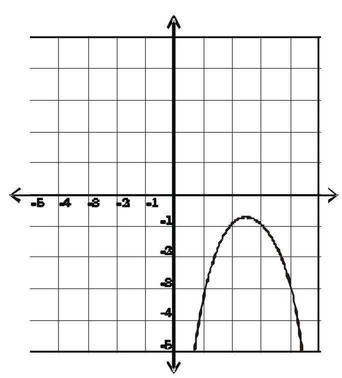


If we observe the graphs, the first graph touches x-axis at $x = -2$ and the second graph at $x=1$.These are the zeroes of the polynomial.

Case – III:

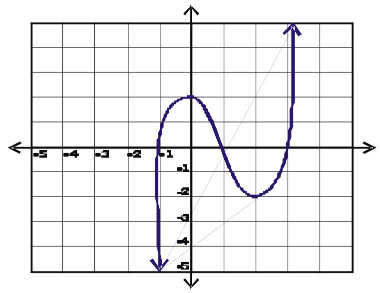
The graph is either completely above the x-axis or completely below the x- axis. So it does not cut the x-axis at any point. In this the quadratic polynomial $ax^2 + bx + c$ **has no zeroes.**





Geometrical Representation of the Zeroes of A Cubic Polynomial

The graph cuts the x-axis at three points. So $y = x^3 - 3x^2 + 2$, has 3 real roots.



Note: In general, given polynomial p(x) of degree n has atmost n zeroes.

Relationship Between Zeros And Coefficients of A Quadratic Polynomial :

Let α and β be the zeros of a quadratic polynomial $f(x) = ax^2 = bx + c$. By factor theorem $(x - \alpha)$ and $(x - \beta)$ are the factors of $f(x)$.

∴ $f(x) = k(x - a)(x - b)$ are the factors of $f(x)$

$\Rightarrow \quad ax^2 + bx + c = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

$\Rightarrow \quad ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + k\alpha\beta$

Comparing the co-efficients of x^2, x and constant terms on both sides, we get

$a = k$, $b = -k(\alpha + \beta)$ and $k\alpha\beta$

$\Rightarrow \quad \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$\Rightarrow \quad \alpha + \beta = -\frac{\text{Co-efficient } x}{\text{Co-efficient of } x^2}$ and $\alpha\beta = \frac{\text{Constant term}}{\text{Co-efficient of } x^2}$

Hence,

Sum of the zeros $= -\frac{b}{a} = -\frac{\text{Co-efficient term}}{\text{Co-efficient of } x^2}$

Product of the zeros $= \frac{c}{a} = \frac{\text{Constant term}}{\text{Co-efficient of } x^2}$

Remarks:

If α and β are the zeros of a quadratic polynomial $f(x)$. The, the polynomial $f(x)$ is given by

$f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

or $f(x) = k\{x^2 - (\text{sum of the zeros})x + \text{Product of the zeros}\}$

Cubic Polynomial

If α, β, γ are the zeroes of the polynomial $ax^3 + bx^2 + cx + d$, then

Sum of the zeroes $= \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$; $\alpha + \beta + \gamma = \frac{-b}{a}$

Sum of product of zeroes taken two at a time $= \dfrac{\text{coefficient of x}}{\text{coefficient of } x^3}$; $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$

Product of the zeroes $= \dfrac{-(\text{constant term})}{\text{coefficient of } x^3}$; $\alpha\beta\gamma = \dfrac{-d}{a}$

Value of a Polynomial

The value of a polynomial f(x) at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$

For example: If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then its value at $x = 1$ is

$f(1) = 2(1)^3 - 13(1)^2 + 17(1) + 12 = 2 - 13 + 17 + 12 = 18$

Note: A real number 'a' is a zero of a polynomial $f(x)$, if $f(a) = 0$, Here 'a' is called a root of the equation $f(x) = 0$.

SOLVED EXAMPLES

Example 1: Factorize $6x^2 + 17x + 5$ by splitting the middle term, and by using the factor Theorem.

Solution: **I :** ***By splitting middle term***

If we can find two numbers p and q such that p + q = 17 and pq = 30, then we can get the factors.

Now, let us see for pairs of factors of 30. Some are 1 and 30, 2 and 15, 3 and 10, 5 and 6 of these pairs, 2 and 15 will give up p + q = 17.

$\therefore\ 6x^2 + 17x + 5 = 6x^2 + (2 + 15)x + 5 = 6x^2 + 2x + 15x + 5 = 2x(3x + 1) + 5(3x + 1)$

Hence, $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$

II: ***Using the factor theorem***

$6x^2 + 17x + 5$

$$p(x) = 6\left(x^2 + \frac{17}{6}x + \frac{5}{6}\right)$$

If a and b are the zeroes of p(x), then

$6x^2 + 17x + 5 = 6(x - a)(x - b)$

$\therefore\ ab = 5/6$

Let us look at some possibilities for a and b.

They could be

$$(a, b)(b, a) = \left(\frac{1}{2}, \frac{5}{3}\right), \left(-\frac{1}{2}, -\frac{5}{3}\right), \left(\frac{1}{3}, \frac{5}{2}\right), \left(-\frac{1}{3}, -\frac{5}{2}\right), \left(\frac{5}{6}, 1\right), \left(-\frac{5}{6}, -1\right)$$

Now, $p(x) = x^2 + \dfrac{17}{6}x + \dfrac{5}{6}$

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + \frac{17}{6}\left(\frac{1}{2}\right) + \frac{5}{6} = \frac{1}{4} + \frac{17}{12} + \frac{5}{6} = \frac{1}{4} + \frac{17}{12} + \frac{5}{6} \neq 0$$

But $$p\left(-\frac{1}{3}\right) = \left(\frac{-1}{3}\right)^2 + \frac{17}{6}\left(-\frac{1}{3}\right) + \frac{5}{6} = \frac{1}{9} - \frac{17}{18} + \frac{5}{6} = \frac{2 - 17 + 15}{18} = \frac{17 - 17}{18} = \frac{0}{18} = 0$$

$\therefore \left(x+\frac{1}{3}\right)$ is a factor of p(x)

Similarly, by trial and error method, we can find that $\left(x+\frac{5}{2}\right)$ is a factor of p(x).

Hence, $6x^2+17x+5=6\left(x+\frac{1}{3}\right)\left(x+\frac{5}{2}\right)=6\left(\frac{3x+1}{3}\right)\left(\frac{2x+5}{2}\right)=(3x+1)(2x+5)$.

Example 2: Factorize the polynomial $2y^3+y^2-2y-1$

Solution : Let $P(y)=2y^3+y^2-2y-1$
then $P(1)=2(1)^3+(1)^2-2(1)-1=2+1-2-1=0$
$\Rightarrow$ (y - 1) is a factor of polynomial P(y) .
Divide P(y) by (y - 1) using long division method.
Hence, $2y^3+y^2-2y-1=(y-1)(2y^2+3y+1)$
$=(y-1)[2y^2+(2+1)y+1]$
(by splitting the terms)
$=(y-1)(y+1)(2y+1)$

$$\begin{array}{r|l} & 2y^2+3y+1 \\ \hline y-1 & 2y^3+y^2-2y-1 \\ & 2y^3-2y^2 \\ & - \quad + \\ \hline & 3y^2-2y \\ & 3y^2-3y \\ & - \quad + \\ \hline & y-1 \\ & y-1 \\ & - \quad + \\ \hline & 0 \end{array}$$

Example 3: If x + 1 and x – 1are factors of $f(x)=x^3+2ax+b$, then find the value of 2a + 3b.

Solution: $p(-1)=0\Rightarrow(-1)^3+2a(-1)+b=0\Rightarrow-1-2a+b=0$

$\Rightarrow -2a+b=1$ … (1)

$p(1)=0\Rightarrow 1^3+2a(1)+b=0\Rightarrow 2a+b=-1$ … (2)

Solving (1) and (2), we get $a=-\frac{1}{2}, b=0$

$\therefore\ 2a+3b=2\left(-\frac{1}{2}\right)+3\times0=-1$

Example 4: If (x – 1), (x + 2) and (x – 2) are factors of x^3+ax^2+bx+c, then find the value of a + b + c.

Solution: $p(1)=0\Rightarrow 1^3+a(1)^2+b(1)+c=0\Rightarrow a+b+c=-1$ … (1)

$p(-2)=0\Rightarrow(-2)^3+a(-2)^2+b(-2)+c=0\Rightarrow 4a-2b+c=8$ … (2)

$p(2)=0\Rightarrow(2)^3+a(2)^2+b(2)+c=0\Rightarrow 4a+2b+c=-8$ … (3)

Solving (1), (2) and (3), we get a = 2, b = –1, c = –2
$\therefore$ a + b + c = 2 – 1 – 2 = –1

Example 5: If $f(x)=2x^3+ax^2+bx-6$, where a and b are constants, when f(x) is divided by (2x – 1) the remainder is –5, when f(x) is divided by (x + 2) the remainder is zero, then find the values of *a* and *b*.

Solution: $f\left(\frac{1}{2}\right)=-5\Rightarrow 2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)-6=-5$

$\Rightarrow\frac{1}{4}+\frac{a}{4}+\frac{b}{2}=1\Rightarrow 1+a+2b=4\Rightarrow a+2b=3$ … (1)

$f(-2)=0\Rightarrow 2(-2)^3+a(-2)^2+b(-2)-6=0\Rightarrow-16+4a-2b-6=0$

$\Rightarrow 4a-2b=22\Rightarrow 2a-b=11$ … (2)

Solving (1) and (2), we get a = 5, b = –1

Example 6: If α and β are the zeros of the quadratic polynomial $f(x)=kx^2+4x+4$ such that $\alpha^2+\beta^2=24$, find the values of k.

Solution: Given $f(x)=kx^2+4x+4$

$\alpha^2+\beta^2=24$

so, $\alpha+\beta=\frac{-4}{k}$, $\alpha\beta=\frac{4}{k}$

$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta \Rightarrow \left(-\frac{4}{k}\right)^2=24+2\times\frac{4}{k}$

$\frac{16}{k^2}=24+\frac{8}{k} \Rightarrow \frac{16}{k^2}-\frac{8}{k}=24$

$24k^2+8k-16=0$

$3k^2+k-2=0$

$(k+1)(3k-2)=0$

$k=-1, \frac{2}{3}$

Example 7 : If α, β are the zeros of the polynomial $f(x)=2x^2+5x+k$ satisfying the relation $\alpha^2+\beta^2+\alpha\beta=\frac{21}{4}$, then find the value of k.

Solution: Given $f(x)=2x^2+5x+k$

$\alpha+\beta=-\frac{5}{2}$, $\alpha\beta=\frac{k}{2}$, $\alpha^2+\beta^2+\alpha\beta=\frac{21}{4}$

$(\alpha+\beta)^2-2\alpha\beta+\alpha\beta=\frac{21}{4} \Rightarrow (\alpha+\beta)^2-\alpha\beta=\frac{21}{4}$

$\left(-\frac{5}{2}\right)^2-\frac{k}{2}=\frac{21}{4} \Rightarrow \frac{25}{4}-\frac{k}{2}=\frac{21}{4} \Rightarrow k=2$

Example 8: Find the zeros of the polynomial $f(x)=x^3-5x^2-16x+80$, if its two zeros are equal in magnitude but opposite in sign.

Solution: Let α, α, γ be the zeros of polynomial f(x)

Then,

$\text{Sum of the zeros}=-\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow \alpha-\alpha+\gamma=-\left(-\frac{5}{1}\right) \Rightarrow \gamma=5$

$\text{Product of the zeros}=-\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

$\Rightarrow \alpha(-\alpha)\gamma=-\frac{80}{1} \Rightarrow -\alpha^2\gamma=-80$

$\Rightarrow \alpha^2\times5=80 \Rightarrow \alpha^2=16 \Rightarrow \alpha=4$

Hence, the zeroes are 4, –4 and 5.

Example 9: If the roots of the equation $\frac{\alpha}{x-\alpha}+\frac{\beta}{x-\beta}=1$ be equal in magnitude but opposite in sign, then find $\alpha + \beta$.

Solution: Given equation is $\frac{\alpha}{x-\alpha}+\frac{\beta}{x-\beta}=1$ can be rewritten as

$x^2 - 2(\alpha + \beta)x + 3\,\alpha\beta = 0$

Let roots be α' and $-\alpha'$

$\Rightarrow \alpha' + (-\alpha') = 2(\alpha + \beta)$

$\Rightarrow 0 = 2(\alpha + \beta)$

$\alpha + \beta = 0$

Example 10: If the zero of the polynomial $x^3 - 3x^2 + x + 1$ are (a-b), a and (a+b), find the value of a and b ?

Solution: As (a – b), a, (a + b) are zeroes of the polynomial $x^3 - 3x^2 + x + 1$

So, a – b + a + a + b = -(-3)/1 = 3

3a = 3

a = 1

Also, (a – b) a(a + b) = -1

(1 – b) (1 + b) = -1

$1 - b^2 = -1$

$b^2 = 2$

$b = \pm\sqrt{2}$

REVISION EXERCISE

SUBJECTIVE QUESTIONS

1. Without actually calculating cubes find the value of $(25)^3+(-17)^3+(-8)^3$.

2. For what value of k, $y^3+ky+2k-2$ is exactly divisible by $(y + 1)$?

3. Factorize:

(i) $\left(\frac{3}{2}x+1\right)^3$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$.

(iii) $27-125a^3-135a+225a^2$ ·

(iv) $6\sqrt{3}a^2-47a+5\sqrt{3}$ ·

(v) $(2a+1)^2-9b^4$.

(vi) $24\sqrt{3}x^3-125y^3$.

(vii) $x(x-y)^3+3x^2y(x-y)$.

(viii) $\left(a-\frac{1}{a}\right)\left(a+\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}\right)\left(a^4+\frac{1}{a^4}\right)$

(ix) $7x^2+2\sqrt{14}x+2$

(x) $x^3+13x^2+32x+20$ ·

(xi) Evaluate $x^3 + 13x^2 - 22x - 8$ when $x = -15$.

4. (i) If $x+\frac{1}{x}=7$ then the value of $x^3+\frac{1}{x^3}$.

(ii) If $x^2+\frac{1}{x^2}=14$. Find $x^3+\frac{1}{x^3}$.

5. (i) What must be subtracted from $3x^5-2x^4+x^2-2$, so that the result is exactly divisible by x^2+x+1?

(ii) What must be added to $x^4+2x^3-2x^2-2x-1$, so that the result is exactly divisible by $x^2 + 2x - 3$?

6. (i) If $x+y+z=1$, $xy+yz+zx=-1$ and $xyz=-1$ find the value of $x^3+y^3+z^3$.

(ii) If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$.

7. Give possible expressions for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$.

8. Use remainder theorem to find remainder when $p(x)$ is divided by $q(x)$ in the following questions:

(a) $p(x) = x^9 - 5x^4 + 1,\ q(x) = x - 1$.

(b) $p(x) = 4x^3 - 12x^2 + 11x - 5,\ q(x) = x - \frac{1}{2}$.

(c) $p(x) = x^3 - 6x^2 - 2x - 4,\ q(x) = 1 - 3x$.

(d) $p(x) = x^3 + 3x^2 + 3x + 1\ q(x) = \left(x - \frac{1}{2}\right)$.

9. Factorise:

(a) $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$.

(b) $3(x+2y)^2 + 5(x+2y) + 2$.

(c) $125(x-y)^3 + (5y-3z)^3 + (3z-5x)^3$.

(d) $x^3 - 3x^2 - 9x - 5$.

10. (a) Using suitable identify, evaluate $(104)^{2}$..

(b) If $x + \frac{1}{x} = \sqrt{3}$ then find the value of $x^3 + \frac{1}{x^3}$.

(c) If $x - \sqrt{3}$ is a factor of the polynomial $ax^2 + bx - 3$ and $a + b = 2 - \sqrt{3}$. Find the values of a and b.

(d) Divide the polynomial $3x^4 - 4x^3 - 3x - 1$ by $x - 1$. Write quotient and remainder.

LEVEL - II

1. If $(y-2)$ and $\left(y - \frac{1}{2}\right)$ are factors of $my^2 + 5y + n$, show the $m = n$.

2. (i) If $x - \sqrt{a}$ is factor of $2x^4 - 2a^2x^2 - 3x + 2a^3 - 2a^2 + 3$, find the value of a.

(ii) If $x + \sqrt{a}$ is factor of $5x^4 + 5\sqrt{a}x^3 + 2x^2 - 3a + 5$, find the value of a^2.

3. If $x = \dfrac{\sqrt{p+2q} + \sqrt{p-2q}}{\sqrt{p+2q} - \sqrt{p-2q}}$ and $q \neq 0$, then prove that $qx^2 - px + q = 0$.

4. (i) If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$, find the value of $a^3 + b^3 + c^3 - 3abc$.

(ii) If $p + q + r = 1$ and $pq + qr + pr = -1$ and $pqr = -1$, find the value of $p^3 + q^3 + r^3$.

5. If the polynomial $2x^3 - 9x^2 + 15x + p$ when divided by $x - 2$ leaves $(-p)$ as remainder. Find the value of p.

6. (i) Determine whether $x - 3$ is a factor of $x^3 - 3x^2 + 6x - 18$.

(ii) Determine whether $x + 5$ is a factor of $3x^3 - x^2 + 6x + 200$.

(iii) Determine whether $x^3 - 5x^2 + 3x + 1$ is divisible by $x - 1$.

(iv) Find the value of b if $x + b$ is a factor of $x^3 - b^2x + x + 2$.

(v) For what value of 'p' is the expression, $2x^3 + 3x^2 + 7x - 2p$ divisible by $x + 1$?

(vi) If $x - 2$ is a factor of $x^3 - 2bx^2 + bx - 1$, then find the value of b.

7. (i) If the expressions $px^4 - 3x^3 + 20$ and $4x^2 + 7x - p$ when divided by $x - 2$ leaves then same remainders, then find the value of p.

(ii) When a polynomial $f(x)$ is divided by $(x - 2)$ and $(x - 5)$, the respective remainders are 17 and 11. What is the remainder when it is divided by $(x - 2)(x - 5)$?

(iii) When a polynomial $f(x)$ is divided by $x - 3$ and $x + 6$, the respective remainders are 7 and 22. What is the remainder when $f(x)$ is divided by $(x - 3)(x + 6)$?

8. Factorize:

(i) $a(b - c)^3 + b(c - a)^3 + c(a - b)^3$.

(ii) $(a + b + c)^3 - (b + c - a)^3 - (c + a - b)^3 - (a + b - c)^3$.

(iii) $a(b - c)^2 + b(c - a)^2 + c(a - b)^2 + 8abc$.

9. Without actual division, prove that $a^4 + 2a^3 - 2a^2 + 2a - 3$ is exactly divisible by $a^2 + 2a - 3$.

10. If $x + 1$ and $x - 1$ are factors of $mx^3 + x^2 - 2x + n$, find the value of m and n.

11. Prove that:: $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a+b)(b+c)(c+a)(a-b)(b-c)(c-a)$.

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\,[(x - y)^2 + (y - z)^2 + (z - x)^2]$ and hence find the value of $27^3 + (-14)^3 + (-13)^2$.

13. If $(3x - 1)^4 = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, then find the value of $a_4 + 3a_3 + 9a_2 + 27a_1 + 81a_0$.

14. (i) Find the zero's of the polynomial: $p(x) = (x - 2)^2 - (x + 2)^2$.

(ii) Show that $p - 1$ is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.

(iii) For what value of m is $x^5 - 2mx^2 + 16$ divisible by $x + 2$?

(iv) If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a.

(v) Find the value of m, so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$.

(vi) Without finding the cubes, factorise:
$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$.

15. (i) If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

(ii) The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$, when divided by $x + 1$ leaves the remainder 19. Find the value of a. Also find the remainder, when $p(x)$ is divided by $x + 2$.

(iii) If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

(iv) If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

(v) Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$

EXERCISE (Single Correct Type)

LEVEL - I

1. The polynomial $2x - x^3 + 5$ is

(a) an equation (b) a trinomial (c) a binomial (d) a monomial

2. –2 is:

(a) bionomial (b) not a polynomial

(c) constant polynomial (d) polynomial of degree one

3. The coefficient of x^3 in the polynomial $5x^2 + 2x - 7 - 4x^3$ is:

(a) 5 (b) 2 (c) 4 (d) –4

4. The co-efficient of x^2 in the polynomial $p(x) = 4x^4 - \frac{1}{3}x^3 - 2x^2 + 7$ is:

(a) 2 (b) –4 (c) –2 (d) 4

5. The co-efficient of x^3 in the polynomial $p(x) = 6x^4 - \sqrt{3}x^3 - \frac{5}{3}$ is:

(a) $3\sqrt{3}$ (b) $-\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

6. The degree of polynomial $p(x) = 4$ is:

(a) 1 (b) 0 (c) 4 (d) not defined

7. The degree of the polynomial $(y^3 - 2)(y^2 + 11)$ is:

(a) 2 (b) 3 (c) 0 (d) 5

8. Degree of the polynomial $2x^3 - x\sqrt{5} + 3x - 4$ is:

(a) 3 (b) $\frac{1}{2}$ (c) 1 (d) none of these

9. Given a polynomial $p(t) = t^4 - t^2 + 6,$ then $p(-1)$ is:

(a) 6 (b) 9 (c) 3 (d) –1

10. Zero of the polynomial $p(x)$, where $p(x) = ax + 1,\ a \neq 0$ is:

(a) 1 (b) $-a$ (c) 0 (d) $-\frac{1}{a}$

11. Which of the following is a zero of the polynomial $x^2 - 5x + 6$.

(a) 3 (b) –3 (c) 5 (d) 6

12. Which of the following polynomials has $(x + 1)$ as a factor?

(i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

(a) (i) (b) (iii) (c) (ii) and (iv) (d) (i) and (ii)

13. If $(x - 1)$ is a factor of $kx^2 - \sqrt{2}x + 1$, then value of k is:

(a) 1 (b) $\sqrt{2} + 1$ (c) $\sqrt{2} - 1$ (d) $-\sqrt{2} - 1$

14. Factors of $x^2 + 11x + 18$ are:

(a) $(x + 9)(x - 2)$ (b) $(x - 9)(x - 2)$ (c) $(x - 9)(x + 2)$ (d) $(x + 9)(x + 2)$

15. Expansion $(x - y)^3$ is:

(a) $x^3 + y^3 + 3x^2y + 3xy^2$ (b) $x^3 + y^3 - 3x^2y + 3xy^2$

(c) $x^3 - y^3 - 3x^2y + 3xy^2$ (d) $x^3 - y^3 + 3x^2y - 3xy^2$

16. Which identity, do we use to factorise $x^2 - \frac{y^2}{100}$?

(a) $(a + b)^2 = a^2 + b^2 + 2ab$ (b) $(a - b)^2 = a^2 + b^2 - 2ab$

(c) $(a - b)^2 = a^3 - 3a^2b + 3ab^2 - b^3$ (d) $a^2 - b^2 = (a - b)(a + b)$

17. Volume of a cuboid is $3x^2 - 27$. Then possible dimensions are:

(a) 3, 3, 3 (b) 3, $(x - 3)$, $(x + 3)$ (c) 3, x^2, 27x (d) 3, x^2, $-27x$.

18. If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of b is

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

19. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ is equal to

(a) 0 (b) abc (c) $3abc$ (d) $2abc$

20. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is

(a) $5 + x$ (b) $5 - x$ (c) $5x + 1$ (d) $10x$

21. The factorisation of $4x^2 + 8x + 3$ is

(a) $(x + 1)(x + 3)$ (b) $(2x + 1)(2x + 3)$ (c) $(2x + 2)(2x + 5)$ (d) $(2x - 1)(2x - 3)$

22. If $\frac{x}{y} + \frac{y}{x} = -1 (x, y \neq 0)$, the value of $x^3 - y^3$ is

(a) 1 (b) -1 (c) 0 (d) $\frac{1}{2}$

23. Zero of the zero polynomial is

(a) 0 (b) 1 (c) Any real number (d) Not defined

24. If $x + 1$ is a factor of the polynomial $2x^2 + kx$, then the value of k is

(a) –3 (b) 4 (c) 2 (d) –2

25. If $x^{51} + 51$ is divided by $x + 1$, the remainder is

(a) 0 (b) 1 (c) 49 (d) 50

26. One of the zero's of the polynomial $2x^2 + 7x - 4$ is

(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) –2

27. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

(a) 3 (b) $2x$ (c) 0 (d) 6

28. The co-efficient of x in the expansion of $(x + 3)^3$ is

(a) 1 (b) 9 (c) 18 (d) 27

29. If x and y be two positive real numbers such that $x > 3y$, $x^2 + 9y^2 = 369$ and $xy = 60$, then the value of $x - 3y$ is

(a) 3 (b) 5 (c) 4 (d) 2

30. The values of l and m so that $lx^4 + mx^3 + 2x^2 + 4$ is exactly divisible by $x^2 - x - 2$, is respectively

(a) $-\frac{5}{2}, \frac{7}{2}$ (b) $\frac{5}{2}, \frac{7}{2}$ (c) $-\frac{5}{2}, -\frac{7}{2}$ (d) $\frac{5}{2}, -\frac{7}{2}$

LEVEL - II

31. If $f(x) = x^3 + ax^2 + bx + 3,$ where a and b are constants, when f(x) is divided by (x +2) the remainder is 7, when f(x) is divided by (x – 1) the remainder is 4, then the values of 3a – b is

(a) 4 (b) –6 (c) 8 (d) –6

32. If $ax^2 + bx + c$ is exactly divisible by (x – 1) (x – 2) and leaves remainder 6 when divided by (x + 1), then the values of a, b and c are respectively

(a) 1,3,2 (b) 1,–3,2 (c) –1,–3,–2 (d) –1,3,–2

33. The factor of $8(a+1)^2 + 2(a+1)(b+2) - 15(b+2)^2$ is/are

(a) $(2a+3b+8)(4a+5b+6)$ (b) $(2a+3b+8)(4a-5b-6)$

(c) $(2a-3b-8)(4a-5b-6)$ (d) $(2a-3b-8)(4a+5b+6)$

34. If the product of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is 2, then the sum of roots is

(a) 1 (b) – 1 (c) 2 (d) – 2

35. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b}$ is equal to

(a) $\frac{2}{a}$ (b) $\frac{2}{b}$ (c) $\frac{2}{c}$ (d) $-\frac{2}{a}$

36. If α, β are the roots of the equation $ax^2 + bx + c = 0$, $\alpha\beta = 3$ and a, b, c are in A.P. then $\alpha + \beta$ is equal to

(a) – 4 (b) – 1 (c) 4 (d) – 2

37. If $P(x + 2) = 2x^3 - 4x^2 + 2x + 3$, then the remainder when P(x) is divided by (x – 3) is

(a) 3 (b) 1 (c) 2 (d) 0

38. 'a' is a real number such that $a^3 + 4a - 8 = 0$. The value of $a^7 + 64a^2$ is

(a) 180 (b) 164 (c) 256 (d) 128

39. The remainder when x^{2017} is divided by $(x^2 - 1)$ is

(a) x (b) x + 1 (c) –x (d) 1

40. If $x = \frac{20172016^2}{20172015^2 + 20172017^2 - 2}$, then the value of $x^{-11} - 2017$ is

(a) 31 (b) 30 (c) 32 (d) 33

41. When f(x) is divided by (x – 1) and (x – 2), it leaves remainder 5 and 7 respectively. What is the remainder when f(x) is divided by (x – 1)(x – 2)?

(a) 2x + 3 (b) x – 3 (c) 2x – 3 (d) x + 3

42. If $p(x) = ax^9 + bx^5 + cx - 11$, where a, b, c are constants and p(1042) = –32 then p(–1042) is equal to

(a) –10 (b) –12 (c) 10 (d) 32

43. If a, b are positive and a + b = 1 the minimum value of $a^4 + b^4$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

44. Let f(x) be a polynomial of degree 1. If f(10) – f(5) = 15, then f(20) – f(5) is equal to

(a) 45 (b) 55 (c) 65 (d) 75

45. The expression $x^2 + px + 2$ with p greater than zero has its minimum value when

(a) x = –p (b) x = p (c) x = p/2 (d) x = –p/2

46. If α, β and γ are the zeroes of the polynomial $f(x) = x^3 - 3x^2 - 7x + 21$, then the value of $\frac{(14 - 7\alpha)(22 - 11\beta)(114 - 57\gamma)}{209}$ is

(a) 21 (b) 63 (c) 62 (d) 60

47. $a^8 - 33a^4b^4 - 108b^8$ on factorization gives

(a) $\left(a^4 + 3b^4\right)\left(a - \sqrt{6}\,b\right)\left(a^2 + 6b^2\right)\left(a + \sqrt{6}\,b\right)$ (b) $\left(a^4 + 3b^4\right)\left(a - 6b\right)\left(a^2 + 6b^2\right)\left(a + 6b\right)$

(c) $\left(a^4 - 3b^4\right)\left(a - \sqrt{6}\,b\right)\left(a^2 + 6b^2\right)\left(a + \sqrt{6}\,b\right)$ (d) $\left(a^4 + 3b^4\right)\left(a - \sqrt{6}\,b\right)\left(a^2 - 6b^2\right)\left(a + \sqrt{6}\,b\right)$

48. If $\left(\sqrt{x}\right)^{\left(\sqrt{x}\right)^{\left(\sqrt{x}\right)\ldots\ldots\infty}} = \frac{1}{16}$, then the value of 'x' is

(a) $\frac{1}{2^{128}}$ (b) $\frac{1}{2^{256}}$ (c) $\frac{1}{2^{48}}$ (d) $\frac{1}{2^{32}}$

49. $a^4 + b^4$ can be factorized as

(a) $\left(a+b+\sqrt{2}\,ab\right)\left(a-b-\sqrt{2}\,ab\right)$

(b) $\left(a+b+2ab\right)\left(a-b-2ab\right)$

(c) $\left[\left(a-b\right)^2+\sqrt{2}\,ab-2ab\right]\left[\left(a+b\right)^2-\sqrt{2}\,ab+2ab\right]$

(d) $\left[\left(a+b\right)^2+\sqrt{2}\,ab-2ab\right]\left[\left(a-b\right)^2-\sqrt{2}\,ab+2ab\right]$

50. If $x = 7^{1/3} + 7^{-1/3}$, then the value of $7x^3 - 21x$ is

(a) $\frac{50}{7}$ (b) $\frac{150}{7}$ (c) 50 (d) 100

MULTIPLE CORRECT ANSWER TYPE

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c), (d), out of which ***ONE or MORE*** *is correct. Choose the correct options.*

51. If $x = \frac{a}{2}$, then which of the following is not the value of $4x^2 + 8x + 18$ is/are

(a) $a^2 + 2a + 8$ (B) $a^2 + 3a + 18$ (c) $a^2 + 4a + 18$ (d) $a^2 + 5a + 18$

52. The polynomial $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$. Then the values of 'a' and 'b' are

(a) $a = -4$ (b) $a = 3$ (c) $b = 1$ (d) $b = 3$

53. The factors of the expression a (x + y + z) + bx + by + bz is/are

(a) ax + ay + az (b) bx + by + bz (c) x + y + z (d) a + b

54. If $x + \frac{1}{x} = 7$, then which of the following is not the value of $x^3 + \frac{1}{x^3}$ is/are

(a) 318 (b) 325 (c) 343 (d) $\sqrt{1296}$

MATRIX MATCH TYPE

Question contains statements given in two columns, which have to be matched. The statements in ***Column I*** *are labeled A, B, C and D, while the statements in* ***Column II*** *are labeled p, q, r, s and t. Any given statement in* ***Column I*** *can have correct matching with* ***ONE OR MORE*** *statements(s) in* ***Column II.***

55. Column II gives the degree of polynomials given in column I match them correctly

Column I	Column II
(A) $2 - y^2 - y^3 + 2y^8$	(p) 2
(B) 2	(q) 1
(C) $5x - \sqrt{7}$	(r) 0
(D) $4 - x^2$	(s) 8

(a) (A-s), (B-r), (C-q), (D-p) (b) (A-q), B-p), (C-s), (D-r)

(c) (A-s), (B-r), (C-p), (D-q) (d) (A-r), B-s), (C-q), (D-p)

56. Column II gives value of k for polynomials given in column I when it is divided x - 1 match them correctly.

Column I	Column II
(A) $kx^2 - 3x + k$	(p) -2
(B) $x^2 + x + k$	(q) 3/2
(C) $2x^2 + kx + \sqrt{2}$	(r) $\sqrt{2} - 1$
(D) $kx^2 - \sqrt{2}x + 1$	(s) $-(2 + \sqrt{2})$

(a) (A-s), (B-r), (C-q), (D-p) (b)(A-q), B-p), (C-s), (D-r)

(c) (A-s), (B-r), (C-p), (D-q) (d) (A-r), B-s), (C-q), (D-p)

57. Column II gives remainder when $x^3 + 3x^2 + 3x + 1$ is divided by expression given in column I, match them correctly

Column I	Column II
(A) x + 1	(p) 27/8
(B) x	(q) -27/8
(C) $x - \frac{1}{2}$	(r) 1
(D) 5 + 2x	(s) 0

(A) (A-s), (B-r), (C-q), (D-p) (B)(A-q), B-p), (C-s), (D-r)

(C) (A-s), (B-r), (C-p), (D-q) (D) (A-r), B-s), (C-q), (D-p)

58. Match the column

Column - I	Column - II
(A) Value of $\frac{(0.337+0.126)^2-(0.337-0.126)^2}{0.337\times0.126}$	(p) 2.60
(B) Value of $\frac{(2.3)^3\times0.027}{(2.3)^2-0.69+0.09}$	(q) 4
(C) Value of $\frac{(1.5)^3+(4.7)^3+(3.8)^3-3\times1.5\times4.7\times3.8}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-4.7\times3.8-3.8\times1.5}$	(r) 10
(D) If $\frac{2p}{p^2+2p+1}=\frac{1}{4}$, then the value of $\left(p+\frac{1}{p}\right)$ is	(s) 10

(a) (A- s), (B - p), (C - q), (D – r) (b) (A- p), (B - q), (C - r), (D – s)

(c) (A- q), (B - p), (C - s), (D – r) (d) (A- r), (B - s), (C - p), (D – q)

59. **Column - I** **Column - II**

Column - I	Column - II
(A) If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$ then the value of $4a - 3b + 5c$ is	(p) $\frac{7}{8}$
(B) If $2x + \frac{2}{x} = 3,$ then the value of $x^3 + \frac{1}{x^3} + 2$ is	(q) 2
(C) If $a^3 - b^3 = 56$ and $a - b = 2$ then value of $(a^2 + b^2)$ is	(r) $\frac{2}{3}$
(D) If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ then the value of $\left(\frac{a}{b} + \frac{b}{a}\right)$	(s) 20

(a) (A- q), (B - p), (C - s), (D – r)
(b) (A- p), (B - q), (C - r), (D – s)
(c) (A- p), (B - q), (C - s), (D – r)
(d) (A- r), (B - s), (C - p), (D – q)

INTEGER TYPE

The answer to each of the questions is a single-digit integer, ranging from 0 to 9.

60. The polynomials $kx^3 + 3x^2 - 3$ and $2x^3 - 5x + k$ are divided by $(x - 4)$ then leave the same remainder in each case, then the value of k is

61. If $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ is divided by $g(x) = x + \frac{2}{3}$, then the remainder is

62. If $A = -8x^2 - 6x + 10$, then its value when 'x' = $\frac{1}{2}$ is

63. Degree of the polynomial $\frac{1}{2}x^5 + 3x^4 + 2x^3 + 3x^2$ is

64. If the polynomials $ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ leave the same remainder when divided by $(x - 3)$, then the value of '–a' is

65. If α, β are the zeroes of $2x^2 + 3x + 10$, then find the value of $\alpha\beta$.

66. If the polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divisible by $(x - 1)$ and $(x + 1)$ then find the value of $(a - b)$.

67. Find the H.C.F. of $(x^2 - xy + 2y^2)$ and $(2x^2 - xy - y^2)$

68. If polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ are divided by $(x - 2)$, the same remainder are obtained. Find the value of $(-a)$

69. If $a + b + c = 0$ then find the value of $\frac{a^2 + b^2 + c^2}{a^2 - bc}$

70. If $x = 1 - \sqrt{2}$, then find the value of $\left(x - \frac{1}{x}\right)^3$.

NTSE QUESTIONS (PREVIOUS YEARS)

1. If α, β and γ are the three zeroes of the polynomial $p(x) = x^3 - 64x - 14$, what is the value of $\alpha^3 + \beta^3 + \gamma^3$?

(a) 36 (b) 40 (c) 42 (d) 64

2. What is the remainder when the polynomial $p(x) = x^{200} - 2x^{199} + x^{50} - 2x^{49} + x^2 + x + 1$ is divided by $(x - 1)(x - 2)$?

(a) 1 (b) 7 (c) $2x + 1$ (d) $6x - 5$

3. One set of factors of $81a^4 + (x - 2a)(x - 5a)(x - 11a)$ is

(a) $x^2 - 13ax + 31a^2$ (b) $x^2 + 13a + 31a^2$

(c) $x^2 + 18a - 31a^2$ (d) $x^2 - 18a + 31a^2$

4. The value of $\dfrac{(2014^2 - 2020)(2014^2 + 4028 - 3)(2015)}{(2011)(2013)(2016)(2017)}$ is

(a) 2014 (b) 2015 (c) 2016 (d) 2017

5. Let $p(x) = x^2 + bx + c$ where b and are integers. If p(x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$ what is P(1)?

(a) 0 (b) 1 (c) 2 (d) 4

6. The coefficient of x^7 in the polynomial expansion of $(1 + 2x - x^2)^4$ is

(a) – 8 (b) 12 (c) 6 (d) – 12

7. Factors of polynomial : $x^2 + 15x - 3250$ will be …………

(a) $(x - 50)(x + 35)$ (b) $(x - 65)(x + 50)$

(c) $(x - 50)(x - 35)$ (d) $(x + 65)(x - 50)$

8. The value of the following $\dfrac{(0.44)^2 + (0.06)^2 + (0.024)^2}{(0.044)^2 + (0.006)^2 + (0.0024)^2}$ is :

(a) 0.100 (b) 0.01 (c) 100 (d) 1

9. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(a) $a - b - 1$ (b) $b - a - 1$ (c) $1 - a + b$ (d) $1 + a - b$

10. If $a^2 + b^2 + 2c^2 - 4a + 2c - 2bc + 5 = 0$ then the possible value of $a + b - c$

(a) 1 (b) 2 (c) –1 (d) –2

11. The value of $\dfrac{(10^4 + 324)(22^4 + 324)(34^4 + 324)(46^4 + 324)(58^4 + 324)}{(4^4 + 324)(16^4 + 324)(28^4 + 324)(40^4 + 324)(52^4 + 324)}$ is

(a) 324 (b) 400 (c) 373 (d) 1024

12. If $xy + yz + zx = 0$, then the value of $\left(\dfrac{1}{x^2 - yz} + \dfrac{1}{y^2 - zx} + \dfrac{1}{z^2 - xy}\right)$

(a) 3 (b) 0 (c) 1 (d) $x + y + z$

3 Euclid's Geometry

Introduction

The word 'geometry' is derived from the greak words 'geo' means 'earth' and 'metron' means 'measuring'. Geometry appears to have originated from the need for measuring land. This branch of mathematics was studied in various forms in every ancient civilisation, be it in Egypt, Babylonia, China, India, Greece, The Incas, etc.

Euclid's definition

1. A **point** is that which has no part.
2. A **line** is breadthless length.
3. The end of a line are points.
4. A **straight line** is a line which lies evenly with the points on itself.
5. A **surface** is that which has length and breadth only.
6. The edges of a surface are lines.
7. A **plane surface** is a surface which lies evenly with the straight lines on itself.

Euclid's Axioms

1. Things which are equal to the same thing are equal to one another.
2. If equals are added to equals, the wholes are equal.
3. If equals are subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.
6. Things which are double of the same things are equal to one another.
7. Things which are halves of the same things are equal to one another.

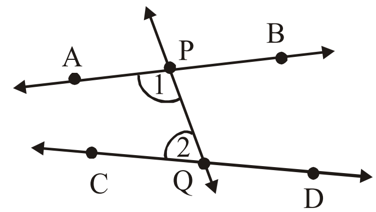
Euclid's postulates

1. A straight line may be drawn from any one point to any other point.

   OR

   Given two distinct points, there is a unique line that passes through them.
2. A terminated line can be produced indefinitely.
3. A circle can be drawn with any centre and any radius.
4. All right angles are equal to one another.

Postulate 5 :



If a straight line falling on two straight lines make the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

For example: the line PQ in figure falls on lines AB and CD such that the sum of the interior angles 1 and 2 is less than 180º on the left side of PQ. Therefore, the lines AB and CD will eventually intersect on the left side of PQ.

Theorem 1: Two distinct lines cannot have more than one point in common.

Proof: Let *m* and *l* be two given different lines.

Let these intersect at two distinct points, say, P and Q.

This implies that both the lines *l* and *m* pass through two distinct points P and Q.

But according to the first postulates of Euclid, *i.e.,* a straight line may be drawn from any one point to any other point. We infer that lines *l* and *m* are one and the same line. But *l* and *m* are two different lines. Which is contradiction to our supposition that *l* and *m* intersects at two different points and therefore, the two lines can not intersect at two different points.

❑❑❑

REVISION EXERCISE

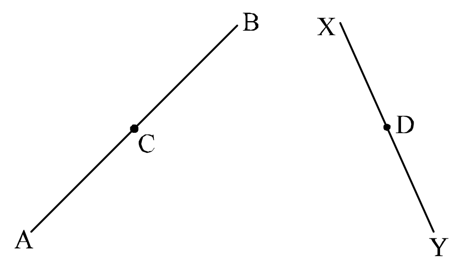
1. Define the following terms:

   (a) Collinear points (b) Concurrent lines

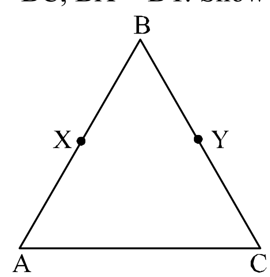
   (c) Points of intersection (d) Parallel lines

2. $\angle AOC$ and $\angle BOC$ are complementary angles. If $\angle AOC = (x + 10°)$ and $\angle BOC = 2(x + 5°)$, then find x.

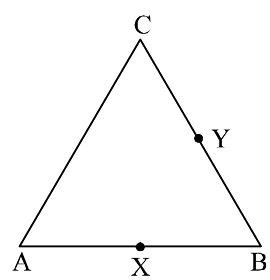
3. In figure we have: AC = XD, C is the mid point of AB and D is the mid-point of XY. Using an Euclid's axiom, show that AB = XY.



4. In the given figure, we have AB = BC, BX = BY. Show that AX = CY.



5. In the given figure, we have X and Y are the mid-points of AB and BC respectively and AX = CY. Show that AB = BC.



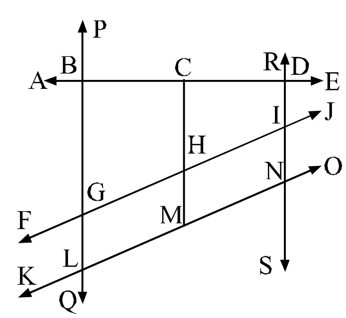
6. In the given figure name the following:

   (a) five rays.

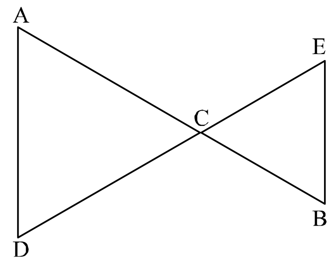
   (b) five line segments.

   (c) two pairs of non-intersecting line segments.

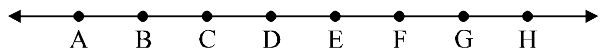
   A, B, C, D and E are collinear points.



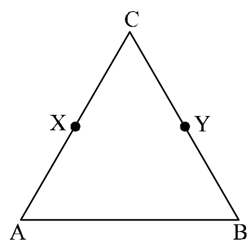
7. In the figure, we have AC = DC, CB = CE. Show that AB = DE.



8. Look at the figure. Show that length AH > Sum of lengths of AB + BC + CD.



9. In the figure, we have X and Y are the mid-points of AC and BC and AX = CY. Show that AC = BC.

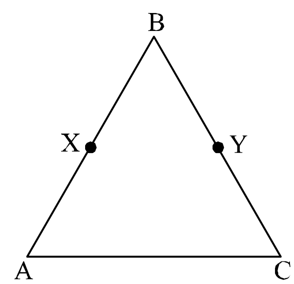


10. In the figure, we have

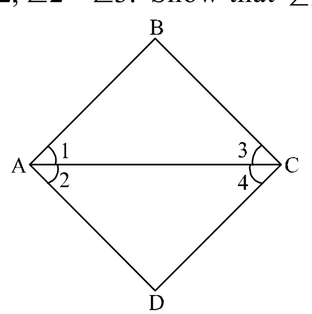
$BX = \frac{1}{2}AB$

$BY = \frac{1}{2}BC$ and AB = BC. Show that

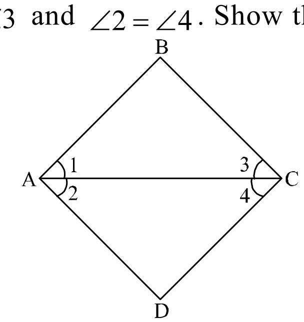
BX = BY



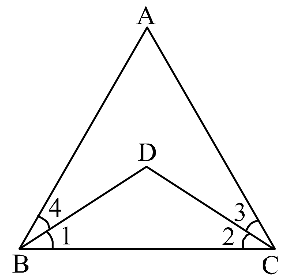
11. In the figure, we have $\angle 1 = \angle 2, \angle 2 = \angle 3.$ Show that $\angle 1 = \angle 3$.



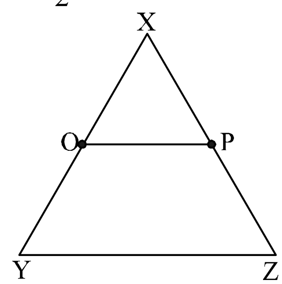
12. In the figure, we have $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Show that $\angle A = \angle C$.



13. In the figure, we have $\angle ABC = \angle ACB$, $\angle 3 = \angle 4$. Show that $\angle 1 = \angle 2$.



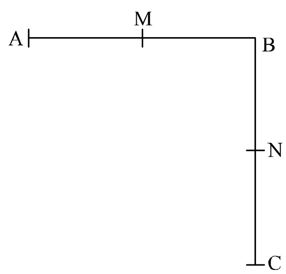
14. In the figure, of $OX = \frac{1}{2}XY, PX = \frac{1}{2}XZ$ and OX = PX, show that XY = XZ.



15. In the figure:

i) AB = BC, M is the mid-point of AB and N is the mid-point of BC. show that AM = NC.

ii) BM = BN, M is the mid-point of AB and N is the mid-point of BC. Show that AB = BC.



16. Read the following statement:

An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all its angles are 60° each.

Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all sides and all angles are equal in a equilateral triangle.

17. Study the following statement:

"Two intersecting lines cannot be perpendicular to the same line."

Check whether it is an equivalent version to the Euclid's fifth postulate.

[**Hint:** Identify the two intersecting lines *l* and *m* and the line *n* in the above statement].

18. Read the following statements which are taken as axioms:

i) If a transversal intersects two parallel lines, then, corresponding angles are not necessarily equal.

ii) If a transversal intersect two parallel lines, then alternate interior angles are equal. Is this system of axioms consistent? Justify your answer.

19. Read the following two statements which are taken as axioms:
i) If two lines intersect each other, then the vertically opposite angles are not equal.
ii) If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°.
In this system of axioms consistent? Justify your answer.

20. Read the following axioms:
i) Things which are equal to the same thing are equal to one another.
ii) If equals are added to equals, the wholes are equal.
iii) Things which are double of the same thing are equal to one another.
Check whether the given system of axioms is consistent or inconsistent.

EXERCISE (Single Correct Type)

1. If B lies between A and C, AC = 15 cm and BC = 9 cm. Then the value of AB^2 is
(a) 30 cm^2 (b) 36 cm^2 (c) 20 cm^2 (d) 26 cm^2

2. P lies in the interior of ∠BAC. If ∠BAC = 70° and ∠BAP = 42°, then the value of ∠PAC is
(a) 20° (b) 30° (c) 28° (d) 18°

3. Which of the following needs a proof?
(a) Postulate (b) Axiom (c) Definition (d) Theorem

4. Number of planes passing through three non-collinear points is
(a) 3 (b) 1 (c) 2 (d) Infinitely many

5. Which of the following is a true statement?
(a) A line has a definite length.
(b) A ray has two end points
(c) A point always determines a unique line.
(d) Three lines are concurrent when they have only one point in common.

6. Which is true?
(a) A line segment $\overline{AB}$ when extended in both directions is called ray $\overrightarrow{AB}$.
(b) Ray $\overrightarrow{AB} = \text{ray}\overrightarrow{BA}$.
(c) Ray $\overrightarrow{AB}$ has one end point A.
(d) Ray $\overrightarrow{AB}$ has two end points A and B.

7. Which is false?
(a) Two circles are equal only when their radii are equal.
(b) A figure formed by line segments is called a rectilinear figure.
(c) Only one line can pass through a single point.
(d) A terminated line can be produced indefinitely on both sides.

8. A point C is the mid – point of the line segment AB, if
   I. AC = AB
   II. C is the interior point of AB.
   III. AC = CB and C is the interior point AB.
   The given statement is true only when
   (a) I holds (b) II holds (c) III holds (d) none holds

9. Let us define a statement as the sentence which can be judged to be true or false. Which of the following is not a statement?
   (a) $3+5=7$
   (b) Kunal is a tall boy.
   (c) The sum of the angles of a triangle is $90°$.
   (d) The angles opposite to equal sides of a triangle are equal.

10. "Lines are parallel if they do not intersect" is stated in the form of
    (a) an axiom (b) a definition (c) a postulate (d) a proof

11. If AC = XD, C is the mid-point of AB and D is the mid-point of XY, then
    (a) AB = XY (b) AB ≠ XY (c) AB < XY (d) AB > XY

12. Number of lines passing through a given point
    (a) 2 (b) 3 (c) 10 (d) un countable

13. Which of these statements are correct?
    (a) Every line has a definite length
    (b) Every line segment has a definite length
    (c) Length of the line $\overleftrightarrow{AB}$ is not same as Length of line $\overleftrightarrow{BA}$.
    (d) Ray $\overleftrightarrow{AB}$ is the same as ray $\overrightarrow{BA}$

14. Euclid's second axiom is:
    (a) The things which are equal to the same thing are equal to one another.
    (b) The equals be added to equals, the whole are equals.
    (c) If equals be subtracted from equals, the remainders are equals.
    (d) Things which coincide with one another are equal to one another.

15. Euclid's fifth postulate is:
    (a) The whole is greater than the part.
    (b) A circle may be described with any centre and any radius.
    (c) All right angles are equal to one another.
    (d) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines it produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

16. The things which are double of the same thing are:
    (a) equal (b) unequal
    (c) halves of the same thing (d) double of the same thing

17. Axioms are assumed:
    (a) Universal truths in all branches of mathematics
    (b) Universal truths specific to geometry
    (c) theorems
    (d) definitions

18. John is of the same age as Mohan. Ram is also of the same age as Mohan. State the Euclid's axioms that illustrates the relative ages of John and Ram.
(a) First Axiom (b) Second Axiom (c) Third Axiom (d) Fourth Axiom

MATRIX MATCH TYPE QUESTIONS

19. Column II gives postulate number by Euclid for statement in Column I match them correct.

Column I		Column II	
(A)	Postulated 1	(p)	A terminated line can be produced indefintely.
(B)	Postulated 2	(q)	All right angles are equal to one another.
(C)	Postulated 3	(r)	A straight line may be drawn from any one point to any other point.
(D)	Postulated 4	(s)	A circle can be drawn with any centre and any radius.

(a) (A-s), (B-r), (C-q), (D-p) (b) (A-r), (B-p), (C-s), (D-q)
(c) (A-s), (B-r), (C-p), (D-q) (d) (A-r), B-s), (C-q), (D-p)

20. Match them correctly

Column I		Column II	
(A)	Only one line can pass through a ….	(p)	One point
(B)	Infinite number of line can pass through ……	(q)	Two point
(C)	Two distinct lines cannot have more than ……. in common	(r)	Same side
(D)	For line segment …… are required.	(s)	Opposite side

(a) (A-s), (B-r), (C-q), (D-p) (b)(A-q), B-p), (C-s), (D-r)
(c) (A-q), (B-p), (C-p), (D-q) (d) (A-r), B-s), (C-q), (D-p)

4 Lines and Angles

DEFINITION OF SOME BASIC TERMS

Ray

We consider a line ℓ and distinct points A, B and C on it as shown in fig. (i)

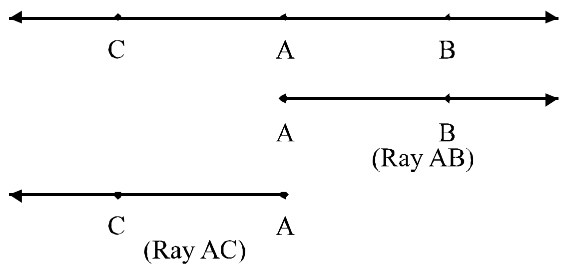
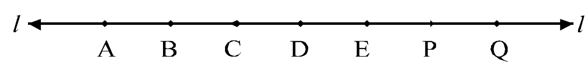


Fig.(i)

Now the part of the line ℓ which has only one end point at A and contains the point C is called ray AC as shown in fig.. The ray AB and the ray AC have only one common point and are parts of the same line. The two rays are called opposite rays. The symbol $\overrightarrow{AB}$ is also used for depicting the ray AB. Thus, in the fig(i). , the rays AB and AC are opposite to each other.

Collinear Points and Non-Collinear Points

Three or more points which lie on the same line are called collinear points



In the above figure the points A,B, C, D, E, P and Q are collinear points because they all lie on same line ℓ .

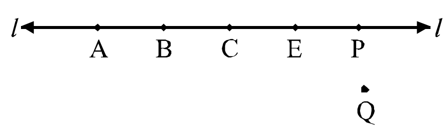
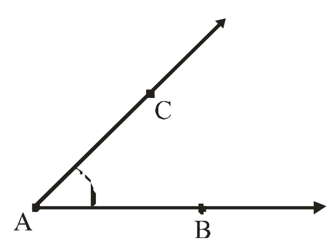


Fig.(ii)

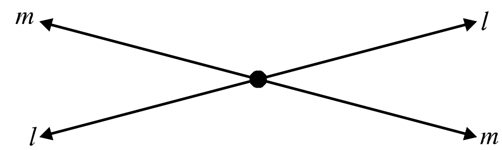
In fig. (ii), we observe that the points A, B, C, E and P are collinear but the points A, B, C, E, P and Q are not collinear.



Thus, there can be three or more points which may not be collinear. The points which are not collinear and called non-collinear points.

Intersecting Lines

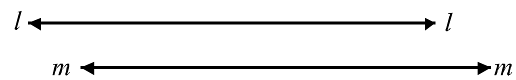
Two lines which are distinct and have a common point are called intersecting lines. The common point is called the point of intersection of two lines.



We can observe in fig. the intersection point of two intersecting lines is a unique point.

Non-Intersecting or Parallel Lines

Two distinct lines which are not intersecting are called non-intersecting lines or parallel lines.

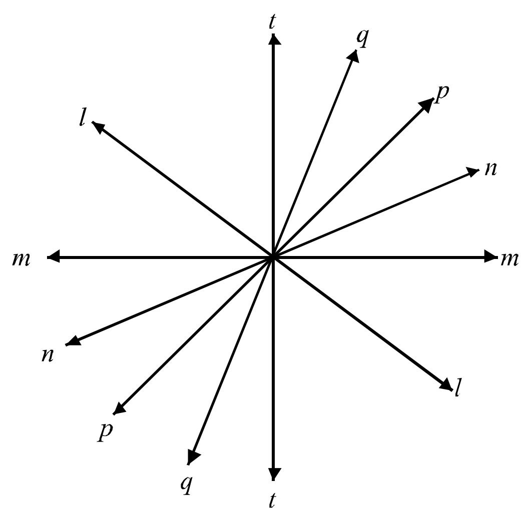


In fig., the lines *l* and m do not intersect and hence the lines *l* and m are parallel to each other.

Concurrent Lines

Three or more lines are concurrent if they all pass through a unique point.

In fig. the lines *l*, *m*, *n*, *p*, *q* and *t* are concurrent lines because all these lines pass through one and only one point.



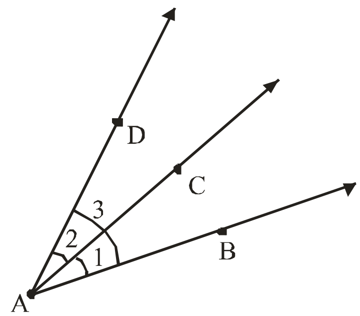
ANGLES AND PAIRS OF ANGLES

An angle is made when two rays are drawn with same end point. It is the measure of the extent to which the two rays are away from each other. The two rays, making the angle are called the arms of the angle and the common end point is called the vertex of the angle.

In the given figure two rays AB and AC originate from the same end point A.

This can be denoted by $\angle BAC$ and read as, the angle BAC. The common end point A of the two rays forming the angle $\angle BAC$ is called the vertex of the angle. Also, the rays AB and AC are called the arms of the angle $\angle BAC$. The angle $\angle BAC$ can also be denoted as $\angle A$.

In fig., ∠BAC, ∠CAD and ∠BAD are made with same vertex A. If we need to write these angles in short form, we can use the symbols ∠1, ∠2 and ∠3 respectively.

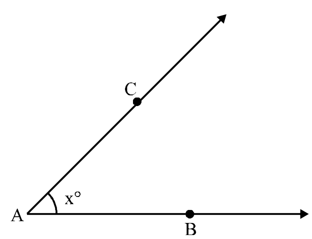


Measurement of Angles

One of the most commonly used unit for the measurement of an angle is one degree measurement and is denoted as 1°.

Acute Angles

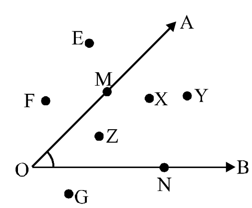
An angle which measures between 0° and 90 is called an Acute angle. Shown in fig.



$m(\angle BAC) = x°$ and $0° < x° < 90°$

Interior and Exterior of an angle

An ∠AOB divides the plane of the paper into 3 parts:

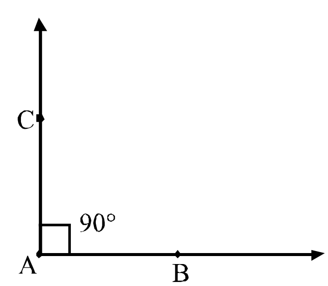


a) The part consisting of all those points which lie inside the angle called the *interior of the angle*.
e.g., X, Y, Z

b) The part consisting of all those points which lie on the boundary of the angle. e.g.: M, N

c) The part consisting of all those points which lie outside the angle called the *exterior of the angle*. e.g., E, F, G.

Right Angle



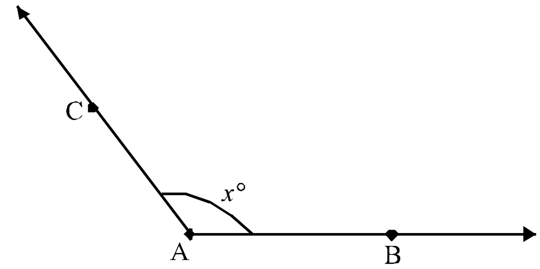
An angle which measures exactly equal to one right angle or 90° is called a right angle. Shown in Fig.

$m(\angle BAC) = 90°$

Therefore, the angle ∠BAC is right angle. Also, we can say AC ⊥ AB.

Obtuse Angles

An angle which measures between 90° and 180° is called an obtuse angle.



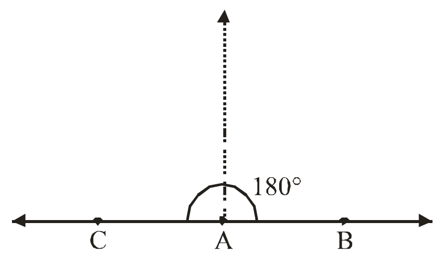
In figure.

m (∠BAC) is an obtuse angle.

For example, if m (∠BAC) = 105°, then the angle ∠BAC is an obtuse angle.

Straight Angles

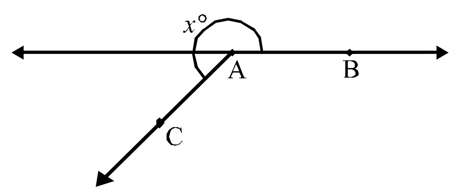
An angle which measures exactly equal to 180° is called a straight angle.



In fig. the rays AB and AC both originate from the same end point A. Also AB an AC are in one line and are opposite to each other. The angle ∠BAC drawn in fig. measures equal to two right angles i.e., 180° and hence it is a straight angle.

Reflex angles

An angle which measures between 180° and 360° is called a reflex angle. In figure.



the reflex ∠BAC = x° and 180° < x° < 360°

Complementary Angles

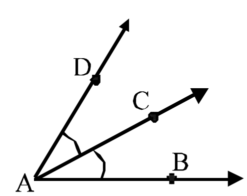
Two angles are said to be complementary if the sum of their measures in degrees is 90°.

Supplementary Angles

Two angles are said to be supplementary if the sum of their measures in degrees is 180°.

Adjacent Angles

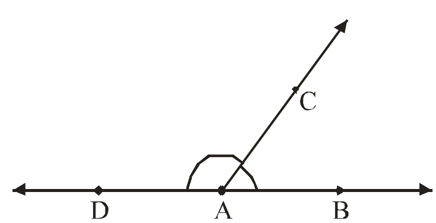
Two angles having same vertex and a common arm are said to be adjacent angles if their non-common arms (i.e. other than the common arm) are on the opposite side of the common arm.



In figure angles ∠BAC and ∠CAD are adjacent angles.

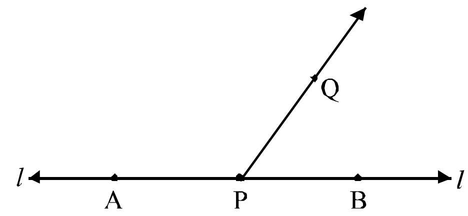
Linear Pairs of Angles

Two adjacent angles make a linear pair of angles, if the non common arms of these angles are two opposite rays (with same end point).



In figure the angles ∠BAC and ∠CAD forms a linear pair of angles because the non-common arms AB and AD of the two angles are two opposite rays. Moreover, ∠BAC + ∠DAC = 180°.

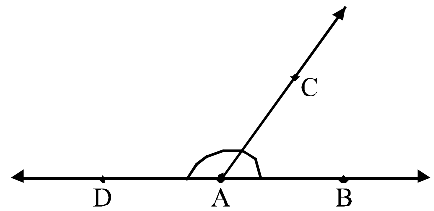
Linear Pair Axiom



If a ray stands on a line, then the sum of the pair of adjacent angles so formed is 180°. In other words, we can say that, the two angles form a linear pair of angles.

The above axiom can be stated in the reverse way as below:

Axiom



If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

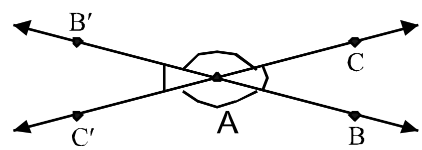
In figure, angles ∠BAC and ∠CAD are adjacent angles such that,

∠BAC + ∠CAD = 180°
Then, we have
∠BAD = 180°
Thus, ∠BAD is straight angle.

Therefore, AB and AD are in one line

The axioms are called linear pair axioms. The two axioms are reverses of each other.

Vertically opposite Angles

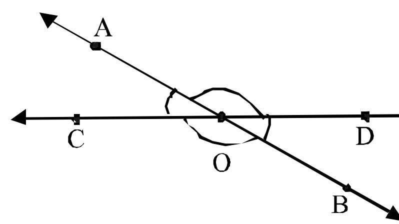


Two angle having same vertex are said to form a pair of vertically opposite angles, if their arms form two pair of opposite rays.

In figure ∠BAC and ∠B′AC′ are pairs of vertically opposite angles.

Similarly, we find that ∠CAB′ and ∠BAC′ is another pair of vertically opposite angles.

If two lines intersect each other, then the vertically opposite angles are equal.

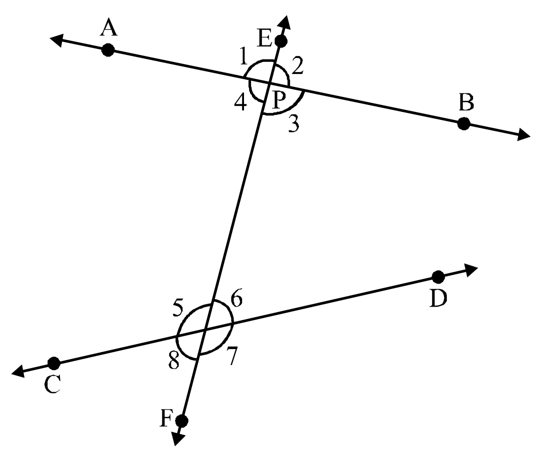


If AB and CD intersect each other at O then, ∠AOC = ∠BOD

and ∠AOD = ∠BOC

i.e., two pairs of vertically opposite equal angles.

LINES AND A TRANSVERSAL



A line intersecting two or more than two distinct lines at distinct points is called a transversal.

Transversal EF intersects line AB and CD at point P and Q respectively.

The angles made by the transversal EF with the lines AB and CD are labeled as ∠1, ∠2, ∠3, ∠4, ∠5, ∠6, ∠7 and ∠8 as shown in figure.

Here ∠1, ∠2, ∠7 and ∠8 are called exterior angle whereas, ∠3, ∠4, ∠5 and ∠6 are called interior angles.

Corresponding angles

In the above figure,the pairs of corresponding angles are :

(i) ∠1 and ∠5 (ii) ∠2 and ∠6 (iii) ∠3 and ∠7 (iv) ∠4 and ∠8

Alternate Interior Angles

In the above figure, the pairs of alternate interior angles are:

(i) ∠3 and ∠5 (ii) ∠4 and ∠6.

Here, in each pair, the two angles are on the opposite sides of the transversal and also one angle lies below the line AB and the other lies above the line CD.

Alternate exterior angles

In the above figure, the pairs of alternate exterior angles are:

(i) ∠1 and ∠7 (ii) ∠2 and ∠8.

Consecutive interior angles

A pair of interior angles on the same side of the transversal are called consecutive interior angles. In the above figure, pairs of consecutive interior angles are:

(i) ∠3 and ∠6 (ii) ∠4 and ∠5

These angles, in pairs are also called pair of **co-interior** angles.

Properties of corresponding angles

Two very important properties relating to the corresponding angles are stated below as axioms.

Axiom (corresponding angles axiom)

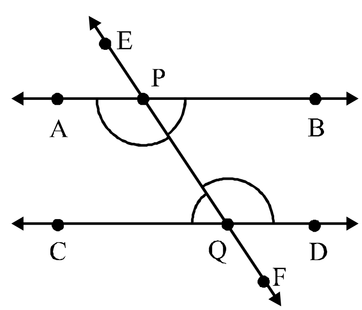
If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

Axiom (converse of corresponding angles axiom)

If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.

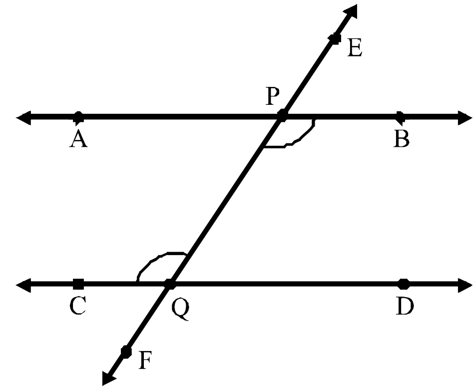
Properties of alternate angles

If a transversal interests two parallel lines, then each pair of alternate interior angles is equal.



If transversal EF intersects two parallel lines AB and CD at points P and Q respectively as shown in figure. Then angle APQ is equal to angle PQD and angle BPQ is equal to angle PQC

(Converse of the Above Result)



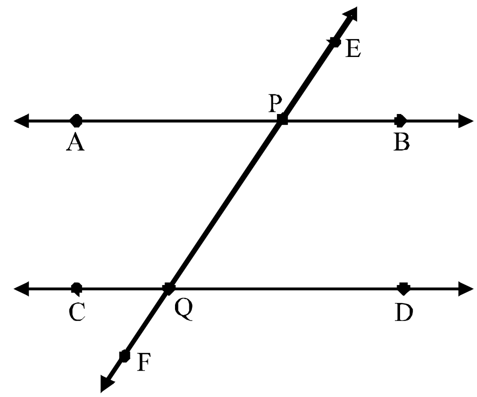
If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.

If transversal EF intersects two lines AB and CD at points P and Q respectively such that

$\angle BPQ = \angle CQP$

Then, AB || CD

Properties of consecutive interior angles



If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal (i.e., consecutive interior angles) is supplementary.

Transversal EF intersects two parallel lines AB and CD at points P and Q respectively.

Then, $\angle APQ + \angle CQP = 180^0$ and $\angle BPQ + DQP = 180^0$

If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

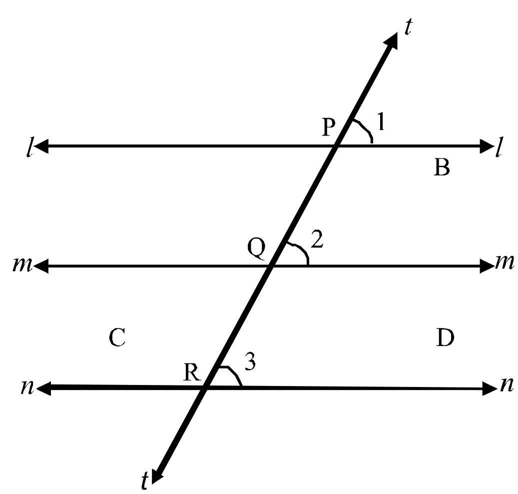
If transversal EF intersects lines AB and CD at points P and Q respectively such that

$\angle APQ + \angle CQP = 180^0$

(i.e., $\angle APQ$ and $\angle CQP$ is a pair of interior angles on the same side of the transversal EF and the angles are supplementary)

Then, AB || CD

Lines parallel to the same line



Lines which are parallel to the same line are parallel to each other.

If transversal t intersects lines l, m and n at points P, Q and R respectively.

Also, $m \,||\, l$ and $n \,||\, l$.

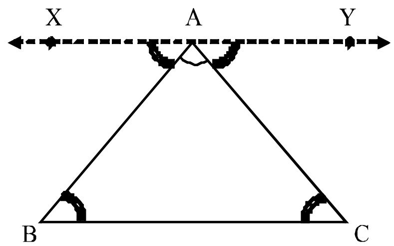
Then $m \,||\, n$.

ANGLE SUM PROPERTY OF A TRIANGLE

The sum of the three interior angles of a triangle is 180^0.

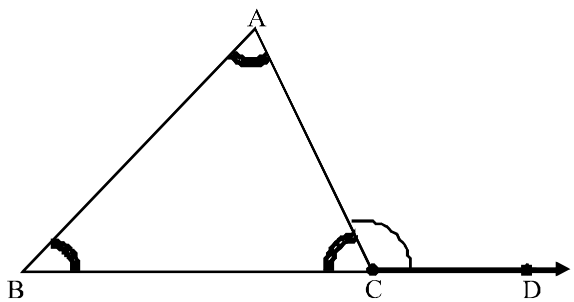
In $\angle ABC$, $\angle BAC$ and $\angle ACB$ are three angles of the triangle.

Then $\angle BAC + ABC + \angle ACB = 180^0$



If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

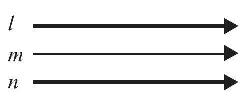
In a triangle ABC, the side BC is produced upto the point D as shown in figure.



Then, $\angle ACD = \angle BAC + \angle ABC$.

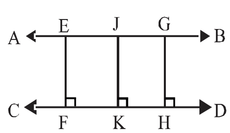
Parallel Lines

i) Two lines which are parallel to a given line are parallel to each other.



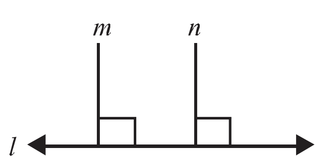
If $l \parallel m$ and $m \parallel n$, then $l \parallel n$

ii) The distance between the parallel lines is always the same.



If AB || CD then EF = GH = JK

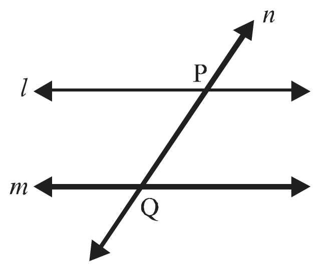
iii) Two lines perpendicular to a given line are parallel to each other.



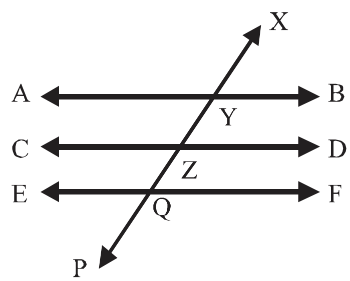
If $m \perp l$ and $n \perp l$ then $m \parallel n$

iv) All coplanar lines perpendicular to a given line are parallel to each other.

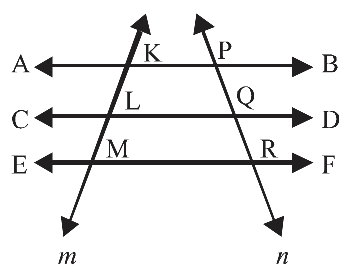
v) In the given figure, the transversal n intersect the two lines l and m at P and Q. $\overline{PQ}$ is the intercepted part of the line.



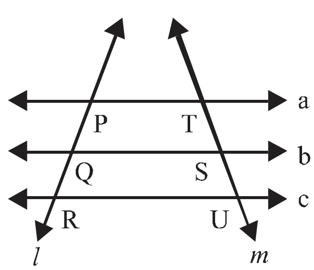
vi) Three lines are intersected by a transversal, two intercepts are formed.



vii) Three lines are intersected by two transversals two sets of intercepts are formed $\overline{KL}$ and $\overline{LM}$, $\overline{PQ}$ and $\overline{QR}$ are the intercepts.



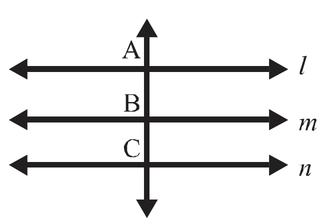
viii) If a system of parallel lines makes equal intercepts on a transversal, then the system makes equal intercepts on any number of transversals drawn to them



If transversal *l* makes equal intercepts on the parallel lines a, b and c i.e, PQ = QR
If transversal m also makes equal intercepts on the parallel lines a, b and c i.e., TS = SU

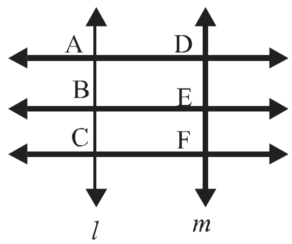
ix) Consider a system of three parallel lines *l*, m and n such that the distance between *l* and m is equal to the distance between m and n.

(a)



$l \parallel m \parallel n$, transversal p is intersected by l, m and n at right angles at A, B and C.

(b)

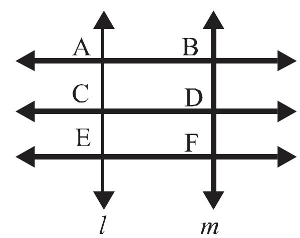


l and m are two transversals such that $\overline{AB} = \overline{BC} = \overline{DE} = \overline{EF}$, then $\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{DE}}{\overline{EF}}$

c) If a system of parallel lines makes equal intercepts on a transversal, then the distance between consecutive lines is the same

d) In a system of equidistant parallel lines, intercepts made in any number of perpendicular transversals are equal.

Intercept Theorem



If three parallel lines make equal intercepts on one transversal then they make equal intercepts on any other transversal as well. Any number of parallel lines which are equidistant from one another make equal intercepts on all transversals drawn to them. When three or more parallel lines are intersected by two transversals they make intercepts on them which are in proportion.

If AC = x cm and CE = y cm, then AC : CE or $\frac{AC}{CE} = \frac{x}{y}$

BD : DF or $\frac{BD}{DF} = \frac{x}{y}$

POLYGON

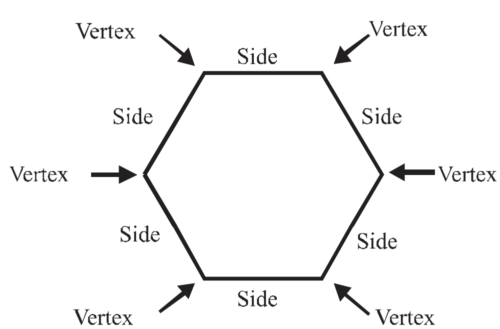
Introduction

i) A plane figure bounded by three or more line segments is called a polygon.

ii) The line segments which bound the polygon are called its sides.

iii) The point of intersection of two consecutive sides of a polygon is called a vertex.

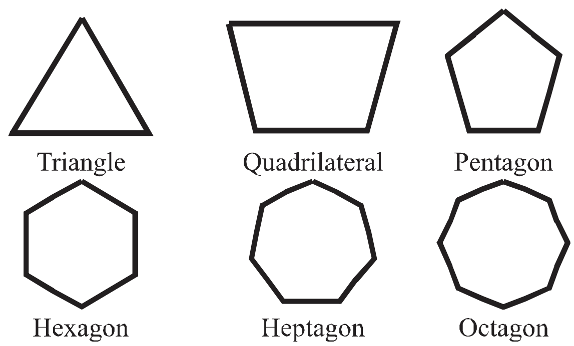
iv) The number of vertices of a polygon is equal to the number of its sides.



Types of Polygons: Polygons are named according to the number of their sides.

Number of sides	3	4	5	6
Name of the polygon	Triangle	Quadrilateral	Pentagon	Hexagon

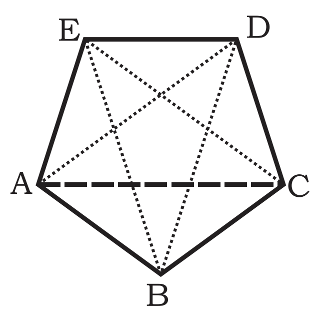
Number of sides	7	8	9	10
Name of the polygon	Heptagon	Octagon	Nonagon	Decagon



A polygon of n-sides is called n-gon. Thus, a polygon of 15 sides is a 15-gon.

Diagonal of a Polygon

A line segment joining any two nonconsecutive vertices of a polygon is called its diagonal.



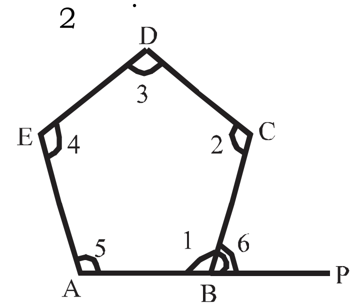
Thus, in the above figure, ABCDE is a polygon and each of the line segments AC,

AD, BD, BE and CE is a diagonal of the polygon.

Let us now have a look at the formula to find the number of diagonals in the polygon with n-sides.

The number of diagonals of the polygon with n-sides is given by the formula $\frac{n(n-3)}{2}$.

Interior and Exterior Angles



An angle formed by two consecutive sides of a polygon is called an interior angle or simply an angle of the polygon.

Thus, in the adjoining figure, each of the angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ and $\angle 5$ is an interior angle of the pentagon ABCDE.

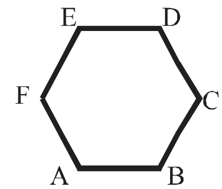
On producing a side of a polygon, an exterior angle is formed.

In the above figure, side AB of the pentagon ABCDE has been produced to P, to form an exterior angle, marked as $\angle 6$.

Clearly, an exterior angle and the adjacent interior angle together form a linear pair.

Thus, in a polygon, we have: Exterior angle + Adjacent interior angle = 180°

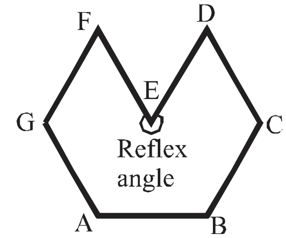
Convex Polygon



If each angle of a polygon is less than 180°, then it is called a convex polygon.

In the above figure, ABCDEF is a convex hexagon.

Concave Polygon



If at least one angle of a polygon is a reflex angle (i.e., more than 180°), then it is called a concave polygon.

In the above figure, ABCDEFG is a concave heptagon.

However, we shall be dealing with convex polygons only and by a polygon, we shall mean only convex polygon.

Regular Polygon

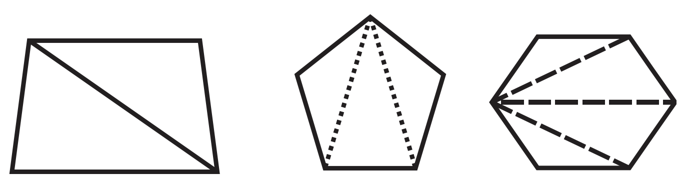
A polygon having all sides equal and all angles equal is called a regular polygon.

In a regular polygon,

i) all sides equal
ii) all interior angles equal
iii) all exterior angles equal

Sum of the interior angles of a polygon of n sides

Observe the following polygons carefully:



What do you notice?

A quadrilateral can be divided into 2 triangles; a pentagon into 3 triangles and a hexagon into 4 triangles. Thus, we conclude that a polygon of n sides can be divided into (n – 2) triangles

Since the sum of the angles of a triangle is 180°, so we have:

Sum of the interior angles of the polygon = $(n - 2) \times 180° = (n - 2) \times 2 \times 90° = (2n - 4)$ rt. Angles

Sum of the interior angles of a polygon of n sides = (2n – 4) rt. Angles

SUM OF THE EXTERIOR ANGLES OF A POLYGON OF n SIDES:

We know that the exterior angles and the adjacent interior angles of a polygon form a linear pair. Thus, we have:

Sum of all exterior angles + Sum of all interior angles = n × 180° = (2n) rt. angles

But, sum of all interior angles = (2n – 4) rt. angles

∴ Sum of all exterior angles = (2n) rt. angles – (2n – 4) rt. angles

= [2n – (2n – 4)] rt. angles = 4 rt. angles = 360°

Sum of all the exterior angle of a polygon is 360°

Important facts

From the above two results, we may derive the following.

For a regular polygon of n sides, we have:

i) Each interior angles $= \left[\dfrac{(n-2)\times 180°}{n}\right]^{\circ}$

ii) Each exterior angles $= \left(\dfrac{360}{n}\right)^{\circ}$

iii) Number of sides, $n = \dfrac{360^{\circ}}{(\text{each exterior angle})}$

iv) Exterior angle = 180° – (interior angle)

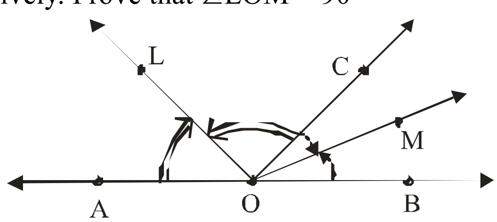
v) Sum of all the interior angles of a polygon of n sides = $(n – 2) × 180°$, $(n \geq 3)$.

vi) Number of sides of polygon $= \dfrac{360°}{180° - \text{each interior angle}}$.

vii) Sum of all the exterior angles formed by producing the sides of polygon = 360°.

SOLVED EXAMPLES

Example 1: In fig., ray OC stands on the line AB, ray OL and ray OM are angle bisectors of ∠AOC and ∠BOC, respectively. Prove that ∠LOM = 90°



Solutions : In figure, we have

∠AOC + ∠BOC = 180° …(i)

Now, ray OL bisects ∠AOC, therefore, $\angle LOC = \frac{1}{2}\angle AOC$

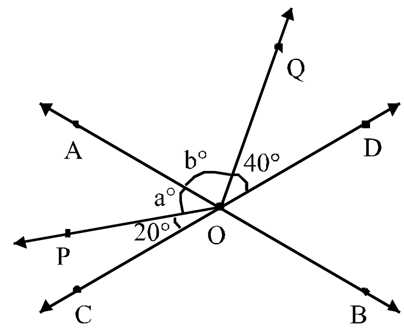
similarly, $\angle MOC = \frac{1}{2}\angle BOC$

$\Rightarrow \angle LOM = \frac{1}{2}\{\angle AOC + \angle BOC\}$

$= \frac{1}{2}\times 180° = 90°$ [by (i)]

Therefore, ∠LOM = 90°

Example 2: In fig. lines AB and CD intersect each other at point O. It is given that, a : b = 1 : 3. Find ∠BOC, ∠BOD and reflex ∠POD.



Solutions: ∠AOC + ∠AOD = 180°

⇒ (a°+20°) + (b° + 40°) = 180°

⇒ a° + b° = 120°

⇒ a + b = 120°.

Also, we are given that,

a : b = 1 : 3

Therefore, $a = \frac{1}{4}\times 120 = 30$ and $b = \frac{3}{4}\times 120 = 90°$.

Then, ∠AOD = ∠AOQ + ∠QOD

$= b° + 40°$ =90° + 40° = 130°

∠BOC = ∠AOD (vertically opposite angles)

∴ ∠BOC = 130°

∠BOD = ∠AOC (vertically opposite angles)

= a° + 20° =30° + 20° = 50°

∴ ∠BOD = 50°

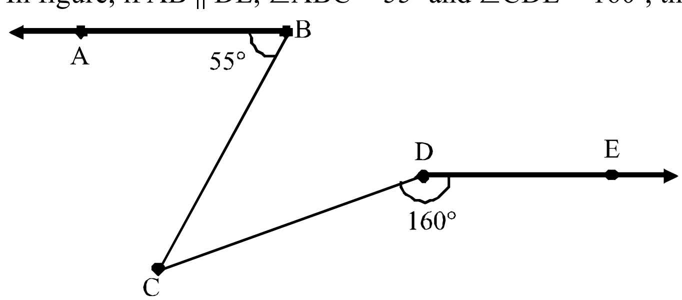
Now, ∠POD = a° + 90° + 40° =160°

Then, reflex ∠POD =360° – ∠POD

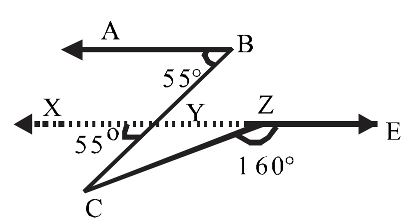
=360° – 160° =200°

∴ Reflex ∠POD = 200°

Example 3: In figure, if AB || DE, ∠ABC = 55^0 and ∠CDE = 160^0, then find ∠BCD.



Solution: ∠ABY = ∠XYC = 55° (corresponding angles)



∠XYC + ∠CYE = 180° (linear pair)

So, ∠CYE = 125°

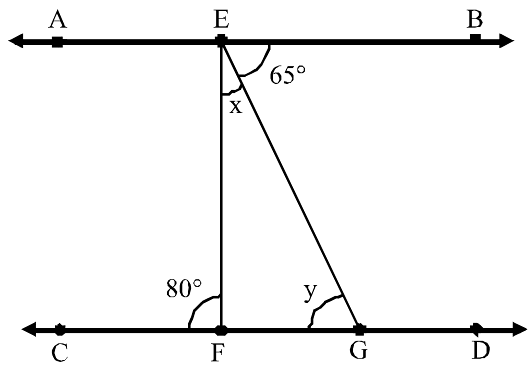
Similarly ∠YZC = 20°

In DYCZ

∠Y + ∠C + ∠Z = 180°

∴ ∠BCD = 35°

Example 4: In figure, if AB || CD, $\angle BEG = 65^0$ and $\angle EFC = 80^0$, then find x and y.



Solution: $x + y = 80°$ [exterior angle theorem

$y = 65^o$ [alternate interior angles]

$\therefore x = 15^0$

Example 5: Find each of the interior angles of a regular Octagon.

Solution: Each of the interior angle of a regular polygon of 'n' = $\frac{2n - 4}{n}$ rt. angles.

For an Octagon n = 8

$\therefore$ Each interior angles = $\frac{2 \times 8 - 4}{8} = \frac{3}{2}$ rt. angle = $\frac{3}{2} \times 90 = 135^o$.

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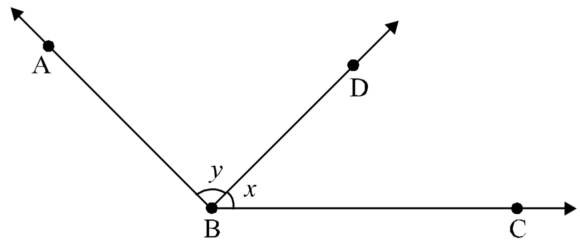
REVISION EXERCISE

LEVEL - I

1. For what value of $x + y$ in figure, will ABC be a line? Justify your answer.



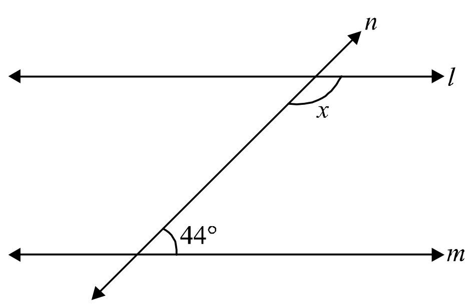
2. Can a triangle have all angles less than 60°? Give reason for your answer.

3. Can a triangle have two obtuse angles?

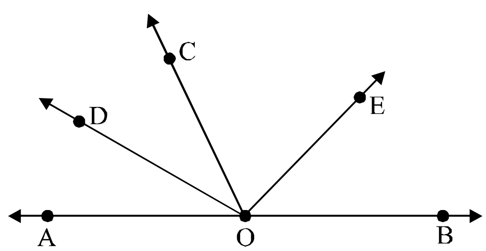
4. How many triangles can be drawn having its angles as 45°, 64° and 72°. Give reason for your answer.

5. How many triangles can be drawn having its angles as 53°, 64° and 63° ? Give reason for your answer.

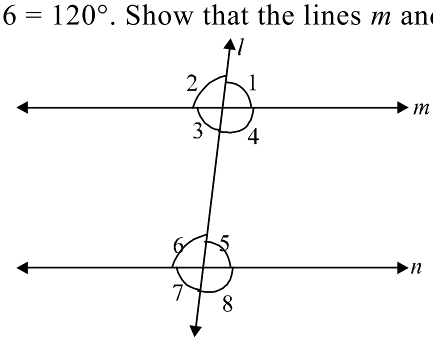
6. In figure, find the value of x for which the lines l and m are parallel.



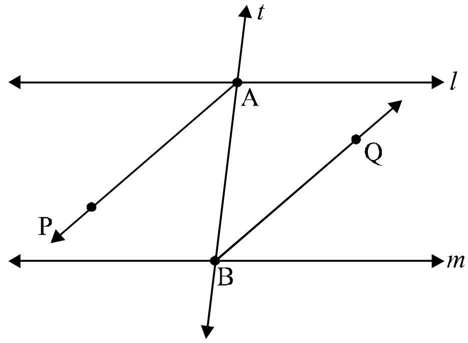
7. In figure OD is a bisector of $\angle AOC$, OE is the bisector of $\angle BOC$ and $OD \perp OE$. Show that the points A, O and B are collinear.



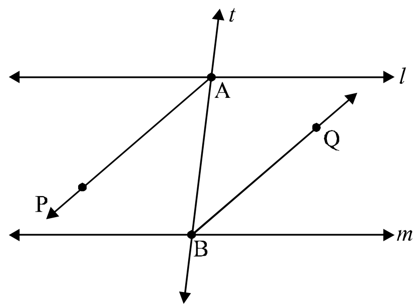
8. In figure $\angle 1 = 60°$ and $\angle 6 = 120°$. Show that the lines m and n are parallel.



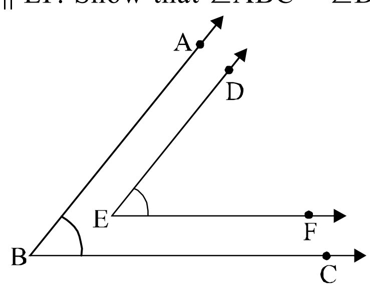
9. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal *t* with parallel lines *l* and *m* in figure. Show that AP || BQ.



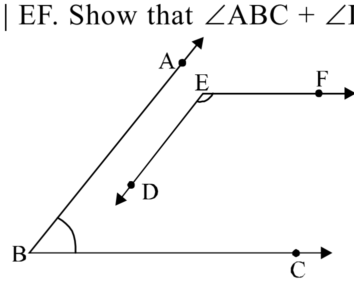
10. If in figure bisectors AP and BQ of the alternate interior angles are parallel, then show that *l* || *m*.



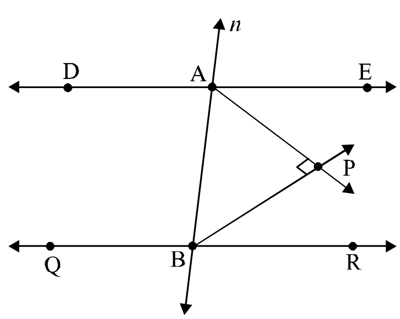
11. In figure BA || ED and BC || EF. Show that ∠ABC = ∠DEF.



12. In figure AB || ED and BC || EF. Show that ∠ABC + ∠DEF =180°.



13. In figure DE || QR and AP and BP are bisectors of ∠EAB and ∠RBA, respectively. Find ∠APB.

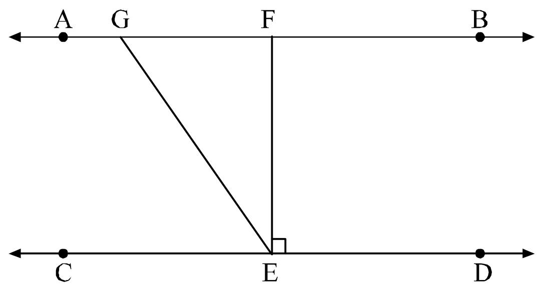


14. The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles of the triangle.

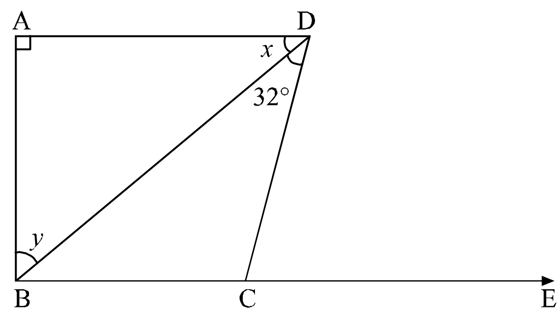
15. A triangle ABC is right angled at A. L is a point on BC such that $AL \perp BC$. Prove that $\angle BAL = \angle ACB$.

16. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

17. In the given figure, if AB || CD, EF $\perp$ CD and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.

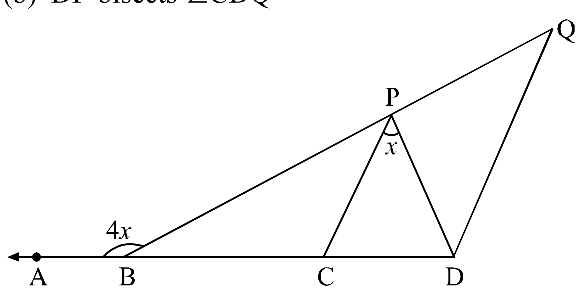


18. In the given figure, $AD \perp AB$, AD || BC. If $\angle BDC = 32°$ and x : y = 11 : 19, then find $\angle DCE$.



19. In the given figure, ABCD and BPQ are lines. BP = BC and DQ || CP. Prove that

(a) CP = CD (b) DP bisects $\angle CDQ$



20. ABCDE is a regular pentagon. Find each angle of ΔBDE.

21. If two lines intersect, prove that the vertically opposite angles are equal.

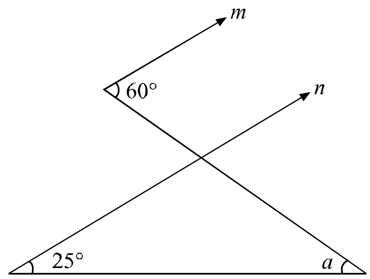
22. Prove that the sum of the three angles of a triangle is 180°.

23. If an angle is 10° more than its complement, then find it.

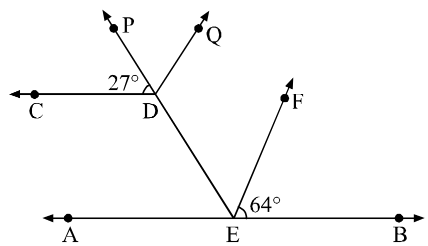
24. The supplement of an angle is one fifth of itself. Determine the angle and its supplement.

25. If two times the measure of one angle is three times the other which is complement of first angle, find the angles.

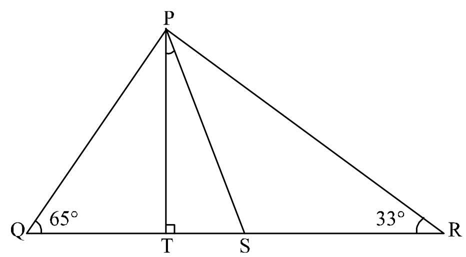
26. In the given figure, $m \parallel n$, then the find the $\angle a$.



27. In the given figure, EF || DQ and AB || CD. If ∠FEB = 64°, ∠PDC = 27° then, find ∠PDQ, ∠AED and ∠DEF.

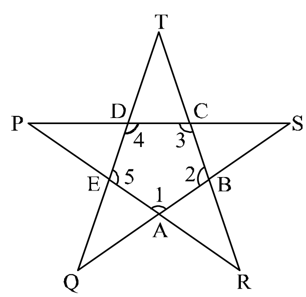


28. In the given figure, PT ⊥ QR and PS bisects ∠QPR. If ∠Q = 65° and ∠R = 33°, find ∠TPS.

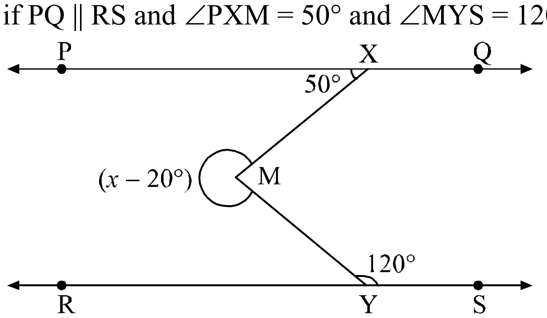


LEVEL - II

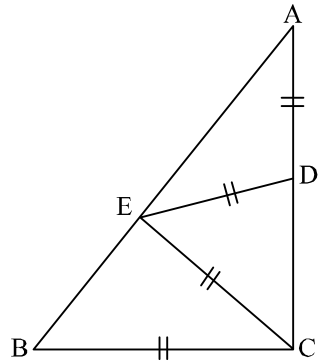
1. In the given figure, determine $\angle P + \angle Q + \angle R + \angle S + \angle T$.



2. In the figure given below, if PQ || RS and ∠PXM = 50° and ∠MYS = 120°, find the value of x.



3. In the given figure, AB = AC. D is a point on AC and E on AB such that AD = ED = EC = BC. Prove that (i) $\angle A : \angle B = 1 : 3$ and (ii) $\angle AED = \angle BCE$.



4. Prove that sum of three angles of a triangle is 180°. Using this result, find value of x and all the three angles of a triangle, if it is given that three angles of triangle are $(2x - 7)°$, $(x + 25)°$ and $(3x + 12)°$ respectively.

5. Bisectors of interior $\angle B$ and exterior $\angle ACD$ of a ΔABC intersect at the point T. Prove that $\angle BTC = \frac{1}{2}\angle BAC$.

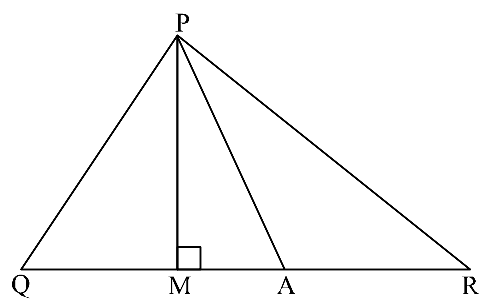
6. A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

7. Prove that through a given point, we can draw only one perpendicular to a given line.

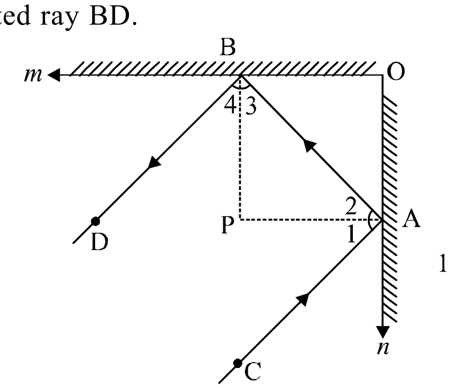
8. Prove the two lines that are respectively perpendicular to two intersecting lines intersect each other.

9. Prove that a triangle must have atleast two acute angles.

10. In figure $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and $PM \perp QR$. Prove that $\angle APM = \frac{1}{2}(\angle Q - \angle R)$



11. In figure m and n are two plane mirrors perpendicular to each other. Show that incident ray CA is parallel to reflected ray BD.

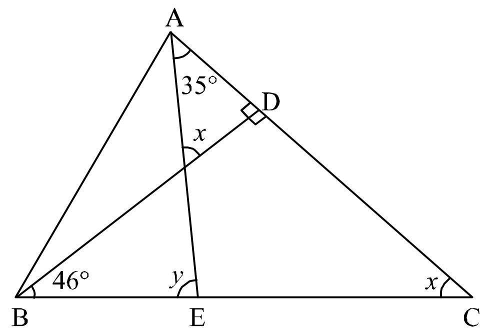


12. Bisectors of angles B and C of a triangle ABC intersect each other at the point O. Prove that $\angle BOC = 90° + \frac{1}{2}\angle A$.

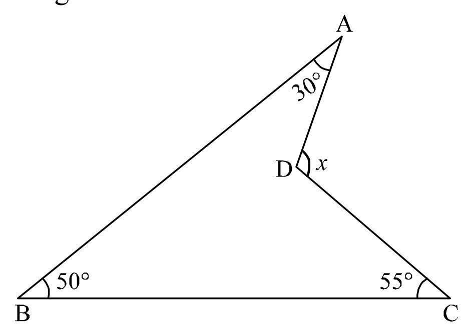
13. The sides BC, CA and AB of a ΔABC are produced in order, forming exterior angles ∠ACD, ∠BAE and ∠CBF. Show that ∠ACD + ∠BAE + ∠CBF = 360°.

14. The measures of angles of a hexagon are $x°$, $(x – 5)°$, $(x – 5)°$, $(2x – 5)°$, $(2x – 5)°$, $(2x +20)°$. Find the value of x.

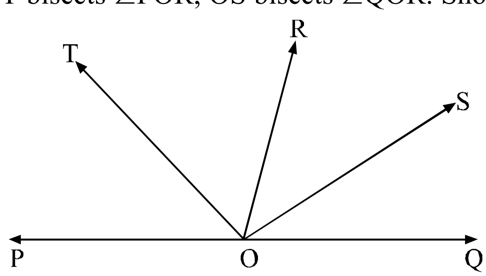
15. In figure, $BD \perp AC$, ∠DBC = 46° and ∠CAE = 35°, then find x and y.



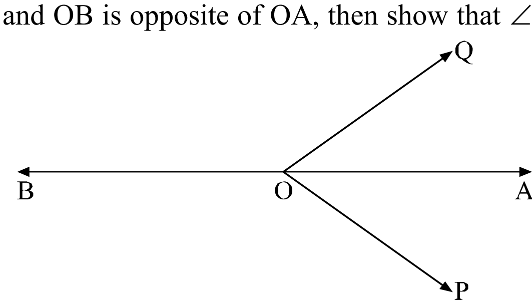
16. Find ∠ADC from the given figure.



17. POQ is a straight line OT bisects ∠POR, OS bisects ∠QOR. Show that ∠SOT is a right angle.

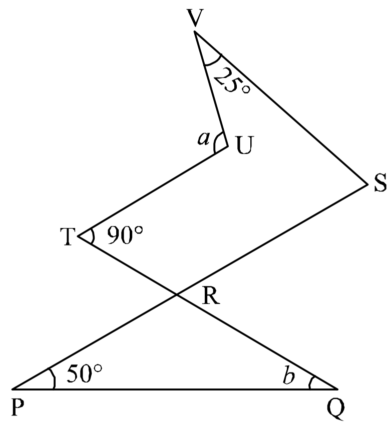


18. If OA bisects ∠POQ and OB is opposite of OA, then show that ∠POB = ∠QOB.

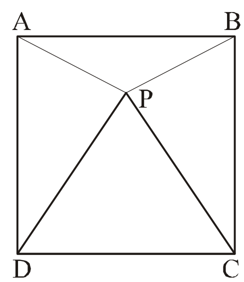


19. If the sides of an angle are respectively parallel to the sides of another angle, then prove that these angles are either equal or supplementary.

20. If in the given figure. TU || SR and TR || SV, then final $\angle a$ and $\angle b$.



21. ABCD is a square. P is a point inside the square such that $\angle PAB = \angle PBA = 15°$. Prove that ΔPDC is equilateral triangle.



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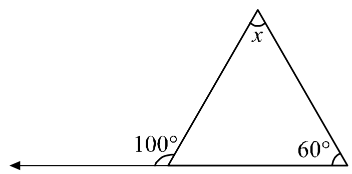
EXERCISE (Single Correct Type)

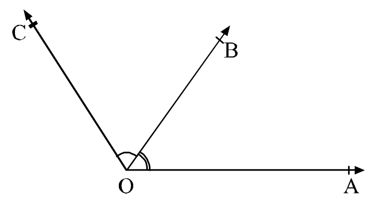
LEVEL - I

1. Value of x in the figure below is



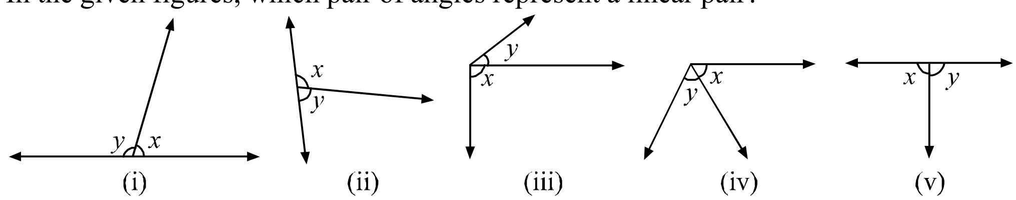
(a) 80° (b) 40° (c) 160° (d) 20°

2. In the given figure, a pair of adjacent angles is:



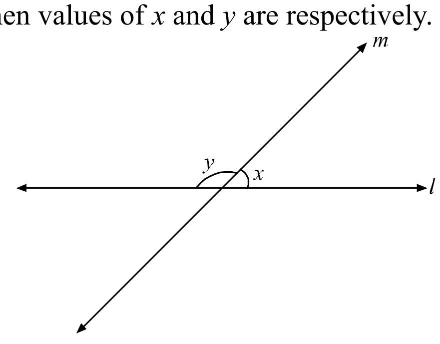
(a) ∠COA and ∠BOA (b) ∠COA and ∠BOC
(c) ∠AOB and ∠BOC (d) ∠AOC and ∠BOA

3. In the given figures, which pair of angles represent a linear pair?



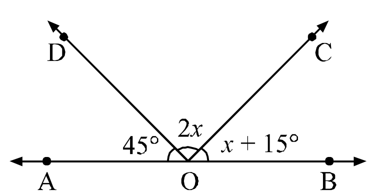
(a) figure (i) and (iii) (b) figure (iii) and (iv) (c) figure (ii) and (v) (d) figures (i), (ii) & (v)

4. In figure if $x : y = 1 : 4$, then values of x and y are respectively.



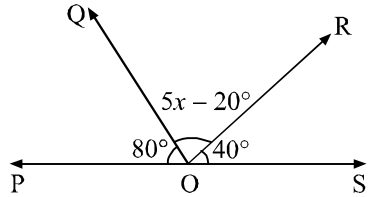
(a) 36° and 144° (b) 18° and 72° (c) 144° and 36° (d) 72° and 18°

5. In the given figure, value of x is



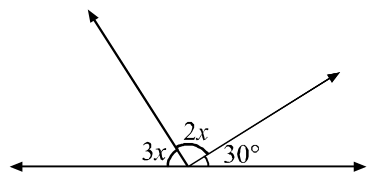
(a) 50° (b) 60° (c) 40° (d) 55°

6. In the given figure, POR is a line then ∠QOR is



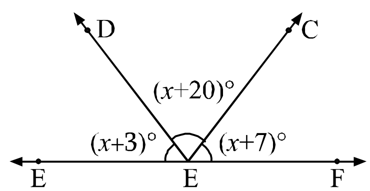
(a) 16° (b) 40° (c) 80° (d) 20°

7. In the given figure, the value of x is



(a) 20° (b) 30° (c) 40° (d) 50°

8. In the given figure, find the value of x.



(a) 50° (b) 90° (c) 60° (d) 80°

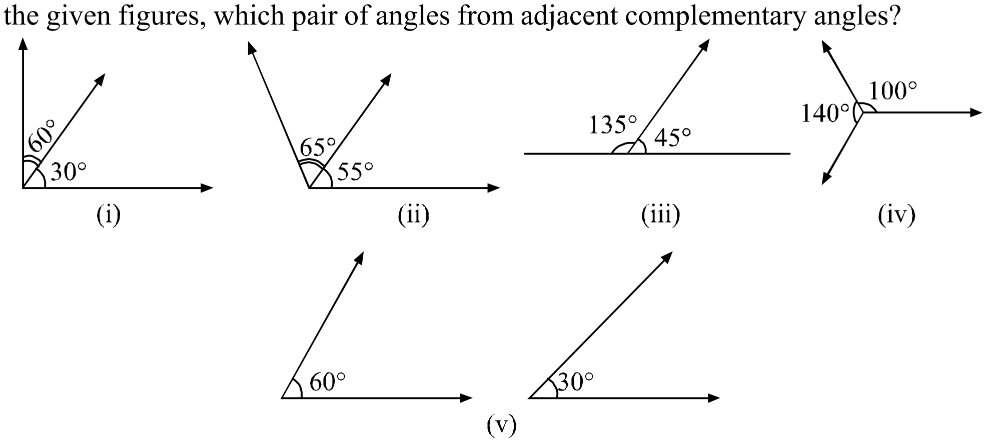
9. If two complementary angles are in the ratio 13 : 5, then the angles are
(a) 65°, 35° (b) 65°, 25° (c) 13°, 5° (d) 25°, 65°

10. Diagonals of a rhombus ABCD intersect each other at O, then, what are the measurements of vertically opposite angles ∠AOB and ∠COD?
(a) ∠ABO = ∠CDO (b) ∠ADO = ∠BCO (c) 60°, 60° (d) 90°, 90°

11. Diagonals of a rectangle ABCD intersect at O. If ∠AOB = 70°, then ∠DCO is
(a) 70° (b) 110° (c) 35° (d) 55°

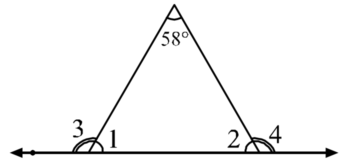
12. If AB = x + 3, BC = 2x and AC = 4x – 5, then for what value of 'x', B lies on AC?
(a) 8 (b) 5 (c) 2 (d) 3

13. In the given figures, which pair of angles from adjacent complementary angles?



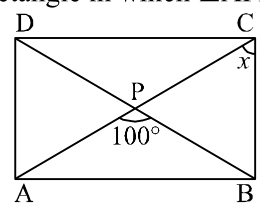
(a) (i) (b) (iii) (c) (ii) and (iv) (d) (i) and (iii)

14. In the given figure, $\angle 1 = \angle 2$ then the measurements of $\angle 3$ and $\angle 4$ are



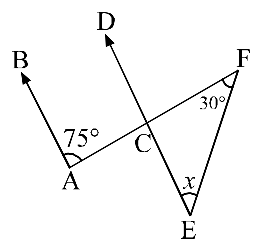
(a) 58°, 61° (b) 61°, 61° (c) 119°, 61° (d) 119°, 119°

15. In the given figure, ABCD is a rectangle in which $\angle APB = 100°$. The value of x is



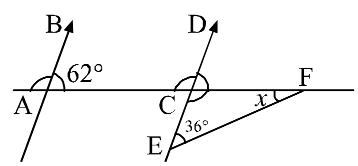
(a) 40° (b) 50° (c) 60° (d) 70°

16. In the given figure, AB || CD the value of x is



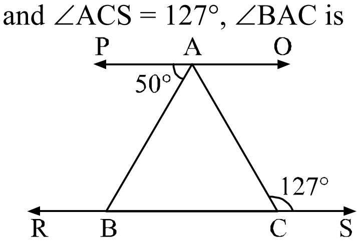
(a) 45° (b) 60° (c) 90° (d) 105°

17. In the figure, AB || ED, the value of x is



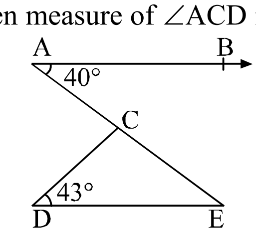
(a) 26° (b) 36° (c) 62° (d) 54°

18. In the given figure, PQ || RS and $\angle ACS = 127°$, $\angle BAC$ is



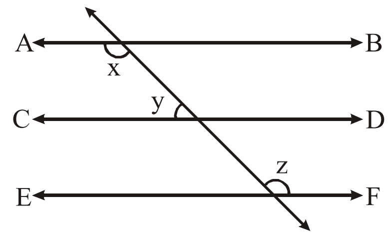
(a) 53° (b) 77° (c) 50° (d) 107°

19. In the given figure, AB || DE, then measure of $\angle ACD$ is



(a) 43° (b) 40° (c) 83° (d) 97°

20. In figure, if AB || CD, CD || EF and y : z = 3 : 7, x = ?

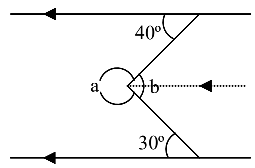


(a) 112 º (b) 116 º (c) 96 º (d) 126 º

MULTIPLE CORRECT ANSWER TYPE

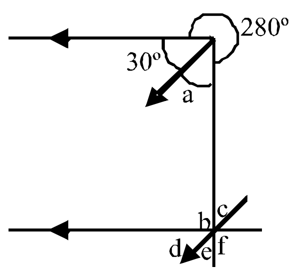
*This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which **ONE or MORE** is correct. Choose the correct options.*

21. In the figure, angle a and b are



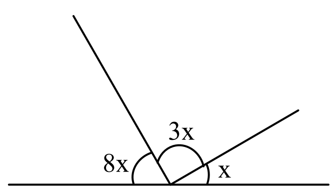
(a) a = 290° (b) b = 70º (c) a = 105° (d) b = 45°

22. In the figure, angle a and b are



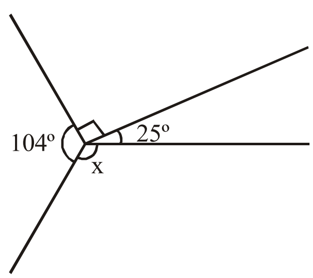
(a) a = 50° (b) a = 70º (c) b = 100º (d) b = 45º

23. Which are not the value of x?



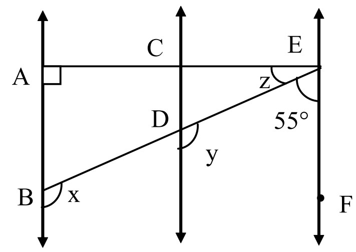
(a) 270 º (b) 15 º (c) 70 º (d) 45 º

24. Which are not the value of x?



(a) 141 º (b) 70 º (c) 105 º (d) 45 º.

25. In the given figure AB || CD and CD || EF. Also, EA ⊥ AB. If ∠BEF = 55°, then the values of x, y and z is

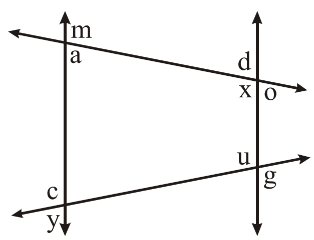


(a) x = y = 125º (b) z = 35º (c) x = y = 120º (d) z = 130º.

MATRIX MATCH TYPE

*Question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labeled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statements(s) in **Column II.***

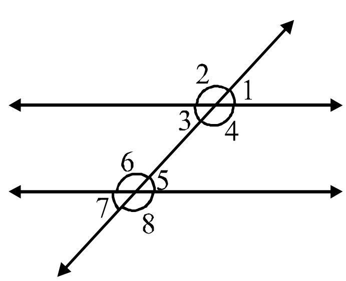
26. Match them correctly



Column - I	**Column - II**
(A) angles m and y	(p) alternate exterior pair
(B) angles a and d	(q) alternate interior pair
(C) angles d and u	(r) corresponding pair
(D) anlges c and g	(s) vertical pair

(a) (A - p), (B - r), (C - q), (D - s) (b) (A - q), (B - p), (C - r), (D - s)
(c) (A - p), (B - q), (C - r), (D - p) (d) (A - r), (B - p), (C - q), (D - s)

27. For the figure shown, Match column–I and II correctly

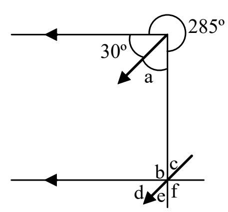


Column - I	**Column - II**
(A) corresponding angles	(p) 1 and 5
(B) alternate interior angles	(q) 4 and 6
(C) alternate exterior	(r) 1 and 7
(D) interior angles on same side of the transversal	(s) 4 and 5

(a) (A - p), (B - q), (C - r), (D - s) (b) (A - q), (B - p), (C - r), (D - s)

(c) (A - s), (B - q), (C - p), (D - r) (d) (A - p), (B - s), (C - q), (D - r)

28. In the figure, the value of angles are



Column - I	Column - II
(A) c =	(p) 30°
(B) d =	(q) 45°
(C) e =	(r) 105°
(D) f =	(s) 70°

(a) (A - q), (B - q), (C - r), (D - s) (b) (A - r), (B - q), (C - p), (D - s)

(c) (A - q), (B - p), (C - q), (D - r) (d) (A - s), (B - q), (C - p), (D - r)

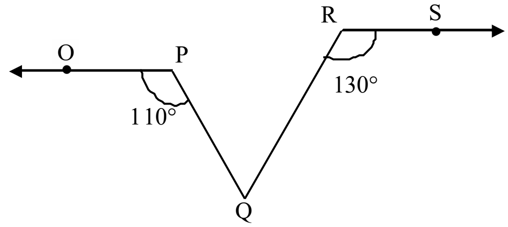
INTEGER TYPE

The answer to each of the questions is a single-digit integer, ranging from 0 to 9.

29. An angle is 14º more than its complementary angle, if angle is $(13k)°$ then k is

30. In twelve hours beginning from past mid-night, the minute hand and hour hand will overlap $(5k + 1)$ times, then k is

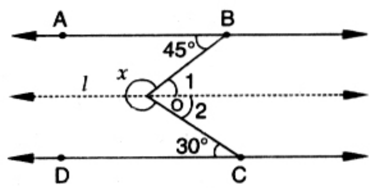
31. In the given figure OP || RS. If ∠PQR is 15K then the value of K is



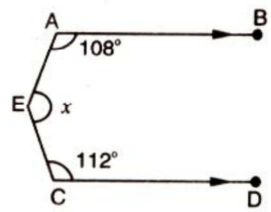
32. Angles of measures 10k and 8 k are a pair of complementary angle, then value of k is

33. Angles of measure 13 k - 2 and 7 k + 2 are a pair of supplementary angles, then the value of k is

34. In given figure, AB || CD. Then, find the value of $\frac{1}{3}(\angle 1 - \angle 2)$



35. In given figure, AB || CD. Then, find the value of $x/140$



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5 Co-ordinate Geometry

Introduction

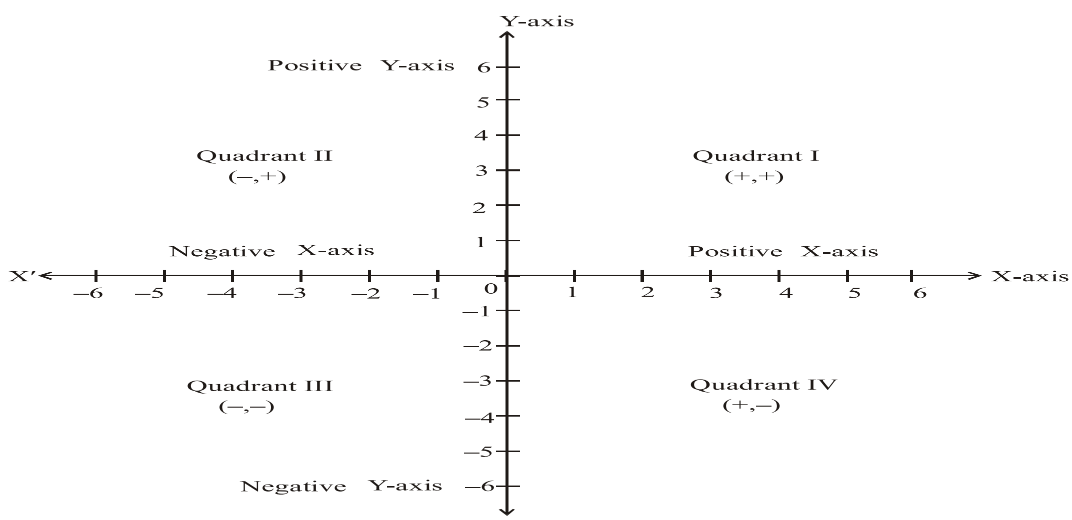
We have already studied how to locate a point on a number line. We also know how to find the position of a point on the number line. In this chapter, we shall study to locate a point on a plane and to find the position of a point on the plane.

Co–ordinate Geometry

It is the branch of Mathematics in which geometric problems are solved through algebra by using the coordinate system. So it is known as coordinate geometry.

Cartesian System

X′X and Y′Y two number lines are taken such that X′X is horizontal and YY is vertical and they are crossing each other at their zeroes or origin. The horizontal line X′X is called X–axis and the vertical line Y′Y is called Y–axis. The point on which X′X and Y′Y intersect each other is called origin and is denoted by O. The positive numbers lie on the directions OX and OY are called the positive directions of the x–axis and y–axis, respectively. Similarly, OX′ and OY′ are called the negative directions of the X–axis and the Y–axis, respectively.



The axes divide the plane into four parts and each part is called quadrant. In anticlockwise they are called quadrant–I, quadrant–II, quadrant–III and quadrant–IV. Hence, the plane consists of the axes and these quadrants. So, this plane is called cartesian plane, or the coordinate plane or the XY–plane. The axes are called the coordinate axes.

X–coordinate

The X–coordinate of a point is its perpendicular distance from Y–axis measured along the X–axis. The X–coordinate is also called the abscissa.

Y–coordinate :

The Y–coordinate of a point is its perpendicular distance from X–axis. The Y–coordinate is also called ordinate.

Note: (i) In quadrant I (+, +) both X–coordinate and Y–coordinate are positive.
(ii) In quadrant II (–, +) X–coordinate is negative where as Y–coordinate is positive.
(iii) In quadrant III (–, –) both X–coordinate and Y–coordinate are negative.
(iv) In quadrant IV (+, –) X–coordinate is positive where as Y–coordinate is negative.
(v) The coordinates of origin are (0, 0) i.e. X–coordinate is zero as well as Y–coordinate is also zero.

Distance Formula

The distance between any two points in the plane is the length of the line segment joining them.

The distance between two points $p(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e. $$PQ = \sqrt{(\text{Difference of abscissae})^2 + (\text{Difference of ordinates})^2}$$

Now, we have to find the distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$. PR and QS perpendiculars have been drawn to the x–axis. A perpendicular from the point P on QS is also drawn which meets at T in figure.

$OR = x_1$, $OS = x_2$

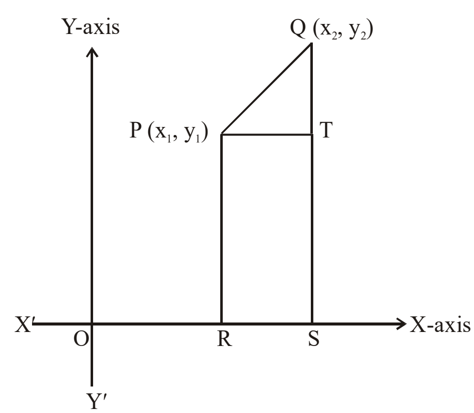
$\therefore RS = x_2 - x_1$

$\Rightarrow PT = x_2 - x_{-1}$ [RS = PT]

Again, $PR = y_1$, $QS = y_2$

$\therefore QT = y_2 - y_1$

Now, in right ΔPTQ, we have



$$PQ^2 = PT^2 + QT^2$$

[By Pythagoras Theorem]

$$\Rightarrow PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Hence, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

It is called distance formula.

The distance is always non–negative, so we take only the positive square root.

Note: (i) The distance of a point P(x, y) from the origin O(0,0) is given by

$$OP = \sqrt{x^2 + y^2}$$

(ii) We can also write :

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

SOLVED EXAMPLES

Example 1: In which quadrant do the following points lie?

(i) (4, 2) (ii) (–3, 5) (iii) (–2, –5) (iv) (4, –2)

Solution: (i) In the point (4, 2) abscissa and ordinate both are positive, so it lies in the first quadrant.

(ii) In the point (–3, 5) abscissa is negative and ordinate is positive, so, it lies in the second quadrant.

(iii) The point (–2, –5) lies in the third quadrant.

(iv) The point (4, –2) lies in the fourth quadrant.

Example 2: Find a point on X–axis which is equidistant from A(2, –5) and B(–2, 9).

Solution: We know that a point on x–axis is of the form (x, 0). So, Let P(x, 0) be the point equidistant from A(2, –5) and B(–2, 9).

Then, PA = PB

$\Rightarrow \sqrt{(x-2)^2 + (0+5)^2} = \sqrt{(x+2)^2 + (0-9)^2}$

$\Rightarrow (x-2)^2 + (5)^2 = (x+2)^2 + (-9)^2$

[Squaring both sides]

$\Rightarrow x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

$\Rightarrow -4x - 4x = 85 - 29$

$\Rightarrow -8x = 56$

$\Rightarrow x = -7$

Hence, the required point is (–7, 0).

Example 3: Show that the points (1, –1), (5, 2) and (9, 5) are collinear.

Solution: Let A(1, –1), B(5, 2) and C(9, 5) be the given points. Then, we have

$AB = \sqrt{(5-1)^2 + (2+1)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

$BC = \sqrt{(5-9)^2 + (2-5)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$

And $AC = \sqrt{(1-9)^2 + (-1-5)^2}$

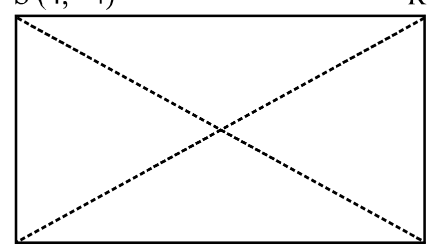
$= \sqrt{64+36} = \sqrt{100} = 10$

Now, it is clear that AC = AB + BC

Hence, A, B, C are collinear points.

Example 4: Prove that the point (–2, –1), (–1, 1), (5, –2) and (4, –4) are the vertices of a rectangle.

Solution : Let P(–2, –1), Q(–1, 1), R(5, –2) and S(4, –4) be the given points



Now, $PQ = \sqrt{[-2-(-1)]^2 + [-1-1]^2}$

$= \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$QR = \sqrt{[5-(-1)]^2 + [-2-1]^2}$

$= \sqrt{(6)^2 + (-3)^2} = \sqrt{45}$

$RS = \sqrt{[4-5]^2 + [-4-(-2)]^2}$

$= \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$ $SP = \sqrt{[4-(-2)]^2 + [-4-(-1)]^2}$

$= \sqrt{(6)^2 + (-3)^2} = \sqrt{45}$

$\therefore$ PQ = RS and QR = SP

Hence, the opposite sides are equal.

Again, diagonal $PR = \sqrt{[5-(-2)]^2 + [-2-(-1)]^2}$

$= \sqrt{(7)^2 + (-1)^2} = \sqrt{50}$ $QS = \sqrt{[4-(-1)]^2 + [-4-1]^2}$

$= \sqrt{(5)^2 + (-5)^2} = \sqrt{50}$

Hence the diagonals are equal.

Hence the given point P, Q, R, S are the vertices of rectangle.

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REVISION EXERCISE

SUBJECTIVE QUESTIONS (LEVEL - I)

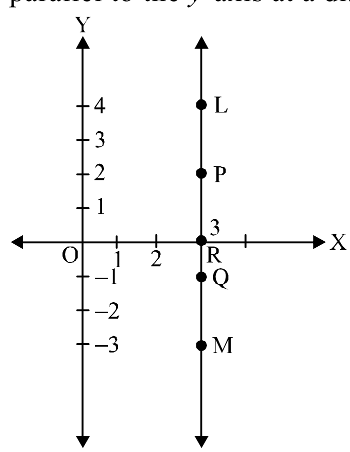
1. Plot the points (x, y) given by the following table:

x	2	4	–3	–2	3	0
y	4	2	0	5	–3	0

2. Plot the following points and check whether they are collinear or not:
(i) (1, 3), (–1, –1),(–2, –3) (ii) (1, 1), (2, –3), (–1, –2)
(iii) (0, 0), (2, 2), (5, 5)

3. Without plotting the points indicate the quadrant in which they will lie, if
(i) ordinate is 5 and abscissa is –3 (ii) abscissa is –5 and ordinate is –3
(iii) abscissa is –5 and ordinate is 3 (iv) ordinate is 5 and abscissa is 3

4. In given figure, LM is a line parallel to the y-axis at a distance of 3 units



(i) What are the coordinates of the points P, R and Q?
(ii) What is the difference between the abscissa of the points L and M?

5. In which quadrant or on which axis each of the following points lie?
(–3, 5), (4, –1), (2, 0), (2, 2), (–3, –6)

6. Which of the following points lie on y-axis?
A(1, 1), B (1, 0), C (0, 1), D (0, 0), E (0, –1), F (–1, 0), G (0, 5), H(–7, 0), I(3, 3)

7. Plot the points (x, y) given by the following table. Use scale 1 cm = 0.25 units.

x	1.25	0.25	1.5	–1.75
y	–0.5	1	1.5	–0.25

8. A point lies on the x-axis at a distance of 7 units from the y-axis. What are its coordinates? What will be the coordinates if it lies on y-axis at a distance of –7 units from x-axis.

9. Find the coordinates of the point
   (i) which lies on x and y axes both
   (ii) whose ordinate is –4 and which lies on y-axis
   (iii) whose abscissa is 5 and which lies on x-axis

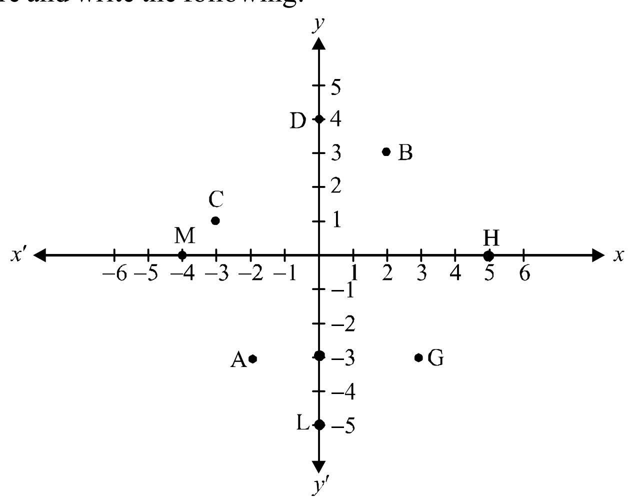
10. (a) Plot the points A (0, 4), B(–3, 0), C (0, –4),D (3, 0).
    (b) Name the figure obtained by joining the points A, B, C and D.
    (c) Also, name the quadrants in which sides AB and AD lie.

11. In which quadrant or on which axis the given points lie?

(a) (–2, 4)	(b) (3, –1)	(c) (–1, 0)	(d) (1, 2)
(e) (–3, –5)	(f) (0, 3)	(g) (0, –4)	(h) (5, 0)

Verify your answer by locating them on the Cartesian plane.

12. See the given figure and write the following:



(a) The coordinate of B.
(b) The coordinate of C.
(c) The point identified by the coordinates (–2, –3).
(d) The point identified by the coordinates (3, –3).
(e) The abscissa of the point D.
(f) The ordinate of the point H.
(g) The coordinates of the point L.
(h) The coordinates of the point M.

13. Plot the following points on a graph paper.

(a) (3, 4)	(b) (–2, 3)	(c) (–1, –2)	(d) (5, –1)
(e) (5, 0)	(f) (–5, 0)	(g) (0, 5)	(h) (0, –8)

14. Plot the points A(–2, 3), B (–2, 0),C(2, 0) and D(2, 6) on the graph paper. Join them consecutively and find the lengths of AC and AD.

15. In which quadrant will the points lie (a) the ordinate is 3 and abscissa is –4, (b) the abscissa is –5 and ordinate is –3, (c) the ordinate is 4 and abscissa is 5?

LEVEL - II

1. Find the area of the triangle whose vertices are (0, 4), (0, 0) and (2, 0) by plotting them on graph.

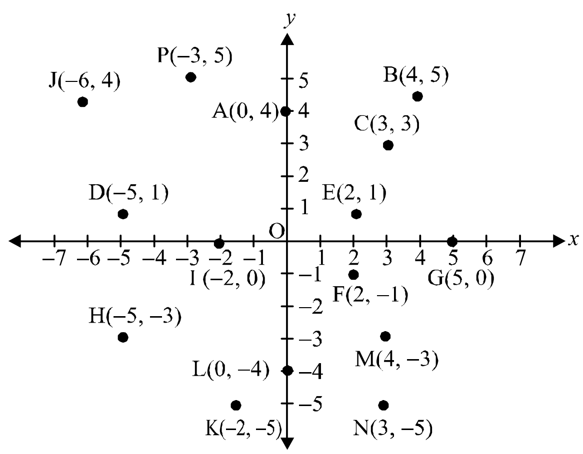
2. Write the coordinate of a point.
   (a) above x-axis lying on y-axis at a distance of 3 units from origin.
   (b) below x-axis and on y-axis at a distance of 8 units from origin.
   (c) right of origin and on x-axis at a distnace of 2 units.

3. Points A (5, 3), B (–2, 3) and D(5, –4) are three vertices of a square ABCD. Plot these points on a graph paper and hence find the coordinates of the vertex C.

4. Write the coordinates of the vertices of a rectangle whose length and breadth are 5 and 3 units respectively, one vertex at the origin, the longer side lies on the x-axis and one of the vertices lies in the third quadrant.

5. Plot the points P (1, 0), Q (4, 0) and S(1, 3). Find the coordinates of the point R such that PQRS is a square.

6. From the figure answer the following:
   (i) Write the points whose abscissa is 0
   (ii) Write the points whose ordinate is 0
   (iii) Write the points whose abscissa is –5.



7. Plot the points A (1, –1) and B (4, 5)
   (i) Draw a line segment joining these points. Write the coordinates of a point on this line segment between the points A and B
   (ii) Extend this line segment and write the coordinates of a point on this line which lies outside the line segment AB.

8. Write two solutions for the equation $2y + x = 6$.

9. Find the equation of a line, parallel to x-axis, at a distance of 2 units below x-axis.

10. Plot the points (5, 4), (0, 0) and (5, 0). Join the points and find the area of the figure.

EXERCISE (Single Correct Type)

1. Points (1, –1), (2, –2), (4, –5), (–3, –4)
   (a) lie in II quadrant (b) lie in III quadrant
   (c) lie in IV quadrant (d) do not lie in the same quadrant

2. If y coordinate of a point is zero, then this point always lies
   (a) in I quadrant (b) in II quadrant (c) on x-axis (d) on y-axis

3. On plotting the points O (0, 0), A (3, 0), B(3, 4), C (0, 4) and joining OA, AB, BC and CO which of the following figure is obtained?
   (a) Square (b) Rectangle (c) Trapezium (d) Rhombus

4. If P (–1, 1), Q (3, –4), R (1, –1), S (–2, –3) and T (–4, 4) are plotted on the graph paper, then the point(s) in the fourth quadrant are
   (a) P and T (b) Q and R (c) Only S (d) P and R

5. If the coordinates of the two points are P(–2, 3) and Q(–3, 5), then (abscissa of P) – (abscissa of Q) is
   (a) –5 (b) 1 (c) –1 (d) –2

6. If P(5, 1), Q(8, 0), R (0, 4), S (0, 5) and O (0, 0) are plotted on the graph paper, then the point(s) on the x-axis are
   (a) P and R (b) R and S (c) Only Q (d) Q and O

7. Abscissa of a point is positive in
   (a) I and II quadrants (b) I and IV quadrants
   (c) I quadrant only (d) II quadrant only

8. The points whose abscissa and ordinate have different signs will lie in
   (a) I and II quadrants (b) II and quadrants
   (c) I and III quadrants (d) II and IV quadrants

9. Which of the points P(0, 3), Q(1, 0), R (0, –1), S(–5, 0), T (1, 2) do not lie on the x-axis?
   (a) P and R only (b) Q and S only (c) P, R and T (d) Q, S and T

10. The point which lies on y-axis at a distance of 5 units in the negative direction of y-axis is
    (a) (0, 5) (b) (5, 0) (c) (0, –5) (d) (–5, 0)

11. The distance of the point (–3, 5) from x-axis in
    (a) –3 units (b) 5 units (c) 3 units (d) –5 units

12. The distance of the point (–1, –6) from y-axis is
    (a) –1 unit (b) –6 units (c) 1 unit (d) 6 unit

13. The point (3, –4) lies in the quadrant
    (a) 1st (b) IInd (c) IIIrd (d) IVth

14. The abscissa of a point is –7 and the ordinate in 2, then the point is
    (a) (2, –7) (b) (–7, 2) (c) (–2, 7) (d) (7, –2)

15. The vertical line drawn to determine the position of a point in the Cartesian plane is
    (a) origin (b) x-axis (c) y-axis (d) xy-plane

16. In which quadrant do the following points lie?
    (a) (3, 2) (b) (–2, 1) (c) (–1, –3) (d) (5, –1)

17. Point (–3, 5) lies in the
    (a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant

18. Signs of the abscissa and ordinate of a point in the second quadrant are respectively:
    (a) –, + (b) –, – (c) –, + (d) +, –

19. Abscissa of all the points on the x-axis is
    (a) 0 (b) 1 (c) 2 (d) any number

20. A point whose both the coordinates are negative will lie in
    (a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant

21. Points (1, –1), (2, –2), (4, –5), (–3, –4)
    (a) lie in II quadrant (b) lie in III quadrant
    (c) lie in IV quadrant (d) do not lie in the same quadrant

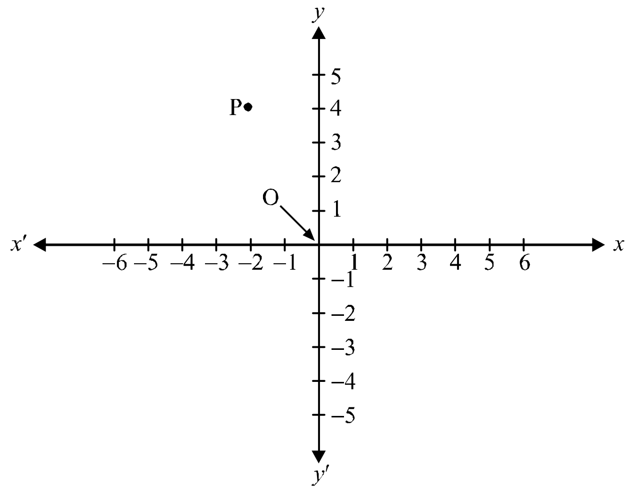
22. The points (–5, 2) and (2, –5) lie in the
    (a) same quadrant (b) II and III quadrants, respectively
    (c) II and IV quadrants, respectively (d) IV and II quadrants, respectively

23. If the perpendicular distance of a point P from the x-axis is 5 units and the foot of the perpendicular lies on the negative direction of x-axis, then the point P has
    (a) x coordinate = –5 (b) y coordinate = 5 only
    (c) y coordinate =–5 only (d) y coordinate = 5 or – 5

24. On plotting the points O(0, 0), A (3, 0), B(3, 4), C(0, 4) and joining OA, AB, BC and CO which of the following figure is obtained?
    (a) square (b) rectangle (c) trapezium (d) rhombus

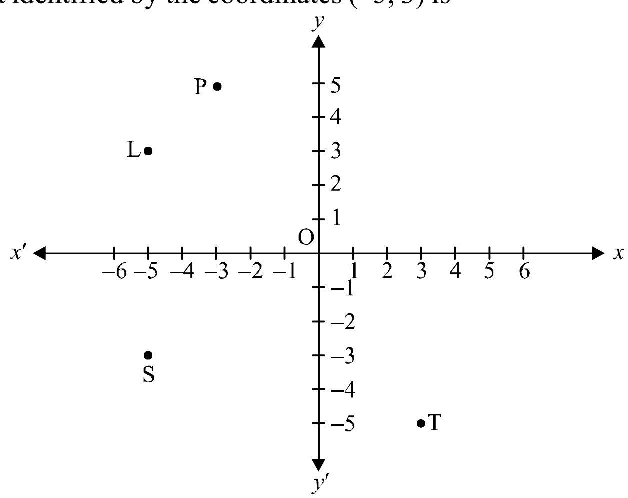
25. If the coordinates of the two points are P(–2, 3) and Q(–3, 5), then (ordinate of P) – (ordinate of Q) is
    (a) –5 (b) 1 (c) –1 (d) –2

26. In figure, coordinates of P are



(a) (–4, 2) (b) (–2, 4) (c) (4,–2) (d) (2, –4)

27. In figure, the point identified by the coordinates (–5, 3) is



(a) T (b) R (c) L (d) S

28. The point which lies on y-axis at a distnace of 5 units in the negative direction of y-axis is
(a) (0, 5) (b) (5, 0) (c) (0, –5) (d) (–5, 0)

29. The co-ordinates of a point are (x, y).If the point lies in the 2nd quadrant, then:
(a) $x > 0, y > 0$ (b) $x < 0, y > 0$ (c) $x < 0, y < 0$ (d) $x > 0, y < 0$

30. If perpendicular distance of a point P from the x-axis be 3 units along the negative direction of the y-axis then the point P has
(a) x-coordinates = – 3 (b) y – coordinate = – 3 (c) y – coordinate = 3 (d) none of these

MULTIPLE CORRECT ANSWER TYPE

*This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which **ONE or MORE** is correct. Choose the correct options.*

31. If A(5, 3), B(11, –5) and P(12, y) are the vertices of a right triangle right angled at P, then y =
(a) 2 (b) –2 (c) -4 (d) 4

32. If (x, 2), (–3, –4) and (7, –5) are collinear, then x =
(a) 60 (b) 63 (c) –63 (d) 3^2 (–7)

33. If the distance between the points $P(11,-2)$ and $Q(a,1)$ is 5 units. Then the value of a
(a) 12 (b) 14 (c) 15 (d) 7

34. If the points (1, 2), (–5, 6) and (a, –2) are collinear, then a is not equal to
(a) –3 (b) 7 (c) 2 (d) – 2

35. The distance between the points (a, 0) and (0, b) is
(a) $a^2 + b^2$ (b) a + b (c) $a^2 - b^2$ (d) $\sqrt{a^2 + b^2}$

36. If points (t, 2t), (–2, 6) and (3, 1) are collinear, then t =
(a) 3/4 (b) 4/3 (c) 5/3 (d) 3/5

37. If x is a positive integer such that the distance between P(x,2) and Q(3, –6) is 10 units, then x will be
(a) 3 (b) –3 (c) 9 (d) –9

38. Find the area of the circle whose centre is $(-3,2)$ and $(2,5)$ is a point on the circle.
(a) 34 π sq.units (b) 17 π sq. units (c) π sq. units (d) 3π units

39. Find the area of square whose one pair of the opposite vertices are $(3,4)$ and (5, 6).
(a) 12 sq. units (b) 8 sq. units (c) 6 sq. units d) 4 sq. units

40. A point is at a distance of 3 units from the x-axis and 5 units from the y=axis. Which of the following may be the co-ordinates of the point ?
(a) (5, 3) (b) (-5, 3) (c) (-5,-3) (d) (3, 5)

41. The point (2,7) is at a distance of _____ units from the y-axis.
(a) 2 (b) 7 (c) 2 + 7 (d) 7 – 2

MATRIX MATCH TYPE

*Question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labeled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statements(s) in **Column II.***

42. Column II given quadrant for points given in column I, match them correctly.

Column I	**Column II**
(A) 4, 4	(p) I quadrant
(B) –3, 7	(q) II quadrant
(C) 2, –3	(r) III quadrant
(D) –1, –3	(s) IV quadrant

(a) (A - q), (B - r), (C - p), (D - s) (b) (A - r), (B - q), (C - p), (D - s)
(c) (A - p), (B - q), (C - s), (D - r) (d) (A - s), (B - q), (C - r), (D - p)

43. Column II gives distance between two points given in column I, match them correctly.

Column I	**Column II**
(A) (–6, 7) and (–1, –5)	(p) $4\sqrt{5}$
(B) (a, 0) and (0, b)	(q) $2\sqrt{5}$
(C) (0, –1) and (8, 3)	(r) 13
(D) (–2, 3) and (0, –1)	(s) $\sqrt{a^2+b^2}$

(a) (A - r), (B - p), (C - q), (D - s) (b) (A - r), (B - s), (C - p), (D - q)
(c) (A - p), (B - q), (C - r), (D - s) (d) (A - q), (B - r), (C - q), (D - s)

INTEGER TYPE

The answer to each of the questions is a single-digit integer, ranging from 0 to 9.

44. If points (1, 2), (–5, 6) and (a, –2) are collinear, then a =

45. The three points (0, 0), $\left(3, \sqrt{3}\right)$ and (3, a) form an equilateral triangle, then a^2 =

46. If the distance between the points (4, p) and (1, 0) is 5, then what is the value of *p* if *p* is a positive integer.

47. If the points A (1, –1), B(5, 2) and C(k, 5) are collinear, then find the value of k.

48. A line segment is of length 10 units. If the coordinates of its one end are (2, –3) and the abscissa of the other end is 10, then what will be the ordinate in first quadrant.

49. What is the distance of the point (2, 5) from *x*-axis?

50. What is the distance of the point (0, 1) from the origin?

6 Linear Equation in Two Variables

Linear Equation in one Variable

An equation of the form $ax + b = 0$, $(a \neq 0)$ where a and b are real numbers and 'x' is a variable, is called a linear equation in one variable.

Linear Equation in two Variable

An equation of the form $ax + by + c = 0$ where $a, b \neq 0$ and x, y are variables, is called a linear equation in two variables. Here 'a' is called coefficient of x, 'b' is called coefficient of y and 'c' is called constant term.

Solution : Any pair of values of x and y which satisfy the equation $ax + by + c = 0$, is called a solution of it.

$x = 2, y = -1$ is a solution of equation $3x + 5y - 1 = 0$

Put $x = 2, y = -1$ in L.H.S. of given equation

$3 \times (2) + 5(-1) - 1$

$= 6 - 5 - 1 = 6 - 6$

$= 0 =$ R.H.S.

$x = 2, y = -1$ satisfies the given equation

Hence, $x = 2, y = -1$ is a solution of equation $3x + 5y - 1 = 0$

BASIC CONCEPTS AND IMPORTANT POINTS

1. An equation of the form $ax + by + c = 0$ where a, b and c are real numbers, such that $a \neq 0$, $b \neq 0$, is called a linear equation in two variables.
2. Any pair of values of x and y which satisfy the equation $ax + by + c = 0$ is called a solution.
3. A linear equation in two variables has infinitely many solutions.
4. The graph of every linear equation in two variables is always a straight line.
5. The graph of $x = a$ is a straight line parallel to y-axis, cutting x-axis at $x = a$.
6. The graph of $x = 0$ is y-axis.
7. The graph of $y = a$ is a straight line parallel to x-axis, cutting y-axis at $y = a$.
8. The graph of $y = 0$ is x-axis.
9. The graph of the equation $y = mx$ is a line passing through the origin.
10. Every point on the graph of a linear equation in two variables is a solution of that equation.

SOLVED PROBLEMS

Example 1: The equation $5x = 2$ is written in two variables as

(a) $5x + y = 2$ (b) $5xy = 2$

(c) $5x = 2y$ (d) $5x + 0y - 2 = 0$

Solution: **(d)** Standard form of the equation in two variables is $ax + by + c = 0$,

So, $5x = 2 \Rightarrow 5x + 0y - 2 = 0$

Example 2: The equation $y = 4x - 7$ has

(a) no solution (b) unique solution

(c) infinitely many solutions (d) exactly two solutions

Solution: **(c)** A linear equation in two variables has infinitely many solution.

Example 3: The graph of the linear equation $4x - 5 = 0$ is

(a) parallel to x-axis (b) lies along x-axis

(c) parallel to y-axis (d) passes through origin

Solution: **(c)** As graph of equation $x = k$ is parallel to y-axis.

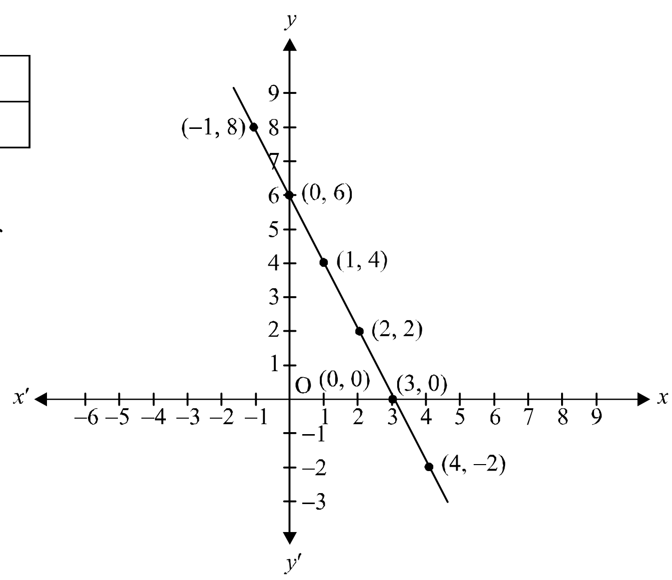
Example 4: Draw the graph for the following equations: (Read a few solutions from the graph and verify the same by actual substitution. In each case, find the points where the line meets the two axis).

(a) $2x + y = 6$ (b) $x - 2y = 4$

(c) $2(x - 1) + 3y = 4$

Solution (a) $2x + y = 6$

x	1	2
y	4	2



Check that $x = 1$, $y = 4$ and $x = 2$, $y = 2$ are solutions of the given equation. So, we use above table to draw the graph

We draw the graph by plotting the two points of the above table and then by joining the same. We find from the graph that $x = -1$, $y = 8$ and $x = 4$, $y = -2$ are also the solutions.

Verification: Putting $x = -1$ and $y = 8$ in L.H.S. of equation.

$2 \times -1 + 8 = 6 =$ R.H.S.

Putting $x = 4$ and $y = -2$ in L.H.S. of equation

$2 \times 4 + (-2) = 8 - 2 = 6 =$ R.H.S.

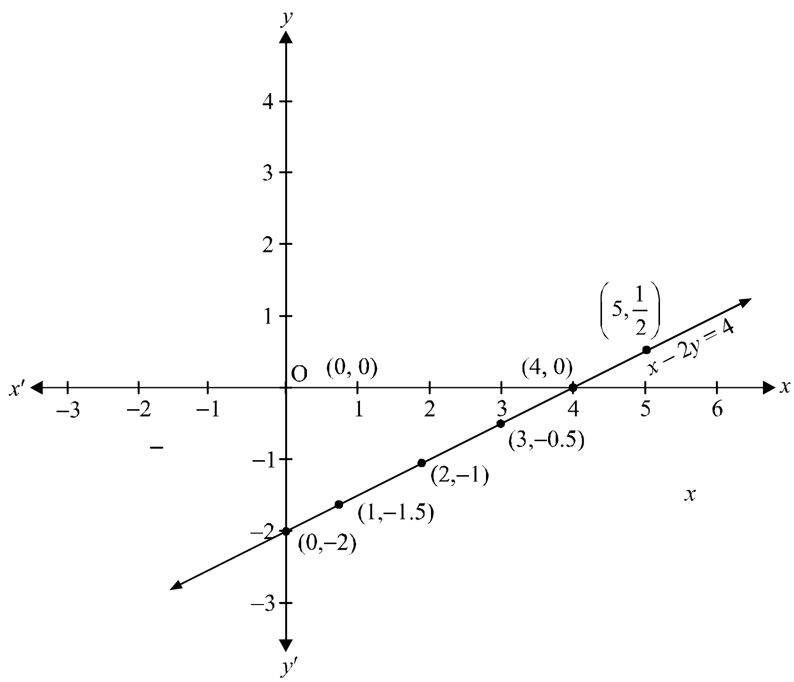
The line meets the x-axis at (3, 0) and y-axis at (0, 6).

(b) $x - 2y = 4$

x	2	3
y	–1	–0.5

Check that $x = 2$, $y = -1$ and $x = 3$, $y = -0.5$ are solutions of the given equation.

So, draw the graph using above table.



Verification: Putting $x = 1$, $y = -1.5$ in L.H.S. of equation.

$1-2\times(-1.5)=1+3=4=$ R.H.S.

Putting $x=5,\ y=\frac{1}{2}$ in L.H.S. of equation

$5-2\times\frac{1}{2}=5-1=4=$ R.H.S.

From the graph, $x = 1$, $y = -1.5$ and $x = 5$, $y=\frac{1}{2}$ are other solution.

The line meets the x-axis at (4, 0) and y-axis at (0, –2).

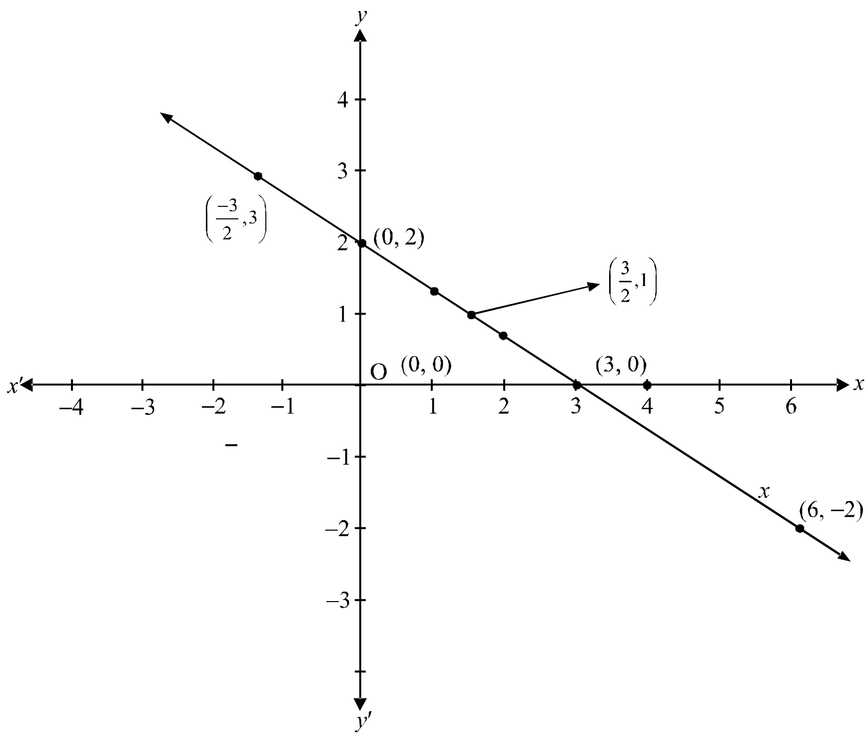
(c) $2(x-1)+3y=4$

$2x-2+3y=4$ or $2x+3y=6$

x	0	$-\frac{3}{2}$
y	2	3

Check that $x = 0$, $y = 2$ and $x = -\frac{3}{2}, y = 3$ are solutions of the given equation.

So, the graph using above table is



Verification: Putting $x = 6$ and $y = -2$ in L.H.S. $2 \times 6 + 3 \times -2 = 12 - 6 = 6 =$ R.H.S.

Putting $x = \frac{3}{2}$ and $y = 1$ in L.H.S. of equation

$2 \times \frac{3}{2} + 3 \times 1 = 3 + 3 = 6 = \text{R.H.S.}$

$x = 6$, $y = -2$ and $x = \frac{3}{2}$, $y = 1$ are the solutions from the graph. The line meets x-axis at (3, 0) and y-axis as (0, 2).

Example 5: Find the value of a. If each of the following equation has $x = 1$, $y = 1$ as a solution

(a) $3x + ay = 6$ (b) $5x + 2ay = 3a$

(c) $9ax + 12ay = 63$

Solution: (a) $3x + ay = 6$

If $x = 1$, $y = 1$ is a solution, then it must satisfy the equation.

$\therefore\ 3(1) + a(1) = 6$

$\Rightarrow\ a = 6 - 3 = 3.$

(b) $5x + 2ay = 3a$

$\therefore 5(1) + 2a(1) = 3a$

$\Rightarrow 5 = 3a - 2a \Rightarrow a = 5.$

(c) $9ax + 12ay = 63$

$\therefore 9a(1) + 12a(1) = 63$

$\Rightarrow 21a = 63 \Rightarrow a = \frac{63}{21} = 3.$

REVISION EXERCISE

1. Draw the graph of $(a)\,x = 3$ $\qquad (b)\,y = -2.$.

2. Draw the graph $y = -2x$. Show that the point (2, –5) is not on the graph.

3. Draw the graph of $y - 2x = -3$ and check, if (2, 3) is on the graph.

4. Express x in terms of y, it is being given that $7x - 3y = 15$. Check if the line represented by the given equation intersects the y-axis at $y = -5$.

5. Show that the points A (1, 2), B(–1, –16), C (0, –7) are on the graph $y = 9x - 7$.

6. If $x = 3$, $y = -2$ is a solution of the linear equation $3x - ky = 1$, then find the value of k.

7. Write the coordinates of any two points which lie on the line $x + y = 8$. How many such points exists?

8. Draw the graph of the equation $3x + 2y = 5$. From the graph, find the value of x, *when* $y = 4$.

9. Check whether $x = -2$ and $y = 6$ is a solution of $3(x-2)+2(y+3) = 6$. Find one more solution. How many more solutions can you find?

10. Draw the graph of $2x + 3y = 1$, and verify from the graph whether $x = 5, y = -3$ is a solution or not. Give reason in support of your answer.

HIGHER ORDER THINKING SKILLS (HOTS)

1. The following observed values of x and y are thought to fulfill the law $y = ax + b$. Find the values of a and b.

x	1	2	–3	0	5
y	12	19	–16	5	40

2. Find m, if point (7, –3) lies on the equation $\left(y - \frac{3}{7}\right) = m\left(x - \frac{2}{7}\right)$.

3. The cost of petrol in a city is ` 40 per litre. Write an equation with x as number of litres and y total cost.

4. Find the coordinates of the points where the line represented by the linear equation $y = 2x - 4$ intersects x-axis and y-axis.

5. If "the cost of 5 tables exceed the cost of eight chairs by 150". Write the linear equation in two variables represent the statement. Also find the cost of one table if cost of one chair is ` 240.

6. Draw the lines $x = 4$, $y = 2$ and $x = y$, on the same graph paper and then identify what type of figure obtained? Also write the point of vertices of this figure formed.

MULTIPLE CHOICE QUESTIONS

1. The equation $2x+5y=7$ has a unique solution, if x, y are:

   (A) natural numbers (B) positive real numbers
   (C) real numbers (D) rational numbers

2. The graph of the linear equation $2x+3y=6$ cuts the y-axis at the point

   (A) (2, 0) (B) (0, 3) (C) (3, 0) (D) (0, 2)

3. Any point on the x-axis is of the form

   (A) (x, y) (B) $(0, y)$ (C) $(x, 0)$ (D) (x, x)

4. Any point on the line $y = x$ is of the form

   (A) (a, a) (B) $(0, a)$ (C) $(–0, a)$ (D) $(a, –a)$

5. The graph of $y = 6$ is a line.

   (A) parallel to x-axis at a distance 6 units from the origin
   (B) parallel to y-axis at a distance 6 units from the origin
   (C) making an intercept 6 on the x-axis
   (D) making an intercept 6 on both the axis.

6. The graph of the linear equation $2x + 3y = 6$ is a line which meets the x-axis at the point:

   (A) (0, 2) (B) (2, 0) (C) (3, 0) (D) (0, 3)

7. The graph of the linear equation $y = x$ passes through the point:

   (A) $\left(\frac{3}{2}, \frac{-3}{2}\right)$ (B) $\left(0, \frac{3}{2}\right)$ (C) (1, 1) (D) $\left(\frac{-1}{2}, \frac{1}{2}\right)$

8. How many linear equation in x and y can satisfy $x = 1$ and $y = 2$?

   (A) only one (B) two (C) infinitely many (D) three

9. The point of the form (a, a) always lies on:

   (A) x-axis (B) y-axis (C) on the line $y = x$ (D) on the line $x + y = 0$

10. The point of the form $(a, –a)$ always lies on the line

    (A) $x = a$ (C) $y = a$ (C) $y = x$ (D) $x + y = 0$.

11. Solve : 37x + 41y = 70
    41x + 37y = 86

    (A) x = 3 and y = – 1 (B) x = 3 and y = 5 (C) x = 4 and y = 7 (D) x = 9 and y = –5

12. 37 pens and 53 pencils together cost `320, while 53 pens and 37 pencils together cost `400. Find the cost of a pen.

    (A) `6.50 (B) `2.50 (C) `3.50 (D) `5.50

13. In two digit number, the ten's digit is three times the unit's digit. When the number is decreased by 54, the digits are reversed. Find the number.

(A) 93 (B) 21 (C) 54 (D) 79

14. The sum of a two digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

(A) 64 (B) 33 (C) 24 (D) 12

15. A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get 18/11, but if the numerator is increased by 8 and the denominator is doubled, we get 2/5. Find the fraction.

(A) $\frac{12}{25}$ (B) $\frac{11}{13}$ (C) $\frac{11}{17}$ (D) $\frac{11}{19}$

16. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.

(A) $\frac{7}{18}$ (B) 5/13 (C) 9/2 (D) 1/5

17. If $\frac{2x+1}{3}+\frac{3y-1}{2}=2$ & $\frac{3x-1}{2}+\frac{2y+1}{3}=2$, then

(A) x = 1, y = 2 (B) x = 1, y = 1 (C) x = 1, y = 1/2 (D) x = 1/2, y = 1/2

18. If 3ax + 2by = 5ab and 5ax – 3by = 2ab, then

(A) x = b, y = a (B) x = a, y = b (C) x = y = a (D) x = y = b

19. If $\frac{a}{x}-\frac{b}{y}=0$ and $\frac{ab^2}{x}+\frac{a^2b}{y}=a^2+b^2$, then

(A) x = y = b (B) x = y = a (C) x = a, y = b (D) x = b, y = a

20. If $\frac{x}{a}=\frac{y}{b}$ and $ax+by=a^2+b^2$, then

(A) x = a, y = b (B) x = b, y = a (C) x = a/2, y = b/2 (D) x = b/2, y = a/2

21. An inconsistent system of two linear equations in two variables will have

(A) one solution (B) two solutions

(C) no solution (D) more than two solutions.

22. $\begin{cases} x+y=0 \\ 2x+2y=0 \end{cases}$ has

(A) no solution (B) one solution

(C) two solutions (D) more than two solutions.

23. The graphs of $2x + 3y - 6 = 0$, $y = 2/3$, $x = 2$ and $4x - 3y = 6$ intersect in
(A) four points (B) one point
(C) in no point (D) in infinite number of points.

24. Yamini and Fatima, two students of class IX of a school, together contributed Rs. 100 towards the Prime Minister's Relief Fund to help the earthquake victims. A linear equation which satisfies this data is
(A) $x - y = 100$ (B) $x + y = 100$ (C) $x + 2y = 100$ (D) $2x + y = 100$.

25. The value of k for which the system of equations
$kx - y = 2$, $6x - 2y = 3$
has a unique solution, is
(A) = 3 (B) $\neq 3$ (C) $\neq 0$ (D) = 0.

26. The value of k for which the system of equations $2x + 3y = 5$, $4x + ky = 10$ has infinite number of solutions, is
(A) 1 (B) 3 (C) 6 (D) 1.

27. The value of k for which the system of equations $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution, is
(A) 10 (B) 6 (C) 3 (D) 1.

28. If $am \neq bl$, then the system of equations $ax + by = c$, $lx + my = n$
(A) has a unique solution (B) has no solution
(C) has infinitely many solutions (D) may or may not have a solution.

29. The present ages of a brother and sister in the ratio of 2 : 3. Ten years hence, the ratio will be 3 : 4, find their present ages.
(A) brother's age = 30 years (B) sister's age = 20 years
(C) brother's age = 20 years (D) sister's age = 30 years .

30. Solving $4x + 3y = 25$ and $5x - 2y = 14$ results in –
(A) $x = 4$ (B) $x = 3$ (C) $y = 3$ (D) $y = 4$.

31. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, the value of a is
(A) $\frac{5}{3}$ (B) $1.\overline{6}$ (C) 1 (D) $\frac{2}{5}$.

MATRIX MATCHING

32. Column–II give solution for equation in column–I match them correctly

Column–I	**Column–II**
(A) $2x + y = 7$	(p) (0, 9)
(B) $x - 2y = 4$	(q) (1, 5)
(C) $x - 4y = 0$	(r) (0, 0)
(D) $px + y = 9$	(s) (4, 0)

33. Match them correctly

Column–I	**Column–II**
(A) $x = a$	(p) x-axis
(B) $y = a$	(q) st. line parallel to x-axis
(C) $y = mx$	(r) st. line parallel to y-axis
(D) $y = 0$	(s) st. line passing through origin.

INTEGER TYPE QUESTIONS

34. In a picnic there are boys and girls. Fifteen girls leave, then the boys and girls are left in the ratio of 2: 1. Later 45 boys leave and the ratio changes to 1 : 5. The number of girls in the beginning was 8λ then find λ.

35. A man can do a piece of work in 30 hours. He and his son together finish it in 20 hours. The son along will finish it in $(8\lambda + 4)$ hours, then find λ.

36. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.

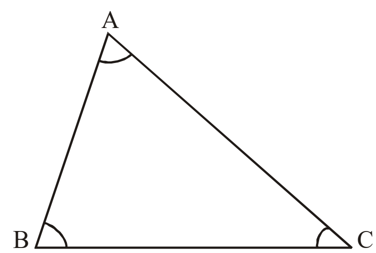
37. In some countries, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius $F = \frac{9}{5}C + 32$, if the temperature is 40º C, the temperature in Fahrenheit is 13λ, then find λ.

7 Triangles

Introduction

We have studied about triangles and their different properties in our previous classes. We know that a closed geometrical figure formed by three intersecting lines is called a triangle. Tri means three, So, a triangle has three sides, three angles and three vertices.

e.g. In figure, ABC is a triangle i.e. in ΔABC, AB, BC and AC are three sides, $\angle A$, $\angle B$ and $\angle C$ are three angles and A, B and C are its three vertices.



Congruence of triangles

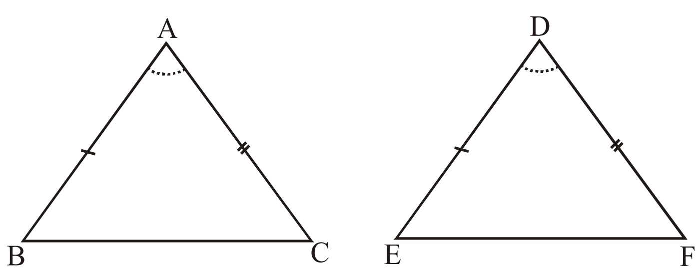
Congruent means equal in all the respect or geometrical figures whose shapes and sizes are same. Let ABC and DEF be two triangles in which AB = DE, BC = EF, AC = DF and $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ respectively. Then, $\Delta ABC \cong \Delta DEF$

"CPCT" means corresponding parts of congruent triangles.

Criteria for Congruence of Triangles

Side-angle-Side

I. **(SAS) Congruence rule:** Two triangles are congruent if two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other triangle.



If in ΔABC and ΔDEF; AB = DE, AC = DF and $\angle BAC = \angle EDF$

Then, $\Delta ABC \cong \Delta DEF$. It is called SAS congruence rule i.e. side-angle-side.

II. Angle-Side-Angle (ASA) Congruence rule: Two triangles are congruent if two angles and the included side of one triangle are equal to two corresponding angles and the included side of other triangle.

Given:

ABC and DEF are two triangles in which $\angle B = \angle E$, $\angle C = \angle F$ and BC = EF

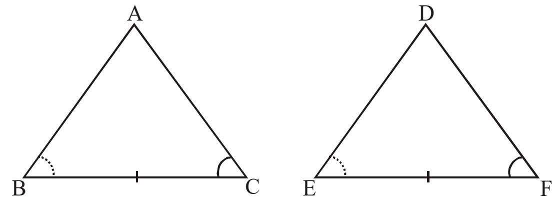


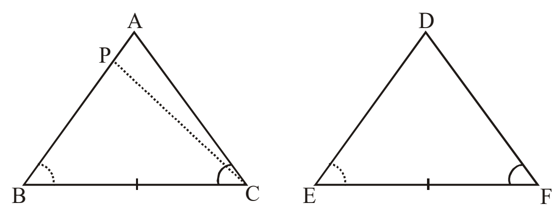
Fig.1

To Prove: $\Delta ABC \cong \Delta DEF$

Proof: Case – (i)

Let AB = DE [Assumed]
$\angle B = \angle E$ [Given]
and BC = EF [Given]
$\therefore$ $\Delta ABC \cong \Delta DEF$ [By SAS rule]

Case – (ii)



Let if possible AB > DE.
Let P be a point on AB such that PB = DE
Now, In ΔPBC and ΔDEF, we have
PB = DE [By construction]
$\angle B = \angle E$ [Given]
and BC = EF [Given]
$\therefore$ $\Delta PBC \cong \Delta DEF$ [By SAS rule]
$\therefore$ $\Delta PCB \cong \Delta DFC$, $\therefore$ $\angle PCB = \angle DFE$ [By CPCT]
But it is given that $\angle ACB = \angle DFE$
$\therefore$ $\angle ACB = \angle PCB$ which is not possible.
i.e. it is only possible if P coincides with A
$\Rightarrow$ BA = ED
Hence, $\Delta ABC \cong \Delta DEF$ [by SAS rule]

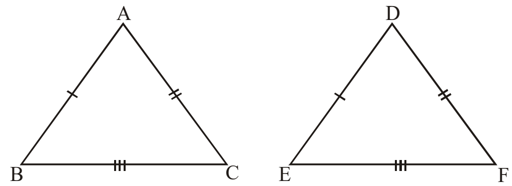
Case – (iii)

If AB < DE, we can choose a point M on DE such that ME = AB and repeating the arguments as given in case (ii), we can conclude that AB = DE and, hence, $\Delta ABC \cong \Delta DEF$

Now, two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal. We may call it as Angle-Angle-Side (AAS).

i.e. **Angle-Side-Angle(ASA)** congruence rule may be called Angle-Angle-Side (AAS) congruence rule.

III Side-Side-Side(SSS) congruence rule:



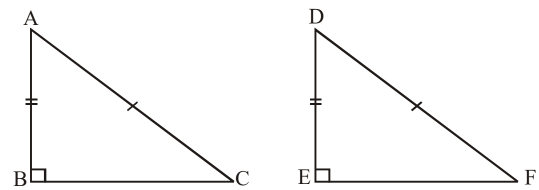
If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

If in ΔABC and ΔDEF, $AB = DE$, $BC = EF$ and $AC = DF$

Then, $\Delta ABC \cong \Delta DEF$

[It is called SSS congruence rule i.e. side-side-side].

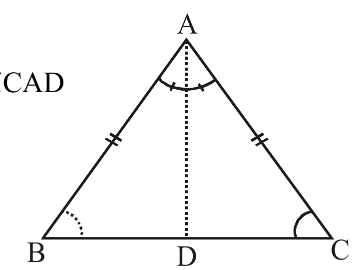
IV Right angle-Hypotenuse-Side (RHS) congruence rule: If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.



If ABC and DEF are two right triangles in which $\angle B = \angle E = 90^o$, $AC = DF$ and $AB = DE$

then, $\Delta ABC \cong \Delta DEF$.

Theorem: Angles opposite to equal sides of an isosceles triangle are equal.



Given: ΔABC is an isosceles triangle i.e. $AB = AC$

Construction: AD bisector of $\angle A$ is drawn at BC i.e. $\angle BAD = \angle CAD$

Proof: In ΔBAD and ΔCAD, we have

$AB = AC$ [Given]

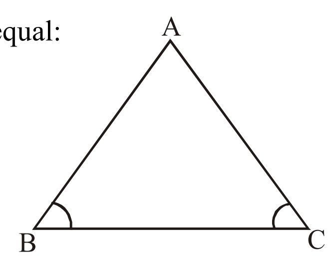
$\angle BAD = \angle CAD$ [By construction]

and $AD = AD$ [Common]

$\therefore$ $\Delta BAD \cong \Delta CAD$ [By SAS rule]

Hence, $\angle B = \angle C$ [By CPCT] Proved.

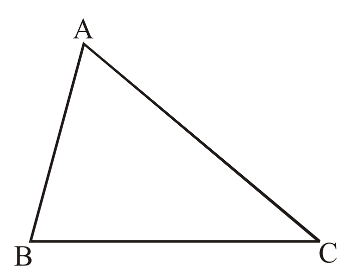
Converse of Theorem: The sides opposite to equal angles of a triangle are equal:



In ΔABC if $\angle B = \angle C$

Then, $AB = AC$

Inequalities in a triangle



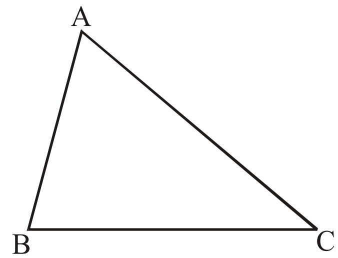
Theorem: If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater)

Let ABC be a triangle in which $AC > AB$ and $AC > BC$.

Then, $\angle B > \angle C$ and $\angle B > \angle A$

The side opposite to the largest angle is the longest.

Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of third side



Let ABC be a triangle and AB, BC and AC are its corresponding sides.

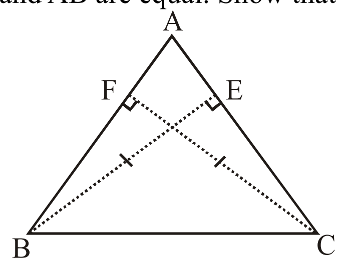
Then, AB + BC > AC, AB + AC > BC and AC + BC > AB.

SOLVED EXAMPLES

Example 1: ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal. Show that

(i) $\Delta ABE \cong \Delta ACF$

(ii) AB = AC; i.e. ABC is an isosceles triangle.



Given:

In ΔABC altitudes BE = CF

To prove: (i) $\Delta ABE \cong \Delta ACF$

(ii) AB = AC ; i.e. ABC is an isosceles triangle.

Solution: (i) In ΔABE and ΔACF, we have

BE = CF [Given]

$\angle AEB = \angle AFC = 90^o$ [$\because$ BE and CF are altitudes: Given]

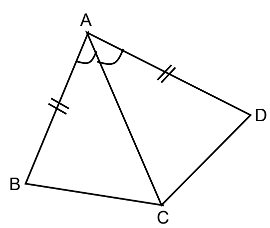
and $\angle A = \angle A$ [Common]

$\therefore$ $\Delta ABE \cong \Delta ACF$ [By AAS congruence rule]

$\therefore$ AB = AC [By CPCT]

Hence, ABC is an isosceles triangle.

Example 2: In figure, ABCD is a quadrilateral such that AB = AD and AC is bisector of the angle A of the quadrilateral. Show that $\Delta ABC \cong \Delta ADC$ and BC = DC.



Solution: For ΔABC and ΔADC, we have AB = AD

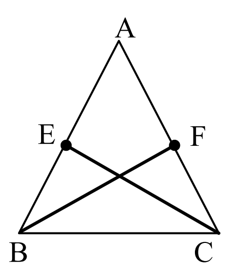
$\angle BAC = \angle DAC$

AC = AC. Therefore, by SAS criteria, we have $\Delta ABC \cong \Delta ADC$

Also, we have corresponding parts of the two triangle equal and it gives

BC = DC.

Example 3: In given figure, AB = AC, E is mid-point of AB and F is mid-point of AC. Show that BF = CE.



Solution: In ΔABC, $AB = AC$. (Given)

E and F are respectively the mid-points of the sides AB and AC.

Now, $BE = \frac{1}{2}AB$ and $CF = \frac{1}{2}AC$

We know that halves of the equal sides are equal.

Therefore, $BE = CF$. ….(1)

Now, for ΔABF and ΔACE, we have $AB = AC$ (Given)

$\angle BAF = \angle CAE$ (Each $= \angle A$)

$BE = CF$ (By 1)

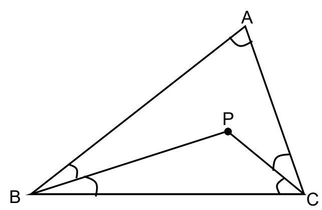
Thus, we conclude that

$\Delta ABF \cong \Delta ACE$

(SAS congruence criteria)

Therefore, $BF = CE$. (By CPCT)

Example 4: In given figure, $AB > AC$, PB and PC are bisectors of $\angle B$ and $\angle C$ respectively. Show that $PB > PC$.



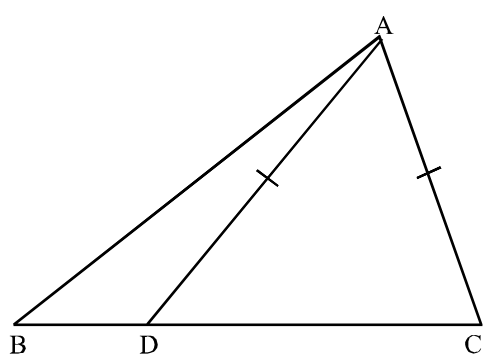
Solution: In ΔABC, $AB > AC \Rightarrow \angle C > \angle B$

$\Rightarrow \frac{1}{2}\angle C > \frac{1}{2}\angle B \Rightarrow \angle PCB > \angle PBC$

$$\left\{ \begin{array}{l} \therefore \angle PCB = \frac{1}{2}\angle C \text{ and } \angle PBC = \frac{1}{2}\angle B \\ \text{as PB and PC are bisectors of } \angle B \text{ and } \angle C \end{array} \right\}$$

$\Rightarrow$ In ΔPBC, $\angle PCB > \angle PBC \Rightarrow PB > PC$

Example 5: In given figure, D is a point on the side BC of ΔABC such that $AD = AC$. Show that $AB > AD$.



Solution: In ΔADC, $AD = AC$

$\Rightarrow \angle ADC = \angle ACD$ ... (1) (Angles opposite to equal sides are equal)

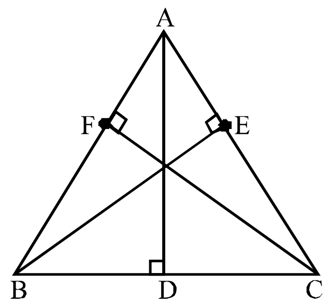
Now, $\angle ADC > \angle ABD$ ... (2) $\left\{\begin{array}{c}\because \text{ An exterior angle of a triangle is} \\ \text{greater than an opposite interior angle}\end{array}\right\}$

From (1) and (2) $\angle ACD > \angle ABD$

i.e., $\angle ACD > \angle ABC$ i.e., in DABC,

$\angle C > \angle B \Rightarrow AB > AC \Rightarrow AB > AD$ ($\because AC = AD$)

Example 6: Show that the perimeter of a triangle is greater than the sum of the lengths of the three altitudes of the triangle.



Solution: In ΔABC, AD, BE and CF are three altitudes.

In right angled DABD, AB is hypotenuse.

$\Rightarrow$ $AB > AD$ ….(1)

Similarly, in right angled DABC and DCFA, we have

$BC > BE$ ….(2) and

$CA > CF$ ….(3)

Adding (1), (2), (3) $AB + BC + CA > AD + BE + CF$

i.e., Perimeter of $\Delta ABC > (AD + BE + CF)$.

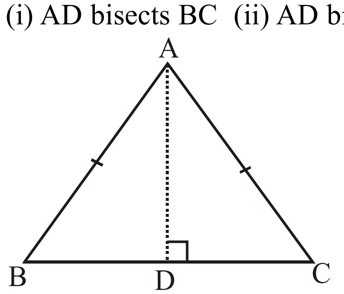
Example 7: AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC (ii) AD bisects $\angle A$

Given: ABC is an isosceles triangle in which AB = AC and AD is an altitude, i.e. $\angle ADC = 90^o$

To prove:

(i) AD bisects BC (ii) AD bisects $\angle A$



Solution: $\because$ AD is an altitude from vertex A on BC

$\therefore$ $\angle ADB = \angle ADC = 90^o$

(i) Now, in right ΔADB and ADC ,

we have $\angle ADB = \angle ADC = 90^o$[Given] and AD = AD [Common]

$\therefore$ $\Delta ADB \cong \Delta ADC$

[By RHS congruence rule]

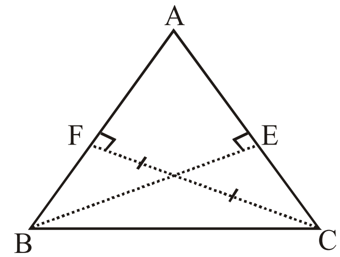
$\therefore$ BD = CD [By CPCT]

Hence, AD bisects BC. Proved.

(ii) $\because$ $\Delta ADB \cong \Delta ADC$ [By proof]

$\therefore$ $\angle BAD = \angle CAD$ [By CPCT]. Hence, AD bisects $\angle A$.Proved.

Example 8: BE and CF are two equal altitudes of a triangle ABC. Prove that the triangle ABC is isosceles.



Given:

BE and CF are two equal altitudes of a triangle ABC, i.e., BE = CF

and ∠AEB = ∠AFC = 90°

To prove:

ΔABC is isosceles.

Solution: In ΔABE and ΔACF, we have

BE = CF [Given]

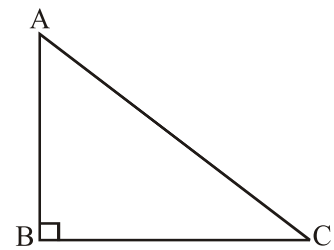
∠AEB = ∠AFC = 90° [Given] and ∠A = ∠A [Common]

∴ ΔABE ≅ ΔACF

[By AAS congruence rule]

∴ AB = AC [By CPCT]. Hence, DABC is isosceles. Proved.

Example 9: Show that in a right angled triangle, the hypotenuse is the longest side.



Given:

ABC is a right angled triangle in which ∠B = 90°,

i.e., ∠B > ∠A and ∠B > ∠C.

To prove:

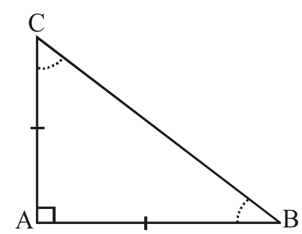
AC is the longest side.

Solution: ∵ ∠B > ∠A [Given] and ∠B > ∠C

∴ AC > AB [The side opposite to the largest angle is the longest]

and AC > BC. Hence, hypotenuse (AC) of right angled triangle ABC is the longest side.

Example 10: ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.



Given:

ABC is a right angled triangle in which ∠A = 90° and AB = AC.

To find: ∠B and ∠C

Solution: In right angled triangle ABC, we have

AB = AC [Given]

∴ ∠B = ∠C

[Angles opposite to equal sides of a triangle are equal] ....(i)

Now, ∠A + ∠B + ∠C = 180° [Angles sum property of a D]

⇒ 90° + ∠B + ∠B = 180° [From (i)

∵ ∠B = ∠C]

⇒ 2∠B = 180° – 90° ⇒ 2∠B = 90°

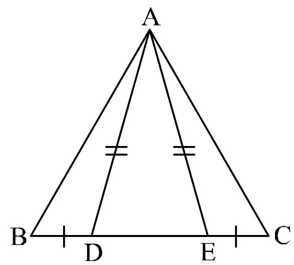
$$\Rightarrow \angle B = \frac{90^\circ}{2} = 45^\circ$$

Hence, ∠B = ∠C = 45°

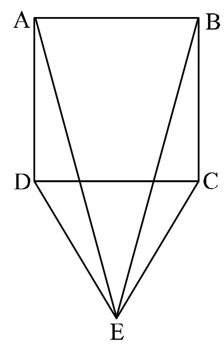
REVISION EXERCISE

SUBJECTIVE QUESTIONS (LEVEL - I)

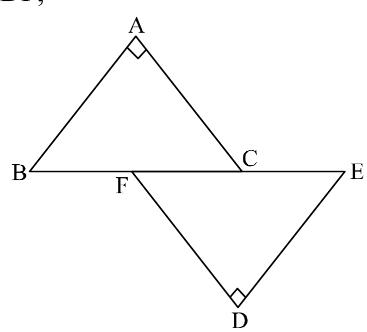
1. In figure D and E are points on side BC of ΔABC such that BD = CE and AD = AE. Show that ΔABD ≅ ΔACE.



2. CDE is an equilateral triangle formed on a side CD of a square ABCD. Show that ΔADE ≅ ΔBCE.



3. In figure, $BA \perp AC$, $DE \perp DF$, such that BA = DE and BF = EC. Show that ΔABC ≅ ΔDEF.

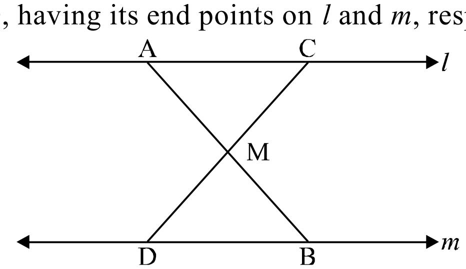


4. Q is a point on the side SR of a ΔPSR such that PQ = PR. Prove that PS > PQ.

5. S is any point on side QR of a ΔPQR. Show that: PQ + QR + RP > 2PS.

6. D is any point on side AC of a ΔABC with AB = AC. Show that CD < BD.

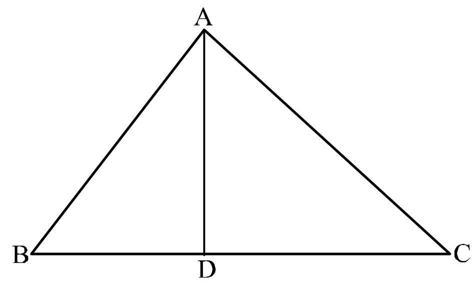
7. In figure $l \parallel m$ and M is the mid point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its end points on l and m, respectively.



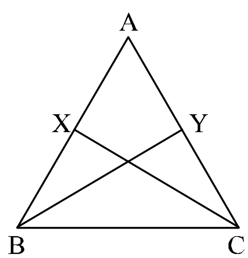
8. Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O. BO is produced to a point M. Prove that ∠MOC = ∠ABC.

9. Bisectors of the angles B and C of isosceles triangle ABC with AB = AC intersect each other at O. Show that external angle adjacent to ∠ABC is equal to ∠BOC.

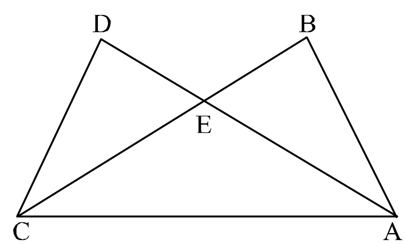
10. In figure AD is bisector of ∠BAC. Prove that AB > BD.



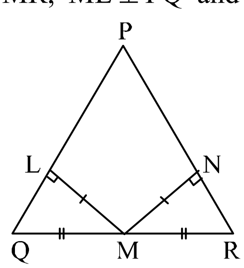
11. In the figure below, ABC is a triangle in which AB = AC. X and Y are points on AB and AC such that AX = AY. Prove that $\Delta ABY \cong \Delta ACX$.



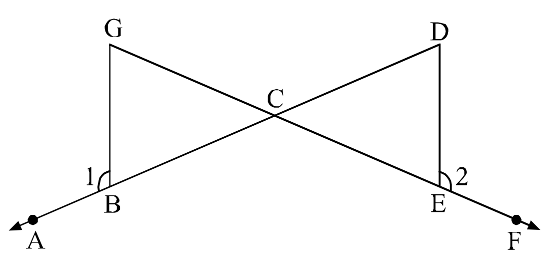
12. In the given figure, AB = CD and AD = BC. Prove that $\Delta ADC \cong \Delta CBA$.



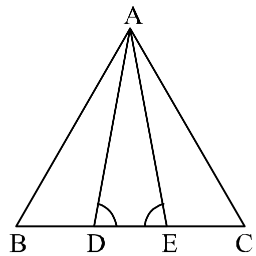
13. In the given figure, LM = MN, QM = MR, $ML \perp PQ$ and $MN \perp PR$. Prove that PQ = PR.



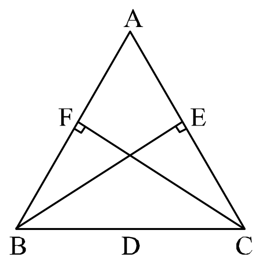
14. In the given figure, BC = CE and ∠1 = ∠2. Prove that $\Delta GCB \cong \Delta DCE$.



15. In the given figure, D and E are points on the base BC of ΔABC such that BD = CE, AD = AE and $\angle ADE = \angle AED$. Prove that $\Delta ABE \cong \Delta ACD$.

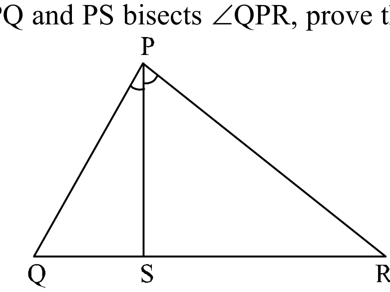


16. In the figure, ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.

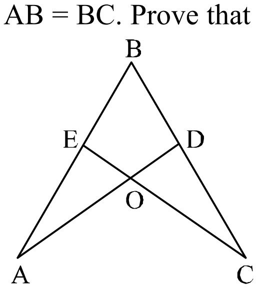


17. A point O is taken inside an equilateral four sided figure ABCD such that its distances from the angular points D and B are equal. Show that AO and OC are in one and the same straight line.

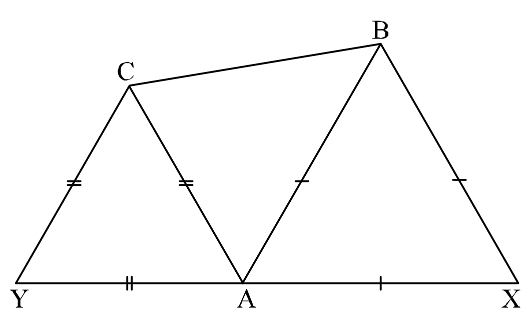
18. In the given figure, PR > PQ and PS bisects $\angle QPR$, prove that $\angle PSR > \angle PSQ$.



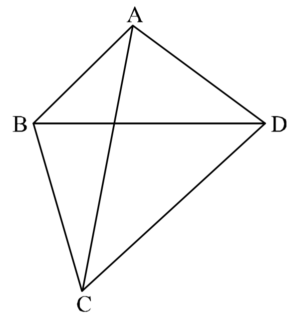
19. In the given figure $\angle A = \angle C$ and AB = BC. Prove that $\Delta ABD \cong \Delta CBE$.



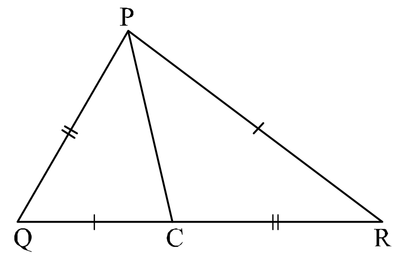
20. In figure, it is given that AX = AB = BX and AC = CY = YA. Prove that $\angle CAX = \angle YAB$ and CX = BY.



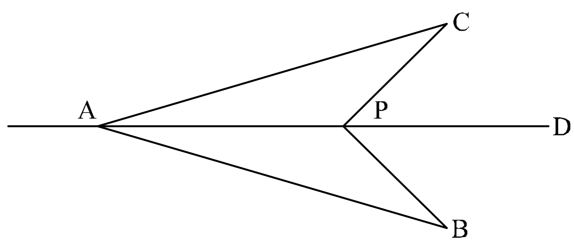
21. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. Prove that $\Delta ABD \cong \Delta BAC$.



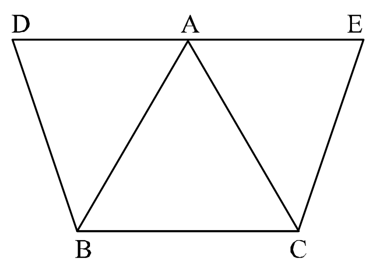
22. In the given figure, triangles PQC and PRC are such that QC = PR and PQ = CR. Prove that ∠PCQ = ∠CPR.



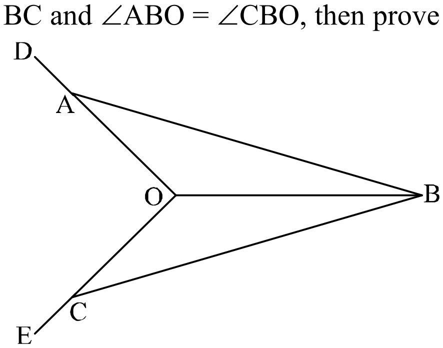
23. In the given figure, AD is bisector of ∠BAC and ∠CPD = ∠BPD. Prove that $\Delta CAP \cong \Delta BAP$.



24. In the given figure, equilateral ΔABD and ΔACE are drawn on the sides of a ΔABC. Prove that CD = BE.

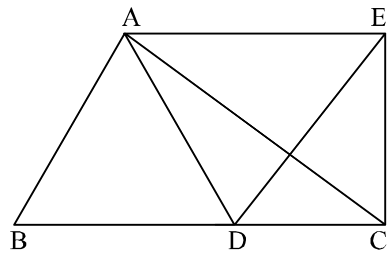


25. In the given figure, AB = BC and ∠ABO = ∠CBO, then prove that ∠DAB = ∠ECB.



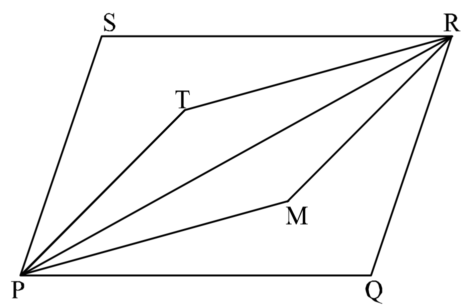
LEVEL - II

1. In the given figure, AB = AD, AC = AE and ∠BAD = ∠EAC, the prove that BC = DE.



2. ΔPQR is given and the sides QP and RP have been produced to S and T such that PQ = PS and PR = PT. Prove that the segment QR || ST.

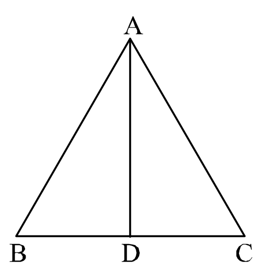
3. In the given figure, T and M are two points inside a parallelogram PQRS such that PT = MR and PT || MR. Then prove that,



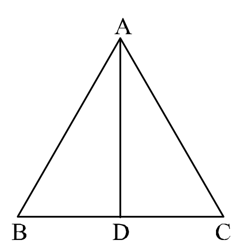
(a) ΔPTR ≅ ΔRMP (b) RT || PM and RT = PM

4. The bisectors of the base angles Q and R of an equilateral triangle PQR meet at S. ST and SM are drawn parallel to the sides PQ and PR. Then show that QT = TM = MR.

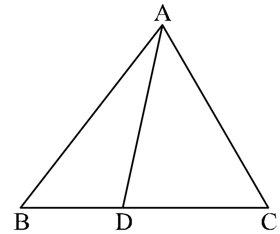
5. In the given figure, AD is the median of ΔABC. Prove that AB + BC + AC > 2AD.



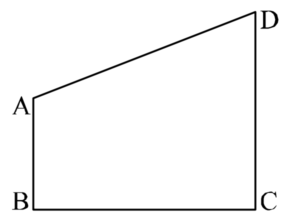
6. In the given figure, D is any point on BC. Prove that $AD < \frac{1}{2}(AB + BC + CA)$.



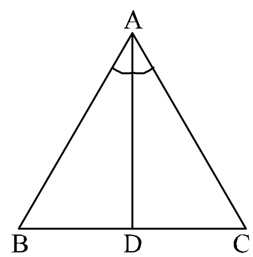
7. In given figure, AB > AC. Show that AB > AD.



8. In the given figure AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD. Show that ∠A > ∠C and ∠B > ∠D.



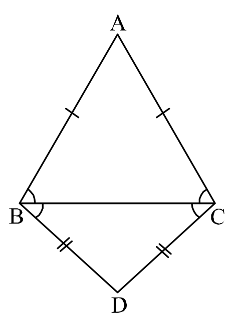
9. In the given figure, if AD is the bisector of ∠BAC then prove that:
   (a) AB > BD (b) AC > CD



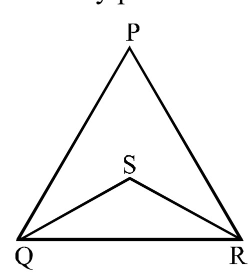
10. Prove that the sum of the lengths of any two sides of a triangle is greater than twice the length of median drawn to the third side.

11. Prove that angles opposite to equal sides of a triangle are equal. Using this result, prove the following:

    ABC and DBC are two isosceles triangles drawn on same base BC such that AB = AC and DB = DC. Prove that ∠ABD = ∠ACD.

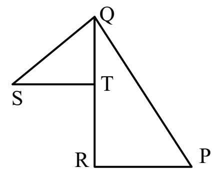


12. In the figure, PQR is a triangle and S is any point in its interior, show that SQ + SR < PQ + PR.

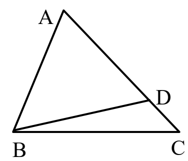


13. Prove that the difference of the lengths of any two sides of a triangle is less than its third side.

14. In figure, T is a point on side QR of ΔPQR and S is a point such that RT = ST. Prove that PQ + PR > QS.



15. In figure, AC > AB and D is the point on AC such that AB = AD. Prove that BC > CD.



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EXERCISE (Single Correct Type)

1. In two triangles, ABC and PQR, $\angle A = 30°$, $\angle B = 70°$, $\angle P = 70°$, $\angle Q = 80°$ and AB = RP, then

(a) $\Delta ABC \cong \Delta PQR$ (b) $\Delta ABC \cong \Delta QRP$ (c) $\Delta ABC \cong \Delta RPQ$ (d) $\Delta ABC \cong \Delta RQP$

2. In two triangles ABC and DEF, AB = DE, BC = DF and AC = EF, then

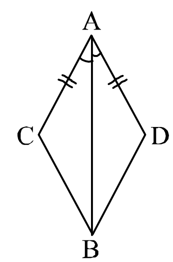
(a) $\Delta ABC \cong \Delta DEF$ (b) $\Delta ABC \cong \Delta EFD$ (c) $\Delta ABC \cong \Delta FDE$ (d) None of these

3. If ΔABC is congruent to ΔDEF by SSS congruence rule, then:

(a) $\angle C < \angle F$ (b) $\angle B < \angle E$

(c) $\angle A < \angle D$ (d) $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

4. In the given figure, the congruency rule used in proving $\Delta ACB \cong \Delta ADB$ is

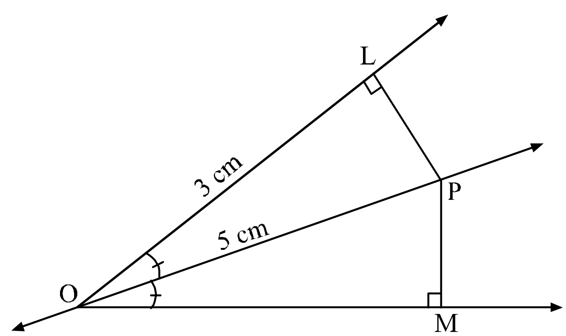


(a) ASA (b) SAS (c) AAS (d) RHS

5. In a triangle PQR, $\angle QPR = 80°$ and PQ = PR, then $\angle R$ and $\angle Q$ are

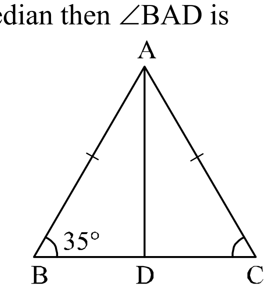
(a) 80°, 70° (b) 80°, 80° (c) 70°, 80° (d) 50°, 50°

6. In the given figure, find PM



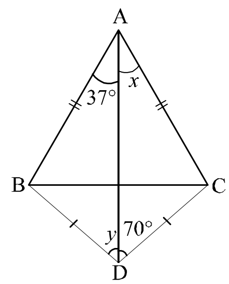
(a) 3 cm (b) 5 cm (c) 4 cm (d) 2 cm

7. In the given figure, AD is the median then $\angle BAD$ is



(a) 35° (b) 70° (c) 110° (d) 55°

8. In the given figure, x and y are



(a) $x = 70°, y = 37°$ (b) $x = 37°, y = 70°$ (c) Both (a) and (b) (d) None of these

9. In ΔABC and ΔDEF, AB = FD, ∠A = ∠D. The two triangles will be congruent by SAS axiom if:
(a) BC = DE (b) AC = EF (c) BC = EF (d) AC = DE

10. ∠x and ∠y are exterior angles of a ΔABC, at the points B and C respectively. Also ∠B > ∠C, then relation between ∠x and ∠y is
(a) ∠x > ∠y (b) ∠x = ∠y (c) ∠x < ∠y (d) None of these

11. In a quadrilateral ABCD, AB = 2 cm, BC = 3 cm, CD = 5 cm and AD = 4 cm, then relation between ∠B and ∠D is
(a) ∠B > ∠D (b) ∠B < ∠D (c) ∠B = ∠D (d) None of these

12. Given two right angled triangles ABC and PRQ, such that ∠A = 20°, ∠Q = 20° and AC = QP. Write the correspondence if triangles are congruent.
(a) ΔABC ≅ ΔPQR (b) ΔABC ≅ ΔPRQ (c) ΔABC ≅ ΔRQP (d) ΔABC ≅ ΔQRP

13. ABC is an isosceles triangle with AB = AC. Altitudes are drawn to the sides AB and AC from vertices C and B. One altitude CF is found to be 4 cm. If BC = 5 cm. Find EC, where BE is altitude to side AC.
(a) 4 cm (b) 5 cm (c) 3 cm (d) None of these

14. If AB = QR, BC = PR and CA = PQ, then
(a) ΔABC ≅ ΔPQR (b) ΔCBA ≅ ΔPRQ (c) ΔBAC ≅ ΔRPQ (d) ΔPQR ≅ ΔBCA

15. In ΔABC, AB = AC and ∠B = 50°. Then ∠C is equal to
(a) 40° (b) 50° (c) 80° (d) 130°

16. In ΔABC, BC = AB and ∠B = 80°. Then ∠C is equal to
(a) 80° (b) 40° (c) 50° (d) 100°

17. In ΔPQR, ∠R = ∠P and QR = 4 cm and PR = 5 cm. Then the length of PQ is
(a) 4 cm (b) 5 cm (c) 2 cm (d) 2.5 cm

18. D is a point on the side BC of ΔABC such that AD bisects ∠BAC. Then
(a) BD = CD (b) BA > BD (c) BD > BA (d) CD > CA

19. It is given that ΔABC ≅ ΔFDE and AB = 5 cm, ∠B = 40° and ∠A = 80°. Then which of the following is true?
(a) DF = 5 cm, ∠F = 60° (b) DF = 5 cm, ∠E = 60°
(c) DE = 5 cm, ∠E = 60° (d) DE = 5 cm, ∠D = 40°

20. Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle cannot be
(a) 3.6 cm (b) 4.1 cm (c) 3.8 cm (d) 3.4 cm

MULTIPLE CORRECT ANSWER TYPE

*This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which **ONE or MORE** is correct. Choose the correct options.*

21. In ΔABC, $\angle B = 45^o$, $\angle C = 55^o$ and bisector of $\angle A$ meets BC at a point D. Find $\angle ADC$.

(a) 75^o (b) $180° - 95°$ (c) 85^o (d) $105°$

22. ABC is a triangle in which $\angle A = 72^o$, the internal bisectors of angles B and C meet in O. Find the magnitude of $\angle BOC$.

(A) 36^o (b) $90° + \angle A/2$ (c) 126^o (d) 44^o.

23. The angles of a triangle are $(x - 40^o)$, $(x - 20)^o$ and $\left(\frac{1}{2}x - 10\right)^\circ$. Then which is not the value of x.

(a) 50^o (b) 45^o (c) 75^o (d) $44°$

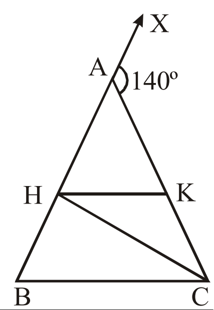
24. In figure, AB = AC, CH = CB and HK || BC. If $\angle CAX$ is 140^o, then $\angle HCK$ is

(a) 40^o

(b) 15^o

(c) 45^o

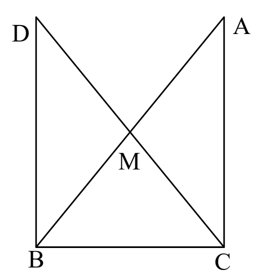
(d) $\pi/6$



MATRIX MATCH TYPE

*Question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labeled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statements(s) in **Column II.***

25. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (figure), match them correctly.

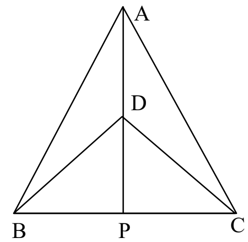


Column I	Column II
(A) ΔAMC	(p) congruent to ΔBMD
(B) $\angle DBC$	(q) a right angle
(C) ΔDBC	(r) congruent to ΔACB
(D) CM	(s) (1/2) AB

(a) (A- p), (B- q), (C- r), (D - s) (b) (A - q), (B - p), (C - s), (D - r)

(c) (A - s), (B - r), (C - p), (D - q) (d) (A - r), (B - s), (C- q), (D - p)

26. ΔABC and ΔDBC are two isosceles triangle on the same base BC and vertices A and D are on the same side of BC (figure). If AD is extended to intersect BC at P. Match them correctly.



Column I	**Column II**
(A) ΔABD	(p) congruent ΔACD
(B) ΔABP	(q) congruent ΔACP
(C) AP bisects	(r) $\angle A$
(D) AP is the perpendicular bisector	(s) BC

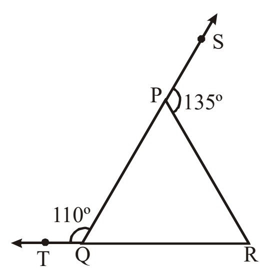
(a) (A - q), (B - p), (C - s), (D - r)
(b) (A- p), (B- q), (C- r), (D - s)
(c) (A - s), (B - r), (C - p), (D - q)
(d) (A - r), (B - s), (C- q), (D - p)

INTEGER TYPE

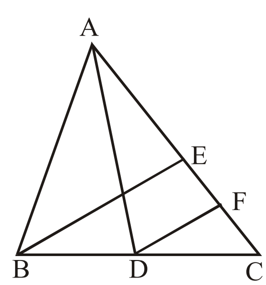
The answer to each of the questions is a single-digit integer, ranging from 0 to 9.

27. In a ΔABC, if $2\angle A = 3\angle B = 6\angle C$, if $\angle C = 5k$ then the value of k is

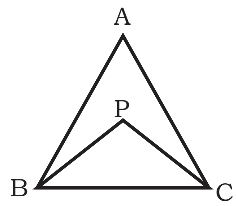
28. In figure sides QP and RQ of ΔPQR are produced to point S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, if $\angle PRQ = 13k$ then the value of k is



29. In figure, AD and BE are medians of ΔABC and BE || DF. If the value of CF is equal to k/4 AC, then the value of k is



30. In the given figure, AB = AC and PB = PC, then find the value of $\frac{\angle ABP}{\angle ACP}$



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ANSWERS

EXERCISE (Single Correct Answer Type)

1. (c)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (d)
6. (c)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (c)
11. (a)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (b)
16. (c)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (d)

MULTIPLE CORRECT ANSWER TYPE

21. (b), (c)	**22.** (c), (b)	**23.** (a), (b), (c), (d)	**24.** (b)

MATRIX MATCH TYPE

25. (a) **26.** (b)

INTERGER TYPE

27. 6	**28.** 5	**29.** 1	**30**. 1

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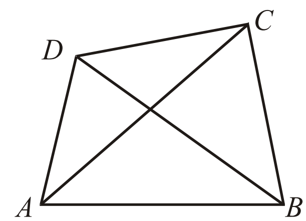
8 Quadrilaterals

Quadrilateral



A plane figure bounded by four line segments such that no two line segments intersect except at their end points is called a quadrilateral. In figure, *ABCD* is a quadrilateral. It is written as quadrilateral *ABCD* or quad. *ABCD*.

AB, *BC*, *CD* and *DA* are called the sides of the quadrilateral. Points *A*, *B*, *C* and *D* are called the vertices of the quadrilateral. *AC* and *BD* are called the diagonals of the quadrilateral.

Two sides of a quadrilateral having a common end point are called its adjacent or consecutive sides. In figure, *AB* and *BC*, *AB* and *AD*, *BC* and *CD*, *AD* and *DC* are four pairs of adjacent sides of quad. *ABCD*.

Two angles of a quadrilateral having a common arm are called its adjacent or consecutive angles. In above figure, $\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, $\angle D$ and $\angle A$ are four pairs of adjacent angles of quad. *ABCD*.

Note:

(i) Two sides of a quadrilateral having no common end points are called its opposite sides.

In above figue, *AB* and *DC* and *BC* and *AD* are two pairs of opposite sides of quad. *ABCD*.

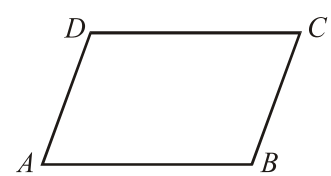
(ii) Two angles of quadrilateral having no common arm are called its opposite angles.

In figure, $\angle A$ and $\angle C$, $\angle B$ and $\angle D$ are two pairs of opposite angles of quad. *ABCD*.

(iii) A quadrilateral is called an equilateral quadrilateral if all its sides are equal.

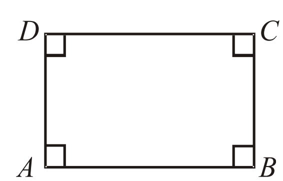
Different Type of Quadrilaterals

(i) **Parallelogram:**



A quadrilateral whose opposite sides are parallel is called a Parallelogram. In figure, *ABCD* is a parallelogram in which *AB* || *DC* and *AD* || *BC*.

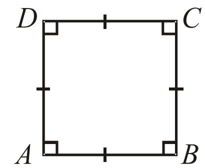
(ii) **Rectangle:**



A parallelogram in which at least one angle is 90° is called rectangle. In figure, *ABCD* is a rectangle.

Note: A rectangle is always a parallelogram but its converse is not necessary true.

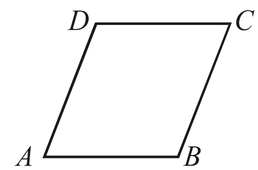
(iii) **Square:**



A rectangle whose all the four sides are equal is called square. In figure, *ABCD* is a square in which *AB* = *BC* = *CD* = *DA* and each angle is 90°.

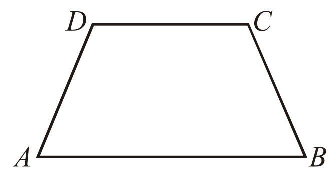
Note: A square is a rectangle but its converse is not necessary true.

(iv) **Rhombus:**



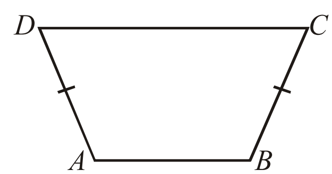
A parallelogram whose all sides are equal is called a rhombus. In figure, *ABCD* is a rhombus in which *AB* = *BC* = *CD* = *AD*.

(v) **Trapezium:**



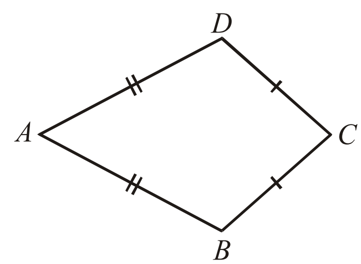
A quadrilateral in which one pair of opposite sides is parallel is called a trapezium. In figure, *ABCD* is a trapezium in which *AB* || *DC*.

Note: (a) A trapezium is called an isosceles trapezium if its non-parallel sides are equal. In figure, *ABCD* is an isosceles trapezium in which *BC* = *AD*.



(b) The line segment joining the mid-points of non-parallel sides of trapezium is called its median.

(vi) **Kite:**



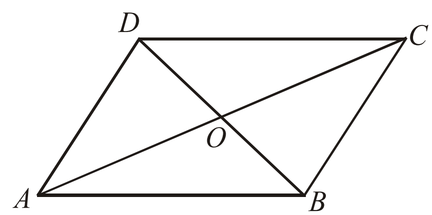
A quadrilateral in which both pair of adjacent sides are equal is called a kite. In figure, *ABCD* is a kite in which *AB* = *AD* and *BC* = *CD*.

Note: In the given figure, *AC* is perpendicular bisector of *BD*.

Properties of a Parallelogram

In figure, *ABCD* is a parallelogram in which *AB* || *DC* and *BC* || *AD*.

On measuring the sides *AB*, *BC*, *CD* and *AD* and the angles ∠*A*, ∠*B*, ∠*C* and ∠*D*, we see that



(a) *AB* = *DC* and *BC* = *AD*

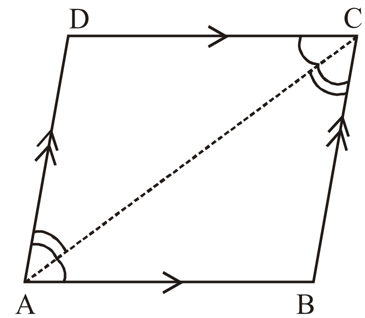
(b) ∠*A* = ∠*C* and ∠*B* = ∠*D*

These properties can be stated as under:

(i) The opposite sides of a parallelogram are equal

(ii) The opposite angles of a parallelogram are equal

Theorem 1: **In a parallelogram, opposite sides are equal.**



Given: A parallelogram $ABCD$ in which $AB \parallel DC$ and $AD \parallel BC$.

To Prove: Opposite sides are equal i.e., $AB = DC$ and $AD = BC$

Construction: Join A and C

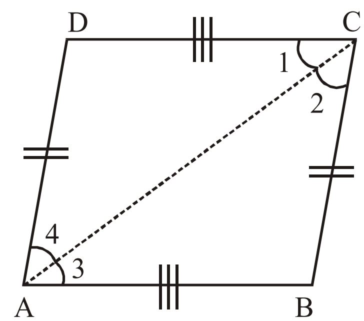
Proof: In ΔABC and ΔCDA

$\angle BCA = \angle DAC$	[Alternate angles]
$AC = AC$	[Common]
$\angle BAC = \angle ACD$	[Alternate angles]
$\therefore \Delta ABC \cong \Delta CDA$	[By ASA]
$\Rightarrow AB = DC$ and $AD = BC$	[By cpctc] **Hence Proved**

Theorem 2: **If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram**

Given: A quadrilateral $ABCD$

To Prove: $ABCD$ is a parallelogram i.e., $AB \parallel DC$ and $AD \parallel BC$



Construction: Join A and C

Proof : In ΔABC and ΔCDA

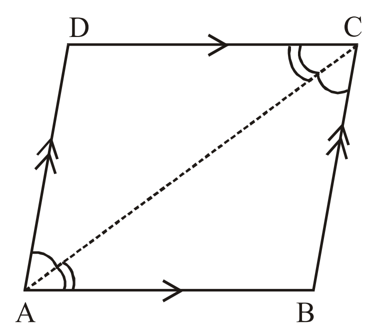
$AB = DC$	[Given]
$AD = BC$	[Given]
And $AC = AC$	[Common]
$\therefore \Delta ABC \cong \Delta CDA$	[By SSS]
$\Rightarrow \angle 1 = \angle 3$	[By cpct]
And $\angle 2 = \angle 4$	[By cpct]

But these are alternate angles and whenever alternate angles are equal, the lines are parallel.

$\therefore AB \parallel DC$ and $AD \parallel BC$

$\Rightarrow ABCD$ is a parallelogram.

Theorem 3: In a parallelogram, opposite angles are equal.



Given: A parallelogram $ABCD$ in which $AB \parallel DC$ and $AD \parallel BC$.

To Prove: Opposite angles are equal
i.e. $\angle A = \angle C$ and $\angle B = \angle D$

Construction: Draw diagonal AC

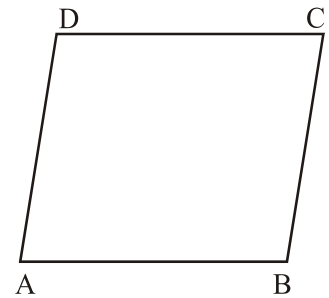
Proof: In ΔABC and ΔCDA :

$\angle BAC = \angle DCA$	[Alternate angles]
$\angle BCA = \angle DAC$	[Alternate angles]
$AC = AC$	[Common]
$\therefore \Delta ABC \cong \Delta CDA$	[By ASA]
$\Rightarrow \angle B = \angle D$	[By cpct]

Similarly, we can prove that $\angle A = \angle C$

Theorem 4: If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

Given: A quadrilateral *ABCD* in which opposite angles are equal.
i.e., $\angle A = \angle C$ and $\angle B = \angle D$



To prove: ABCD is a prallelogram i.e,., $AB \parallel DC$ and $AD \parallel BC$.

Proof: Since, the sum of the angles of quadrilateral is 360º

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow \angle A + \angle D + \angle A + \angle D = 360^\circ$ [$\angle A = \angle C$ and $\angle B = \angle D$]

$\Rightarrow 2\angle A + 2\angle D = 360^\circ$

$\Rightarrow \angle A + \angle D = 180^\circ$ [Co-interior angle]

$\Rightarrow AB \| DC$

Similarly,

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow \angle A + \angle B + \angle A + \angle B = 360^\circ$ [$\angle A = \angle C$ and $\angle B = \angle D$]

$\Rightarrow 2\angle A + 2\angle B = 360^\circ$

$\Rightarrow \angle A + \angle B = 180^\circ$ [$\therefore$ This is sum of interior angles on the same side of transversal AB]

$\therefore AD \parallel BC$

So, $AB \parallel DC$ and $AD \parallel BC$

$\Rightarrow$ *ABCD* is a parallelogram. **Hence Proved.**

Theorem 5: In a parallelogram, the diagonals bisect each other.

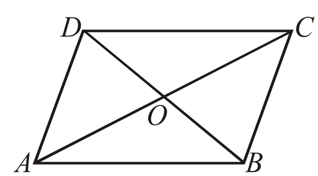
Given: A parallelogram *ABCD* such that diagonals *AC* and *BD* intersect at *O*.

To Prove $OA = OC$ and $OB = OD$

Proof Since *ABCD* is a parallelogram. Therefore

$AB \,||\, DC$ and $AD \,||\, BC$

AC is the transversal for $AB \,||\, CD$



$\therefore \angle BAC = \angle DCA$

$\Rightarrow \angle BAO = \angle DCO$ ...(i)

Again *BD* is the transversal for $AB \,||\, DC$

$\therefore \angle ABD = \angle CDB$

$\Rightarrow \angle ABO = \angle CDO$ ...(ii)

Now, in ΔAOB and ΔCOD

$\angle BAO = \angle DCO$ ...(from (i))

$AB = CD$ ...(Given)

$\angle ABO = \angle CDO$ ...(from (ii))

Hence $\Delta AOB \cong \Delta COD$ ...(by ASA)

$\Rightarrow OA = OC$ and $OB = OD$

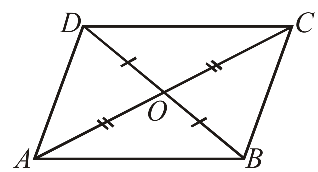
Hence $OA = OC$ and $OB = OD$

Theorem 6: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Given A quadrilateral $ABCD$ in which diagonals AC and BD bisect each other at O. i.e., $OA = OC$ and $OB = OD$.

To Prove $ABCD$ is a parallelogram

Proof In ΔAOB and ΔCOD,



$OA = OC$ ...(Given)

$OB = OD$ ...(Given)

and $\angle AOB = \angle COD$ ...(Vert. opp. $\angle$s)

$\therefore\ \Delta AOB \cong \Delta COD$ ...(SAS)

$\therefore\ \angle OAB = \angle OCD$

But this is a pair of alternate interior angles made by transversal AC on the lines AB and CD

$\therefore\ AB \,||\, CD$

Similarly, by taking ΔAOD and ΔCOB we can prove that

$$AD \,||\, BC$$

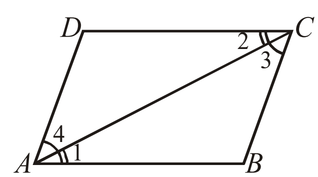
Hence quadrilateral $ABCD$ is a parallelogram.

Theorem 7: A quadrilateral is a parallelogram, if a pair of its opposite sides is parallel and of equal length.

Given A quadrilateral $ABCD$ in which $AB \,||\, CD$ and $AB = CD$.

To Prove $ABCD$ is a parallelogram

Construction Join AC



Proof In ΔABC and ΔCDA,

$AB = CD$ ...(Given)

$\angle 1 = \angle 2$ ...(Alt. int. angles)

and $AC = AC$ ...(Common)

$\therefore\ \Delta ABC \cong \Delta CDA$ ...(SAS)

$\therefore\ \angle 3 = \angle 4$ and $BC = AD$

But these are alternate interior angles

$\therefore\ AD \,||\, BC$

$\because\ AD = BC$ and $AD \,||\, BC$

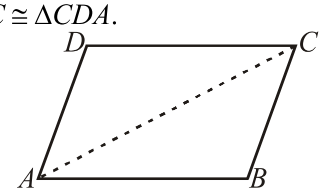
Hence $ABCD$ is a parallelogram.

Theorem 8: A diagonal of a parallelogram divides it into two congurent triangles.

Given: A parallelogram $ABCD$.

To Prove: Diagonal AC divides it in to two congurent triangles i.e., $\Delta ABC \cong \Delta CDA$.

Construction: Join A and C.



Proof Since $ABCD$ is parallelogram, therefore

$AB \,||\, DC$ and $AD \,||\, BC$

Now $AD \mid\mid BC$ and transversal AC intersect them at A and C respectively.

$\therefore \angle DAC = \angle BCA$ ...(i)

Again $AB \mid\mid DC$ and transversal AC intersect them at A and C respectively. Therefore,

$\angle BAC = \angle DCA$ ...(ii)

Now in ΔABC and ΔCDA

$AC = AC$ ...(Common)

$\angle BCA = \angle DAC$ ...(From (i))

$\angle BAC = \angle DCA$ ...(From (i))

Hence $\Delta ABC \cong \Delta CDA$ ...(by ASA)

Corollary: The adjacent angles of a parallelogram are supplementary

From the foregoing theorems, we conclude that

A quadrilateral is a parallelogram, if one of the following is true

(i) The diagonals bisect each other

(ii) One pair of opposite sides are equal and parallel

(iii) Each pair of opposite angles are equal

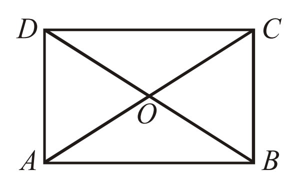
(iv) Each pair of opposite sides are equal

Properties of rectangle, squares and rhombuses

We have already learnt that a rectangle, a square or a rhombus is a special type of a parallelogram. It implies that these figures have all the properties of a parallelogram. But there are some additional properties which these figures possess. We discuss them below.

Theorem 9: A parallelogram is a rectangle if its diagonals have equal length

Given: A parallelogram $ABCD$ in which $BD = AC$



To Prove: $ABCD$ is a rectangle

Proof: In ΔDAB and ΔCBA,

$AD = BC$ ...(Opposite sides of a parallelogram)

$AB = BA$ ...(Common)

and $BD = AC$ ...(Given)

$\therefore \Delta DAB \cong \Delta CBA$ ...(SSS)

$\therefore \angle DAB = \angle CBA$ *(C. P. C. T.)*

But these are interior angles on the same side of the transversal and are supplementary.

$\therefore \angle DAB = \angle CBA = 90°$

Hence $ABCD$ is a rectangle

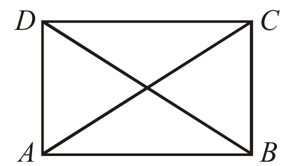
Problem: Draw a rectangle $ABCD$

Join its diagonals AC and BD.

Measure AC and BD

You will get $AC = BD$

[$\because$ $AC^2 = AB^2 + BC^2$ and $BD^2 = AB^2 + AD^2 = AB^2 + BC^2$]



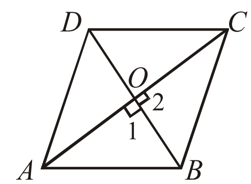
Draw two more rectangles and repeat the above steps. You will get the same result.

Theorem 10: A parallelogram is a rhombus if its diagonals are perpendicular.

Given The diagonals AC and BD of a parallelogram $ABCD$ are perpendicular to each other.

To Prove $ABCD$ is a rhombus

Proof In ΔAOB and ΔCOB,



$AO = CO$ ...(Diagonals bisect each other)

$BO = BO$ ...(Common)

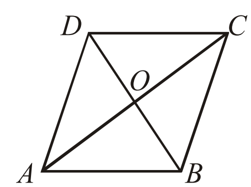
and $\angle 1 = \angle 2 = 90°$ ...(Given)

$\therefore$ $\Delta AOB \cong \Delta COB$ ...(SAS)

$\therefore$ $AB = BC$ (C. P. C. T.)

Hence $ABCD$ is a rhombus

Problem: Draw a rhombus $ABCD$



Join its diagonals AC and BD intersecting each other at O.

Measure $\angle AOD$

You will find

$\angle AOD = 90°$

i.e., $OD \perp AC$

Draw two more rhombus and repeat these steps. You will get the same result.

Thus we conclude that

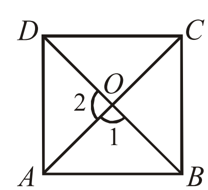
The diagonals of a rhombus are perpendicular to each other.

Theorem 11: A parallelogram is a square if its diagonals are equal and are at right angles.

Given: $ABCD$ is parallelogram in which diagonal AC = diagonal BD and $AC \perp BD$.

To Prove: $ABCD$ is a square

Proof: In ΔABO and ΔADO,



$BO = DO$

$AO = AO$ ...(Common)

and $\angle 1 = \angle 2 = 90°$ ...(Given)

$\therefore$ $\Delta ABO \cong \Delta ADO$ ...(SAS)

$\therefore$ $AB = AD$ (C. P. C. T.)

In ΔABD and ΔABC,

$AD = BC$ ...(Opp. sides of a parallelogram)

$AB = BA$ ...(Common)

and $BD = AC$ ...(Given)

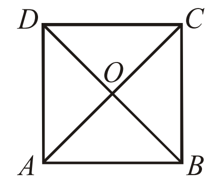
$\therefore\ \Delta ABD \cong \Delta BAC$ ...(SSS)

$\therefore\ \angle DAB = \angle CBA$ (C. P. C. T.)

But $\angle DAB + \angle CBA = 180°$ ...(Interior angles on the same side)

$\therefore\ \angle DAB = \angle CBA = 90°$

Hence $ABCD$ is a square



Problem: Draw a square $ABCD$

Join its diagonals AC and BD intersecting each other at O.

Measure AC and BD

You will find

$AC = BD$

Now measure $\angle AOD$

You will find

$\angle AOD = 90° \Rightarrow$ $DO \perp AC$

$\Rightarrow BD \perp AC$

Repeat these steps by taking two more squares.

You will get the same results

Theorem 12: In a triangle, the line segment joining the mid-points of any two sides is parallel to the third side and is half of it.

Given A ΔABC in which D and E are the mid-points of the sides AB and AC respectively.

To Prove (i) $DE \parallel BC$ (ii) $DE = \frac{1}{2} BC$

Construction: Produce DE to F, such that $EF = DE$. Join CF

Proof In ΔADE and ΔCFE,

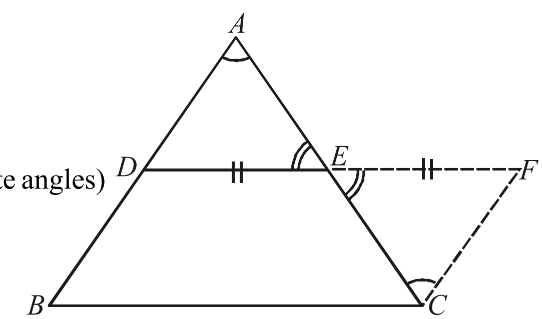
$AE = CE$ ...(Given)

$DE = FE$ ...(Construction)

and $\angle AED = \angle CEF$ . ..(Vertically opposite angles)

$\therefore\ \Delta ADE \cong \Delta CFE$ ...(SAS)

$\therefore\ \angle EAD = \angle ECF$ (C. P. C. T.)



But these are alternate interior angles

$\therefore\ CF \parallel AB$

and $FC = AD$ (C. P. C. T.)

But $AD = DB$ ...(Given)

$\therefore\ FC = DB$

Thus FC and DB are equal and parallel

$\therefore\ BCFD$ is a parallelogram

$\therefore$ DF and BC are also equal and parallel

or $DE \parallel BC$

and $DE = \frac{1}{2} DF$ ...(Construction)

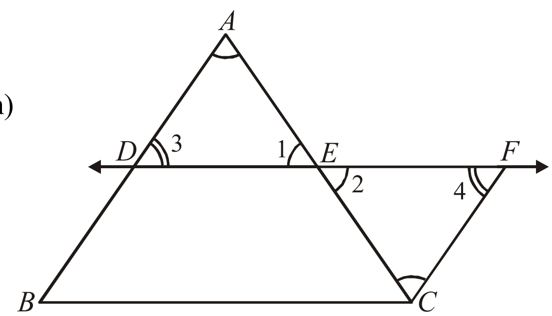
$\therefore$ $DE = \frac{1}{2} BC$

Theorem 13: The line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Given: A ΔABC. D is the mid-point of AB. Line DE is parallel to BC and intersects AC at E.

To Prove: $AE = EC$

Construction: Draw $CF \parallel BD$ meeting DE produced at F



Proof $\because$ $BC \parallel DE$ ...(Given)

and $BD \parallel CF$ ...(Construction)

$\therefore$ $BCFD$ is a parallelogram

$\therefore$ $BD = CF$

but $BD = AD$ ...(Given)

$\therefore$ $AD = CF$

Now in ΔADE and ΔCFE,

$AD = CF$ ...(Proved)

$\angle 1 = \angle 2$ ...(Vertically opposite angles)

and $\angle 3 = \angle 4$ ...(Alternative angles)

$\therefore$ $\Delta ADE \cong \Delta CFE$ ...(SAA)

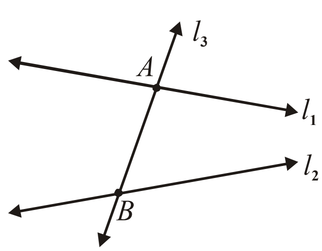
Hence $AE = CE$

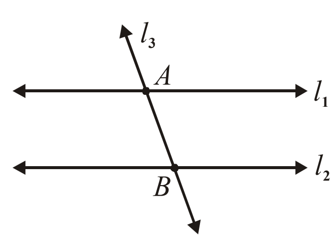
Note: The above theorem is also stated as under:

"The line drawn through the mid-point of one side of triangle parallel to another side intersects the third side at its mid-point".

Intercept

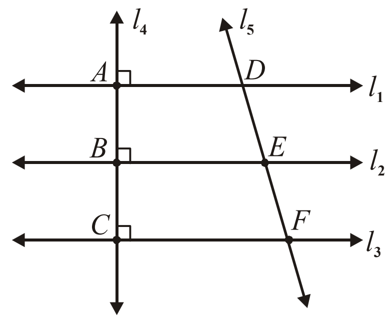
If there are two lines l_1 and l_2 (intersecting or parallel) in a plane and a third line l_3 cuts them at A and B, then the line segment AB is said to be the intercept made by l_1 and l_2 on l_3.





Now we esablish an important result on intercepts.

Problem: Draw three lines l_1, l_2 and l_3 such that $l_1 \,||\, l_2 \,||\, l_3$ and l_2 is equidistant from l_1 and l_3.



Draw a transversal l_4 perpendicular to them. Let l_4 meet l_1, l_2 and l_3 at A, B and C respectively.

$\because$ l_2 is equidistant from l_1 and l_3

$\therefore$ $AB = BC$ i.e., the intercepts made by l_1, l_2 and l_3 on l_4 are equal.

Now draw any other transversal l_5 meeting l_1, l_2 and l_3 at D, E and F respectively. Measure DE and EF. You will find that $DE = EF$ i.e., the intercepts made by l_1, l_2 and l_3 on l_5 are also equal.

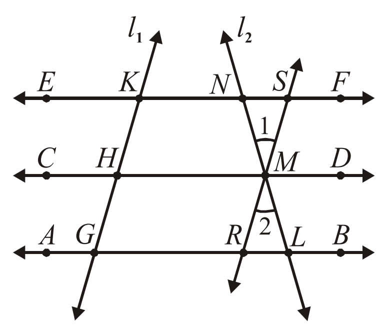
Now we state and prove an important theorem.

Theorem 14: If there are three or more parallel lines and the intercepts made by them on one transversal are equal, the corresponding intercepts on any other transversal are also equal

Given A transversal l_1 intersects three parallel lines AB, CD and EF at G, H and K respectively such that $GH = HK$. Another transversal l_2 intersects these lines at L, M and N respectively.

To Prove $MN = ML$

Construction: Through M draw a line $RS \,||\, l_1$ meeting AB at R and EF at S.



Proof

$\because$ $KS \,||\, HM$ ...(Given)

and $KH \,||\, SM$ ...(Construction)

$\therefore$ $KHMS$ is a parallelogram

$\therefore$ $HK = MS$ ...(i)

Similarly

$\because$ $HM \,||\, GR$ ...(Given)

and $HG \,||\, MR$

...(Construction)

$\therefore$ $HGRM$ is a parallelogram

$\therefore$ $GH = RM$ ...(ii)

From (i) and (ii), we get

$MS = RM$ ...($\because$ $GH = HK$ Given)

Now in ΔMNS and ΔMLR,

$MS = MR$ ...(Proved)

$\angle 1 = \angle 2$ ...(Vertically opposite angles)

and $\angle MNS = \angle MLR$ ...(Alternative angles)

$\therefore$ $\Delta MNS \cong \Delta MLR$ ...(SAA)

Hence $MN = ML$ (C. P. C. T.)

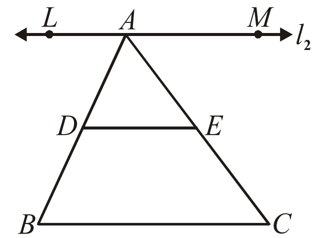
now we prove the following theorem in another simply way.

Theorem 15 : The line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Given A ΔABC. D is the mid-point of AB and $DE \parallel BC$.

To Prove $AE = EC$

Construction: Through A draw $LM \parallel BC$



Proof LM, DE and BC are parallel lines and transversal AB cuts them at A, D and B respectively such that

$AD = DB$ ...(Given)

Another transversal AC cuts them at A, E and C respectively

$AE = EC$

Hence DE bisects AC also.

BASIC CONCEPTS AND IMPORTANT POINTS

1. A quadrilateral which has both pairs of opposite parallel sides is called a parallelogram.
2. In a parallelogram, opposite sides are equal.
3. The opposite angles of a parallelogram are equal.
4. The diagonals of a parallelogram bisect each other.
5. If all sides of a parallelogram are equal, then it is a rhombus.
6. If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.
7. A parallelogram with one of its angle as a right angle is a rectangle.
8. If a pair of consecutive sides of parallelogram are equal, then parallelogram is a rhombus.
9. A square is a rectangle with a pair of consecutive sides equal.
10. The diagonals of rectangle are of equal length.
11. The diagonals of a rhombus are perpendicular bisectors to each other.
12. The diagonals of a square are equal and perpendicular to each other.
13. The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
14. The line drawn through the mid-point of one side of a triangle, parallel to another side, intersects the third side at its mid-point.
15. If there are three parallel lines making equal intercepts on any transversal then the intercepts made by them on any other transversal are also equal.

SOLVED PROBLEMS

Example 1: In a quadrilateral three angles are in the ratio 3 : 3 : 1 and one of the angles is 80°, then other angles are

(a) 120°, 120°, 40° (b) 100°, 100°, 80°

(c) 110°, 110°, 60°

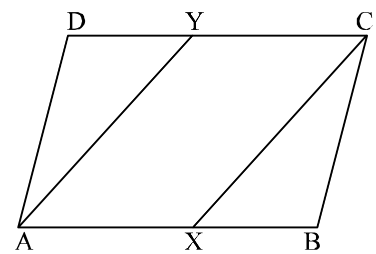
Solution: **(a)** Sum of angles of a quadrilateral is 360°, one angle is 80°. So sum of other three angles is 280°, which is to be divided in the ratio 3 : 3 : 1.

Example 2: ABCD is a parallelogram and X and Y are the mid-points of sides AB and CD respectively. Then the quadrilateral AXCY is

(a) a parallelogram (b) a rectangle

(c) a rhombus (d) a square

Solution: **(a)** X and Y are the mid-points of AB and CD.



$\therefore \quad AX = \frac{1}{2}AB$ and $CY = \frac{1}{2}CD$

$\Rightarrow \quad AX = CY \qquad [\because AB = CD]$

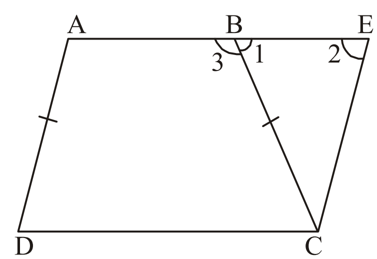
Also AX || CY $\qquad [\because AB \parallel CD]$

$\therefore$ AXCY is a parallelogram

[If a pair of opposite sides is equal and parallel then, quadrilateral is a parallelogram]

Example 3: If ABCD is a trapezium in which AB || CD and AD = BC, prove that ∠A = ∠B.

Solution: Given: AB || CD and AD = BC



To prove: ∠A = ∠B

Proof: Since AB ||CD

AE || CD

So, AECD is a parallelogram

⇒ AD = EC (Opposite sides of parallelogram)

But AD = BC [Given]

⇒ EC = BC

⇒ ∠1 = ∠2 ...(i) (Angles opp. equal sides of a triangle)

Now, ∠A + ∠2 = 180° ...(ii)

[Interior angles on the same side of transversal]

∠3 + ∠1 = 180° [Linear pair] ..,(iii)

From (ii) and (iii), we get

∠A + ∠2 = ∠3 + ∠1

From (i), ∠A = ∠B

Hence proved.

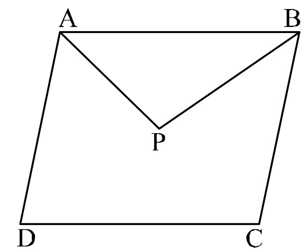
Example 4: In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.

Solution: Given: ABCD is a parallelogram such that angle bisectors of adjacent angles A and B intersect at point P.

To prove: $\angle APB = 90°$

Proof: $\angle A + \angle B = 180°$

[AD ∥ BC and ∠A and ∠D are consecutive interior angles]



$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$$

But $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle APB = 180°$

[Sum of angles of a triangle is 180°]

$\Rightarrow 90° + \angle APB = 180°$

$\Rightarrow \angle APB = 90°$

Hence proved.

Example 5: AB and CD are two parallel lines and a transversal *l* intersects AB at X and CD at Y. Prove that the bisectors of the interior angles form a rectangle.

Solution: Given: AB ∥ CD and transversal *l* intersects AB at X and CD at Y.

To prove: PYQX is a rectangle

Proof: $\angle AXY = \angle DYX$ [Alternate angles]

$$\frac{1}{2}\angle AXY = \frac{1}{2}\angle DYX$$

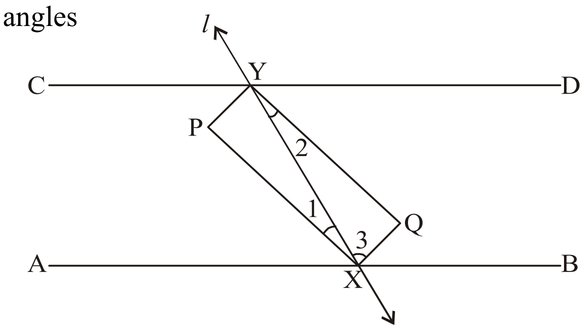
$\therefore \angle 1 = \angle 2$

and these angles are alternate interior angles

$\Rightarrow$ PX ∥ QY

Similarly, PY ∥ QX

PYQX is a parallelogram



Now, $\angle BXY + \angle DYX = 180°$

[Consecutive interior angles]

or $2\angle 2 + 2\angle 3 = 180°$

$\angle 2 + \angle 3 = 90°$

$\Rightarrow \angle 1 + \angle 3 = 90°$ [$\because \angle 2 = \angle 1$]

$\Rightarrow \angle QXP = 90°$

$\therefore$ PYQX is a rectangle. Hence proved.

Example 6: In the given figure, E is the mid-point of side AD of a trapezium ABCD with AB ∥ CD. A line through E parallel to AB meets BC in F. Show that F is the mid-point of BC.

Solution: Given: ABCD is a trapezium E is the mid-point of AD and AB || CD, EF || AB.

To prove: F is the mid-point of BC.

Construction: Join AC to intersect BF at point G

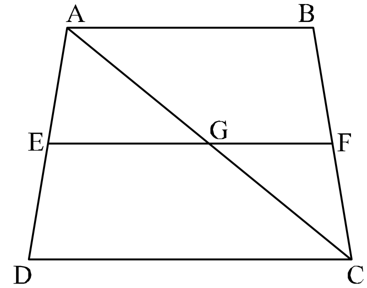
Proof: AB || DC, EF || AB $\Rightarrow$ EF || DC

In ΔADC, EG || DC and E is mid-point of AD.

$\therefore$ G is the mid-point of AC.

[By converse of mid-point theorem]

In ΔABC, FG || AB [$\because$ EF || AB]



G is mid-point of AC

$\therefore$ F is mid-point of BC. [by converse of mid-point theorem]

Example 7: ABCD is a rhombus and AB is produced to E and F such that AE = AB = BF. Prove that ED and FC are perpendicular to each other.

Solution: Given: ABCD is a rhombus. AB produced to E and F such that AE = AB= BF.

Construction: Join ED and FC and produce it to meet at G.

To prove: $ED \perp FC$

Proof: AB is produced to points E and F such that

AE = AB = BF ...(i)

Also, since ABCD is a rhombus

AB = CD = BC = AD ...(ii)

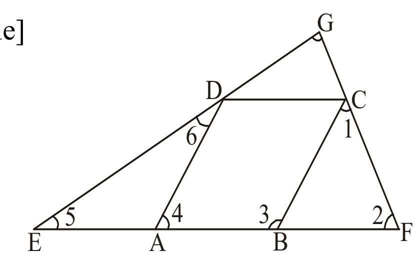
In ΔBCF,

BC = BF [From (i) and (ii)]

$\Rightarrow$ $\angle 1 = \angle 2$

$\angle 3 = \angle 1 + \angle 2$ [Exterior angle]

$\angle 3 = 2\angle 2$ ...(iii)



Similarly,

AE = AD

$\angle 5 = \angle 6$

$\Rightarrow$ $\angle 4 = \angle 5 + \angle 6 = 2\angle 5$ ...(iv)

Adding (iii) and (iv) we get:

$\angle 4 + \angle 3 = 2\angle 5 + 2\angle 2$

$\Rightarrow$ $180° = 2(\angle 5 + \angle 2)$

[$\because$ $\angle 4$ and $\angle 3$ are consecutive interior angle]

$\Rightarrow$ $\angle 5 + \angle 2 = 90°$

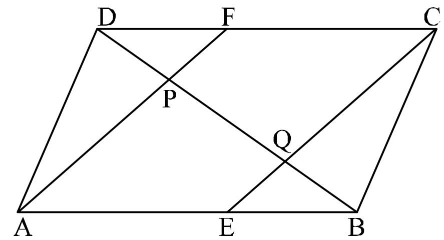
$\therefore$ $BG \perp FC.$ Now in ΔEGF

$\angle 5 + \angle 2 + \angle EGF = 180°$

$\Rightarrow$ $\angle EGF = 90°$

Hence proved

Example 8: In a parallelogram ABCD, E and F are the mid-points of sides AB and CD. Prove that the line segments AF and CE trisect the diagonal BD.



Solution: Given: ABCD is a parallelogram. E and F are the mid-points of AB and CD.

To prove: DP = PQ = QB

Construction: Join AF and CE

Proof: Since ABCD is a || gm

AB || CD and AB = CD

AE || CD and AB = CD

$\therefore$ AE = FC $\quad$ $[AE = \frac{1}{2}AB, FC = \frac{1}{2}CD]$

$\Rightarrow$ AFCE is a || gm

[A pair of opposite sides is equal and parallel]

Now, in ΔDQC,

FP || CQ and F is mid-point of CD

$\Rightarrow$ P is the mid point of DQ (converse of MPT)

i.e. PQ = DP ...(i)

Similarly, from ΔAPB, E is mid-point of BP and QE || AP.

$\Rightarrow$ Q is the mid point of AB (converse of MPT)

i.e., PQ = BQ ...(ii)

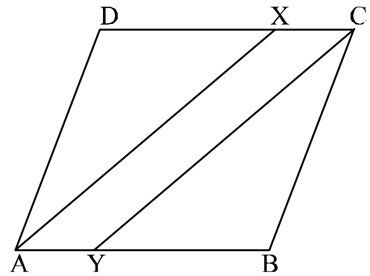
From (i), and (ii),

PQ = DP = BQ

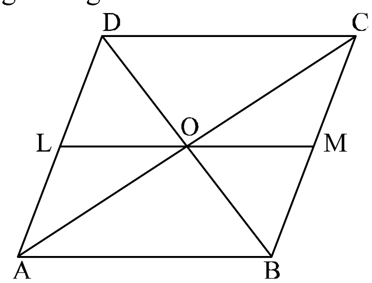
$\therefore$ AF and CE trisect the diagonal BD.

REVISION EXERCISE

1. In the given figure, ABCD is a parallelogram and line segments AX and CY bisect the angles A and C respectively. Show that AX || CY.

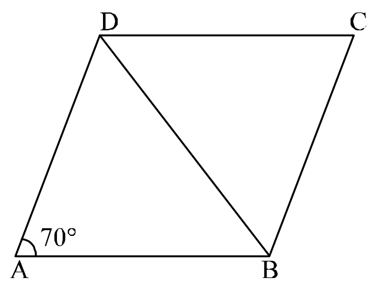


2. In the given figure, ABCD is a parallelogram in which diagonals AC and BD intersect at O. A line segment LM is drawn passing through O. Prove that LO = OM.



3. PQRS is a rhombus with ∠PQR =58°. Determine ∠PRS.

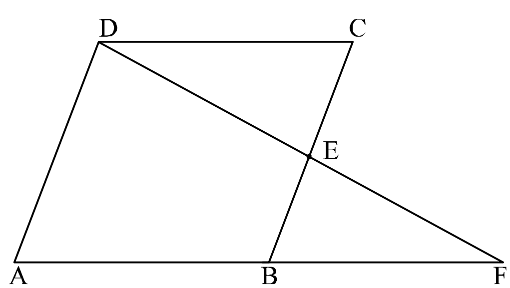
4. In the given figure, ABCD is a rhombus. If ∠DAB = 70° then find ∠CDB.



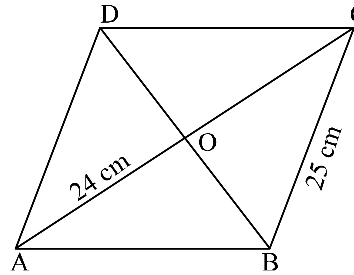
5. D, E and F are respectively the mid-points of sides, BC, CA and AB of an equilateral ΔABC. Prove that DEF is also an equilateral triangle.

6. Prove that the figure formed by joining the mid-points of the pairs of consecutive sides of a quadrilateral is a parallelogram.

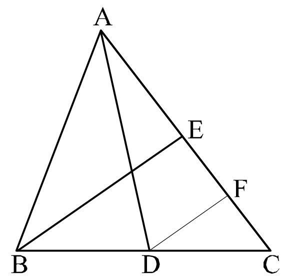
7. ABCD is a parallelogram and E is the mid-point of side BC. DE and AB on producing meet at F. Prove that AF = 2 AB.



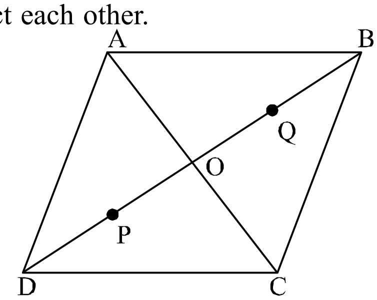
8. In the given figure, ABCD is a rhombus in which BC = 25 cm, and AO = 24 cm. Find the sum of the lengths of the diagonals.



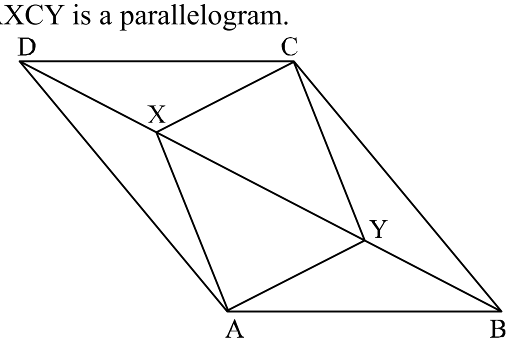
9. In the figure, AD and BE are medians of ΔABC and BE || DF. Prove that $CF = \frac{1}{4}AC$.



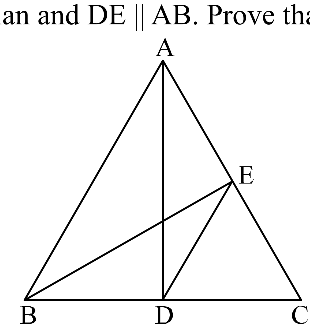
10. In a parallelogram ABCD, two points P and Q are taken on the diagonal BD such that DP = BQ. Prove that PQ and AC bisect each other.



11. In the given figure, ABCD is a parallelogram and X and Y are points on the diagonal BD such that DX = BY. Prove that AXCY is a parallelogram.

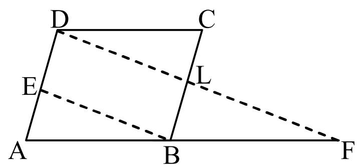


12. In the given figure, AD is median and DE || AB. Prove that BE is the median.

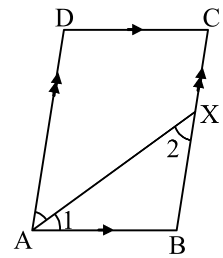


13. In the adjoining figure, ABCD is a parallelogram and E is the midpoint of AD. A line through D, drawn parallel to EB, meets AB produced at F and BC at L. Prove that

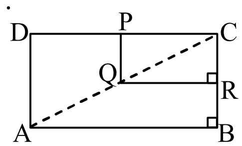
(i) AF = 2DC, (ii) DF = 2DL.



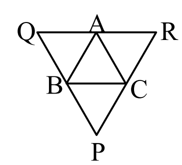
14. In the adjoining figure, ABCD is a parallelogram and the bisector of $\angle A$ bisects BC at X. Prove that AD = 2AB.



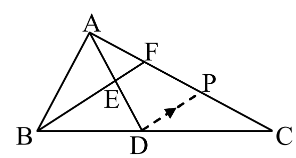
15. In the adjoining figure, ABCD and PQRC are rectangles, where Q is the midpoint of AC. Prove that (i) DP = PC, (ii) PR = $\frac{1}{2}AC$.



16. A ΔABC is given. If lines are drawn through A, B, C parallel respectively to the sides BC, CA and AB, forming ΔPQR, as shown in the adjoining figure, show that $BC = \frac{1}{2}QR$.

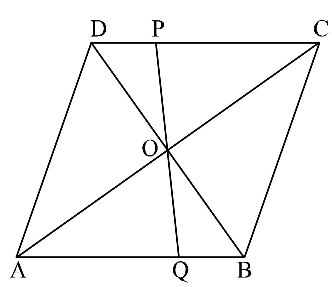


17. In the adjoining figure, AD is a median of ΔABC and E is the midpoint of AD. Also, BE produced meets AC in F. Prove that $AF = \frac{1}{3}AC$.

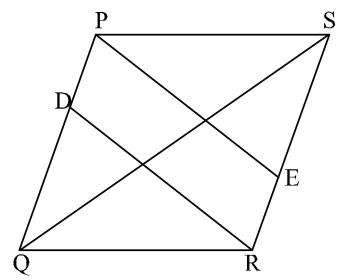


HIGHER ORDER THINKING SKILLS (HOTS)

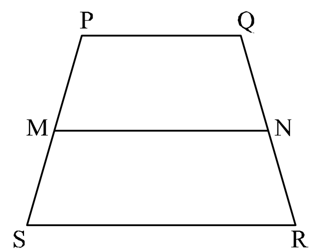
1. In the given figure, ABCD is a parallelogram whose diagonals intersect each other at O. Through O, PQ is drawn. Then prove that OP = OQ.



2. PQRS is a parallelogram. D is a point on PQ such that $PD = \frac{1}{3}PQ$ and E is a point on RS and $RE = \frac{1}{3}RS.$ Then prove that quadrilateral PDRE is a parallelogram.



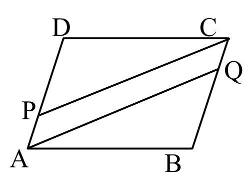
3. M and N are the mid-points of non parallel sides of a trapezium PQRS. Prove that (a) MN || PQ, (b) $MN = \frac{1}{2}(PQ + RS)$.



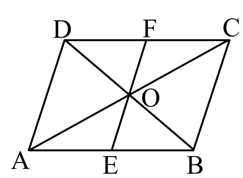
4. Show that the four triangles formed by joining the mid-points of the three sides of a triangle are congruent to each other.

5. Show that the line segment joining the mid-point of opposite sides of quadrilateral bisect each other.

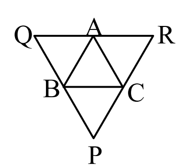
6. In the adjoininig figure, ABCD is a parallelogram. If P and Q are points on AD and BC respectively such that AP = $\frac{1}{3}AD$ and CQ = $\frac{1}{3}BC$, prove that AQCP is a parallelogram.



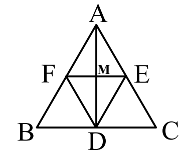
7. In the adjoining figure, ABCD is a parallelogram whose diagonals intersect each other at O. A line segment EOF is drawn to meet AB at E and DC at F. Prove that OE = OF.



8. In the adjoining figure, ΔABC is a triangle and through A, B, C lines are drawn, parallel respectively to BC, CA and AB, intersecting at P, Q and R. Prove that the perimeter of ΔPQR is doble the perimeter of ΔABC.

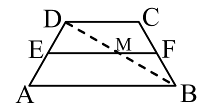


9. In the adjoining figure, ΔABC is an isosceles triangle in which AB = AC and D, E, F are the midpoints of BC, CA and AB respectively. Show that $AD \perp FE$ and AD is bisected by FE.



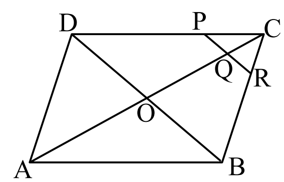
10. Let ABCD be a trapezium in which AB || DC and let E be the midpoint of AD. Let F be a point on BC such that FE || AB. Prove that

(i) F is the midpoint of BC, (ii) EF = $\frac{1}{2}(AB + DC)$.

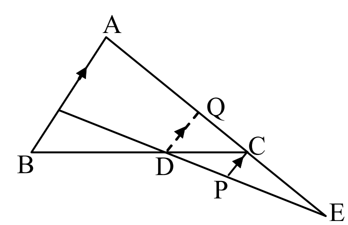


11. Prove that the line segment joining the midpoints of the diagonals of a trapezium is parallel to the parallel sides and equal to half their difference.

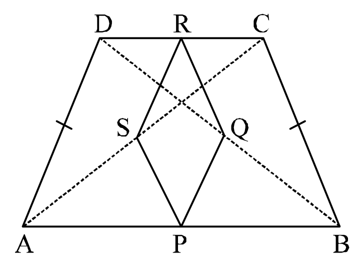
12. In the adjoining figure, ABCD is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC.$ Also, PQ when produced meets BC at R. Prove that R is the midpoint of BC.



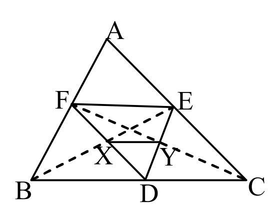
13. In the adjoining figure, the side AC of ΔABC is prodcued to E such that CE = $\frac{1}{2}AC.$ If D is the midpoint of BC and ED produced meets AB at F, and CP, DQ are drawn parallel to BA, prove that $FD = \frac{1}{3}FE$.



14. In the adjoining figure, ABCD is a trapezium in which AB || DC and AD = BC. If P, Q, R, S be respectively the midpoints of BA, BD and CD, CA then show that PQRS is a rhombus.



15. In the adjoining figure, D, E, F are the midpoints of the sides BC, CA and AB of ΔABC. If BE and DF intersect at X while CF and DE intersect at Y, prove that $XY = \frac{1}{4}BC.$



MULTIPLE CHOICE QUESTIONS

1. A diagonal of a rectangle is inclined to one side of the rectangle at 25°. The acute angle between the diagonals is

   (a) 55° (b) 50° (c) 40° (d) 60°

2. ABCD is a rhombus such that ∠ACB =40°. Then ∠ADB is

   (a) 40° (b) 45° (c) 50° (d) 60°

3. The quadrilateral formed by the lines joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if

   (a) PQRS is a rhombus (b) PQRS is a parallelogram

   (c) diagonals of PQRS are perpendicular (d) diagonals of PQRS are equal

4. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3 : 7 : 6 : 4, then ABCD is a

   (a) rhombus (b) parallelogram (c) trapezium (d) kite

5. If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a

   (a) rectangle (b) rhombus

   (c) parallelogram (d) quadrilateral whose opposite angles are supplementary

6. The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is

   (a) a rhombus (b) a rectangle (c) a square (d) any parallelogram

7. The figure formed by joining the mid-points of the side of a quadrilateral ABCD, taken in order, is a square only if.

   (a) ABCD is a rhombus

   (b) diagonals of ABCD are equal

   (c) diagonals of ABCD are equal and perpendicular

   (d) diagonals of ABCD are perpendicular

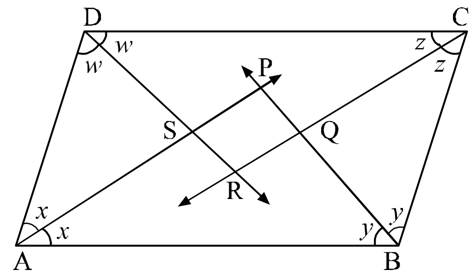
8. The diagonals AC and BD of a parallelogram ABCD intersect each other at point O. If ∠DAC = 32° and ∠AOB =70°, then ∠DBC is equal to

   (a) 24° (b) 86° (c) 38° (d) 32°

9. D and E are the mid-points of the sides AB and AC respectively of ΔABC, DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is

   (a) ∠DAE = ∠EFC (b) AE = EF (c) DE = EF (d) ∠ADE = ∠ECF

10. In the given figure, ABCD is a parallelogram.



The quadrilateral PQRS is exactly

(a) a square (b) a parallelogram (c) a rectangle (d) a rhombus

MATRIX MATCHING

*This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**.*

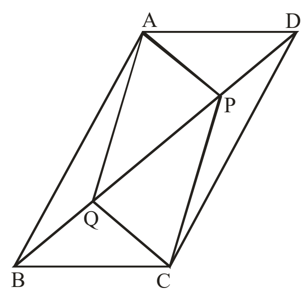
11. Match them correctly.

Column–I	**Column–II**
(A) Parallelogram	(p) Opposite sides are equal
(B) Rectangle	(q) Opposite angles are equal
(C) Square	(r) Diagonal bisect each other
(D) Trapezium	(s) unequal sides.

(A) (A-s), (B-r), (C-q), (D-p) (B) (A-q), (B-r), (C-s), (D-p)

(C) (A-p), (B-q), (C-r), (D-s) (D) (A – p,q,r), (B – p, q, r), (C – p,q,r) (D –s)

12. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ.



Column–I	**Column–II**
(A) Δ APD is congruent to	(p) Δ CQB
(B) AP is equal to	(q) CQ
(C) Δ AQB is congruent to	(r) Δ CPD
(D) AQ is equal to	(s) CP

(A) (A-s), (B-r), (C-q), (D-p) (B) (A-q), (B-r), (C-s), (D-p)

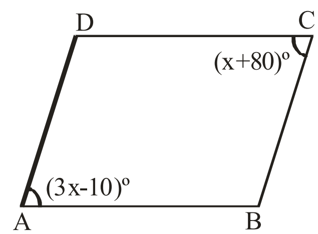
(C) (A-q), (B-p), (C-r), (D-s) (D) (A – p), (B – q), (C – r) (D –s)

INTEGER TYPE QUESTIONS

The answer to each of the questions is a single-digit integer, ranging from 0 to 9.

13. In ΔABC, the mid-points of BC, CA and AB are D, E and F respectively. If the lengths of side AB, BC and CA are 17cm, 18cm and 19 cm respectively. If the perimeter of ΔDEF is equal to $4\lambda + 3$, then λ is

14. In figure, ABCD is a parallelogram if the value of x is 5λ, then λ is



CHAPTER

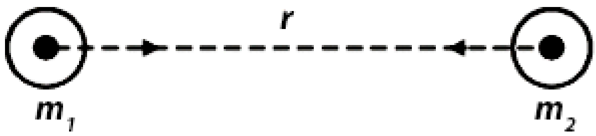
3

Gravitation

Universal Law of Gravitation

Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the separation between the two objects. The force is directed along the line joining the two objects.

$$F_g = G\frac{m_1 m_2}{r^2} \quad \ldots\ldots(i)$$



F_g is the gravitational force, m_1 & m_2 are the masses of the two objects, r is the separation between the objects, G is the universal gravitational constant.

The constant of proportionality G is known as the *universal gravitational constant*. It is termed as "universal constant" because it is same at all places and all times, and thus universally characterizes the intrinsic strength of the gravitational force. The SI unit of G is $Nm^2\ kg^{-2}$. The accepted value of G is $6.673 \times 10^{-11}\ Nm^2 kg^{-2}$.

If the m_1 and m_2 of the two bodies are $1\ kg$ each and the distance *r* between them is m_1, then putting $m_1 = 1kg$; $m_2 = 1\ kg$ and $r = 1\ m$ in equation (i) we get

$$F = G$$

Thus, the gravitational constant G is numerically equal to the force of gravitation which exists between two bodies of unit masses kept at a unit distance from each other.

Gravitation and Gravity

Gravitation	Gravity
(i) Gravitation is the force of attraction between any two bodies of the universe.	(i) Gravity is the earth's gravitational pull on the body lying on or near the surface of the earth.
(ii) The gravitational force on a body A of mass m_1 due to a body B of mass m_2 placed at a distance r is	(ii) The force of gravity on a body of mass m is, $F = mg$

$F = G\,\frac{m_1 m_2}{r^2}$ [where G = universal gravitational constant]	[where g = acceleration due to gravity]
(iii) The force of gravitation between two bodies can be zero, if the separation between them becomes infinity.	(iii) The force of gravity on a body is zero at the centre of the earth.

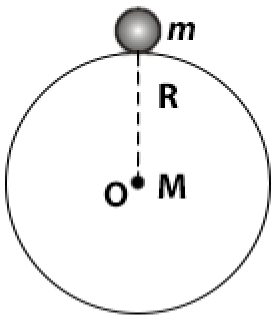
The uniform acceleration produced in a freely falling body due to the gravitational pull of the earth is known as acceleration due to gravity.

Relation between *g* and G

Consider the earth to be spherical body of mass *M*, radius *R* with centre *O*. Suppose a body of mass *m* is placed on the surface of the earth, where acceleration due to gravity is g.

According to Newton's law of gravitation, the force exerted by the earth on the body is given by

$$F = \frac{GMm}{R^2} \ldots\ldots\ldots\ldots (i)$$



This force involves acceleration in the body which is equal to the product of the mass of the body and acceleration due to gravity *i.e.,*

$$F = m \times g \ldots\ldots\ldots \text{(ii)}$$

From equation (i) and (ii) we get

$$m \times g = \frac{GMm}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}} \ldots\ldots\ldots\ldots \text{(iii)}$$

[where g is a acceleration due to gravity. G is gravitational constant, M is mass of the earth and R is radius of earth.

Now, to calculate the value of *g*, we put the above values in equation (iii),

$$G = 6.7 \times 10^{-11}\ Nm^2kg^{-2}$$

$$M = 6 \times 10^{24}\ \text{kg}$$

$$R = 6.4 \times 10^{6}\ m$$

$$g = \frac{6.7 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^{6})^{2}} \text{ ms}^{-2} = 9.8 \text{ ms}^{-2}$$

Variation in the value of *g*

We take the value of *g* to be constant near the surface of earth for the sake of our convenience in calculations. But the value of *g* varies from place to place on the surface of earth. Thus, the value of g depends on the following factors:

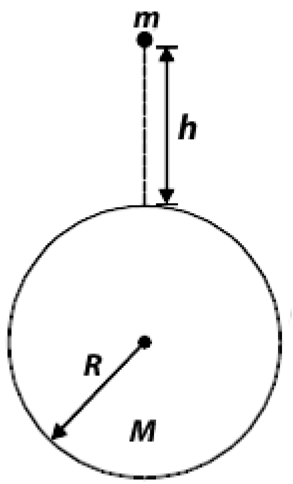
(i) **Shape of Earth:** the earth is not a perfect sphere. It is somewhat flat at the two poles. The equatorial radius is approximately 21 km more than the polar radius. And since

$$g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$$

The value of *g* is minimum at equator and maximum at the poles.

(ii) **Height above the surface of the earth.**

The force of gravity on an object of mass *m* at a height *h* above the surface of earth is:



$$F = \frac{GMm}{(R + h)^2}$$

∴ Accelerraion due to gravity at this height

$$g' = \frac{F}{m} = \frac{GM}{(R + h)^2}$$

Or $$g' = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

or, $$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} \qquad \left[as, \frac{GM}{R^2} = g\right]$$

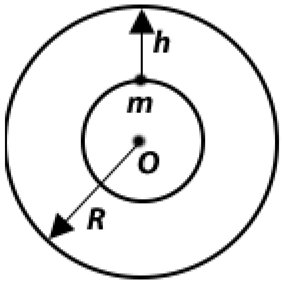
∴ g' < g

$$g' = g\left(1 + \frac{h}{R}\right)^{-2}$$

If $h << R$

$$g' \approx g\left(1 - \frac{2h}{R}\right)$$

(iii) **Depth below the surface of earth:** Let on object of mass ***m*** is situated at the depth ***h*** below the earth's surface. It's distance from the centre of earth (*R* – *h*). This mass is situated at the surface of the inner solid sphere and lies inside the outer spherical shell. The gravitational force of attraction on a mass inside a spherical shell is always zero. Therefore, the object experiences gravitational attraction only due to inner solid sphere. The mass of this sphere is:



$$M' = \left(\frac{M}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi(R-h)^3$$

or, $M' = \frac{(R-h)^3}{R^3} . M$

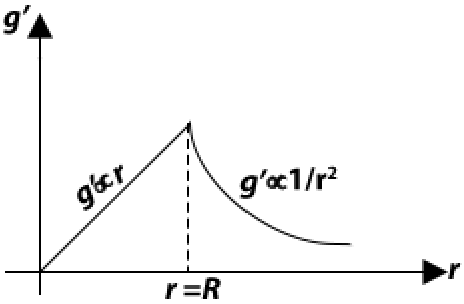
$\therefore \quad F = \frac{GM'm}{(R-h)^2} = \frac{GmM(R-h)}{R^3}$

and $g' = \frac{F}{m}$

Substituting the values, we get

$$\boxed{g' = g\left(1 - \frac{h}{R}\right)}$$

i.e. $g' < g$

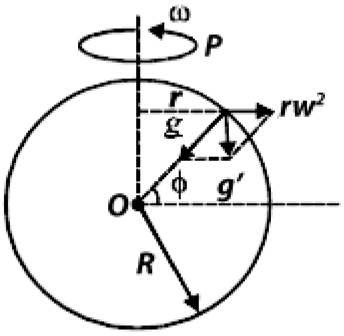


(iv) **Axial rotation of earth.**

If a particle is at rest on the surface of earth, in latitude ϕ. The effective acceleration due to gravity is the resultant of *g* and *rw*^{2} (centrifugal acceleration) and is given as:

$$\boxed{g' = g - Rw^2 \cos^2 \phi}$$

Following conclusions can be drawn from the above discussion.



(a) The effective value of g is not truly vertical.

(b) The effect of centrifugal force due to rotation of earth is to reduce the effective value of g.

(c) At equators

$\phi = 0^o$

$\therefore g' = g - Rw^2$

At polar

$\phi = 90^o$

$\therefore g' = g$

Thus, at equator *g*' is minimum while at poles *g*' is maximum.

Motion of Objects Under the Influence of Gravitational Force of the Earth:

The equations are:

Case - I

When we drop the object from a certain height, then $u = 0$, $a = +g$ (g is along the direction of v)

$v = u + at$ or $v = 0 + gt \Rightarrow v = gt$

$s = ut + 1/2\, at^2$ or $s = 0 \times t + 1/2\, gt^2 \Longrightarrow s = 1/2\, gt^2$

$v^2 = u^2 + 2as$ or $v^2 = 0^2 + 2\, gs \Longrightarrow v^2 = 2\, gs$

Case-II

When we throw an object upwards with certain velocity, then final velocity v = 0 and $a = -g$. Therefore,

$v = u - gt$

$0 = u - gt \Rightarrow u = gt$ (at highest point, $v = 0$)

$t = \frac{u}{g}$ (time taken to attain maximum height)

$s = ut - 1/2\ gt^2$

$v^2 = u^2 - 2gs$

$0^2 = u^2 - 2gs \Rightarrow u^2 = 2gs$ (at highest point, $v = 0$)

$s = \frac{u^2}{2g}$ gives maximum height attained

Mass and Weight

Mass	Weight
(i) Mass of a body is the quantity of matter Contained in it.	(i) Weight of a body is the force with which it is attracted towards the centre of the earth.
(ii) Mass is a scalar quantity. The SI unit of mass is kilogram (kg).	(ii) Weight is a vector quantity. The SI unit of weight is newton (N).
(iii) Mass of a body is constant and does not change from place to place.	(iii) Weight of a body is not constant; it varies from place due to the variation of *g*.
(iv) Mass of a body cannot be zero.	(iv) Weight of a body can be zero.

Weight of an Object on the Moon

Let the mass of an object be *m*. Let its weight on the moon be W_m

Also, let the mass of the moon is M_m and its radius R_m.

According to Universal law of gravitation, the weight of the object on the moon will be

$$W_m = G\frac{M_m \times m}{R_m^2} \quad .....(i)$$

Let the weight of the same object on earth be W_e. The mass of the earth is M_e and its radius is R_e. Then weight of the object on the earth will be

$$W_e = G\frac{M_e \times m}{R_e^2} \quad ....(ii)$$

Now, we know that the mass of the earth is about 100 times that of the moon and the radius of the earth is about 4 times that of the moon.

$$i.e\ M_e \approx 100\ M_m$$

$$R_e \approx 4R_m$$

Now dividing equation (*i*) and (*ii*) and using these conditions in (*i*) and (*ii*) we get ,

$$\frac{W_m}{w_e} = \frac{GM_m m}{R_m^2}\frac{R_e^2}{GM_e m}$$

or $$\frac{W_m}{W_e} = \frac{M_m R_e^2}{M_e R_m^2} = \frac{M_m 16 R_m^2}{100 M_m R_m^2} = \frac{16}{100} \approx \frac{1}{6}\ or\ W_m = \frac{1}{6}\ W_e$$

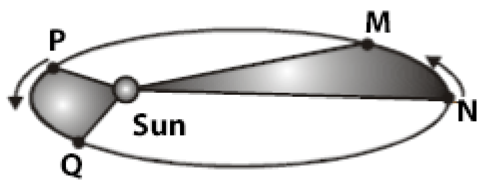
or Weight of the object on moon = $\left(\frac{1}{6}\right)^{th}$ weight of the object on earth.

Centripetal Force: The force towards the centre is called centripetal force. In the absence of this force, the body flies off along a straight line which is tangent to the circular path.

Kepler's Laws of Planetary Motion: Johannes Kepler formulated three laws, which govern the motion of the planets. These laws are:

i) **Law of Orbit:** Every planet revolves around the sun in an elliptical orbit with the sun situated at one of the foci.

ii) **Law of Area:** The line joining the planet and the sun sweeps out equal areas in equal intervals of time *i.e.,* the areal velocity of the planet around the sun is constant.



iii) **Law of Time Period:** The square of time period of revolution of a planet around the sun is directly proportional to the cube of the mean distance of a planet from the sun *i.e.,*

$$T^2 \propto r^3$$

where, T is the time period and *r* is mean distance of the planet from the sun.

Motion of Satellites

Just as the planets resolve round the sun, in the same way few celestial bodies revolve around the planets.

These bodies are called satellites. They can be of two types

(a) Natural Satellites: Moon is a natural satellite of earth.

(b) Artificial Satellites: Such satellites are launched from earth by human and are used for telecommunication, weather forecast and other applications.

Orbital Speed: The minimum velocity required to put the satellite into a given orbit around the earth is called orbital speed of the satellite.

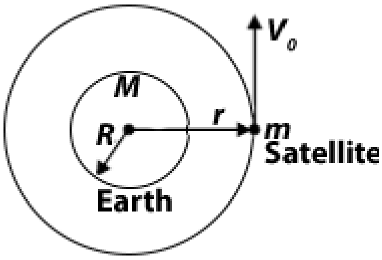
$$V_o = \sqrt{\frac{GM}{r}} = \sqrt{gr}$$

M = mass of the Earth

R = radius of the Earth

r = radius of the orbit of satellite

m = mass of the satellite



For a satellite close to earth's surface

$$r \approx R$$

$$\therefore\ V_o = \sqrt{gR} = 7.9\ km/s$$

Escape Velocity

The minimum velocity with which a body is thrown vertically upwards from the surface of the planet so that it escapes the planet's gravitational field and never return back on its own is called escape velocity. It is given by

$$V_e = \sqrt{2gR}$$

$$\approx 11.2\ km/s$$

REVISION EXERCISE - LEVEL - I

1. If radius of earth is *R* and acceleration due to gravity on its surface is *g*, at what height acceleration due to gravity becomes half of that on surface of earth?

(a) $R(\sqrt{2}+1)$ (b) $R(\sqrt{2}-1)$

$(c)\ \sqrt{2}R$ $(d)\ \frac{R}{\sqrt{2}}$

2. If radius of earth is R, and acceleration due to gravity on its surface is *g*, at what depth acceleration due to gravity becomes half of that on its surface?

$(a)\ \frac{R}{4}$ $(b)\ \frac{R}{2}$ $(c)\ \frac{3R}{4}$ $(d)\ \frac{R}{8}$

3. If change in value of *g* at height *h* above the surface of earth is same as at the depth *y* below it when both *y* and *h* are much smaller than the radius of earth, then

(a) $y = h^2$ (b) $2y = h$ (c) $y = 2h$ (d) $y = h$

4. If radius of earth were to shrink by 1%, and the mass remains same, acceleration due to gravity on surface of earth would

(a) decrease by1% (b) decrease by 2%

(c) increase by 1% (d) increase by 2%

5. At what depth from the surface of earth, acceleration due to gravity will be $\frac{1}{4}$th value of g on the surface of the earth? Take R as the radius of the earth.

(a) 0.5 *R* (b) 0.75 *R* (c) 0.8 *R* (d) *R*

6. The acceleration due to gravity on earth depends upon

(a) size and shape of body (b) mass of body

(c) volume of body (d) mass of earth

7. Two bodies *P* and *Q* having masses '*m*' and '3*m*' respectively are kept at a distance *x* apart. A small particle is to be kept so that net gravitational force on it due to *P* and *Q* is zero. Its distance from mass *P* will be

$(a)\ \frac{x}{1+\sqrt{4}}$ $(b)\ \frac{x}{1-\sqrt{3}}$ $(c)\ \frac{x}{1+\sqrt{2}}$ $(d)\ \frac{x}{1+\sqrt{3}}$

8. At what height from the earth will the value of acceleration due to gravity be 64% of the value at the surface? (Radius of earth is 6400 km)

(a) 1020 km (b) 1100 km

(c) 1450 km (d) 1600 km

9. A body weighs 18 kg wt on earth's surface. How much will it weigh on a planet whose mass is 1/9 th mass of earth and radius is 1/2 radius of earth?

(a) 18 kg wt. (b) 4 kg wt

(c) 8 kg wt (d) 6 kg wt

10. The period of revolution of a certain planet in an orbit of radius *R* is *T*. Its period of revolution in an orbit of radius 4R will be

(a) 8T (b) 4T (c) $2\sqrt{2}$ T (d) 2T

11. The value of acceleration due to gravity at a certain place is 'g'. Assume that earth shrinks suddenly uniformly to half of its present size without any change in its mass. The value of acceleration due to gravity at same position (assuming that distance of point from centre of earth remains same) will be

(a) $\frac{g}{4}$ (b) $\frac{g}{2}$ (c) g (d) 2g

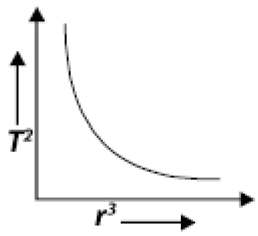
12. Reading of a spring balance when an object hangs from it is 2 kg weight. Spring balance is released. Its reading will be

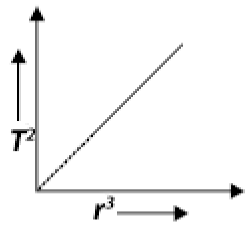
(a) 4 kg wt (b) > 5 kg wt (c) < 5 kg wt (d) zero

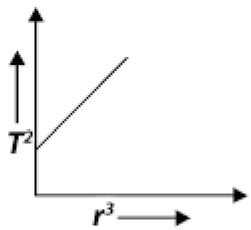
13. A body is moving up. Acceleration due to gravity will be

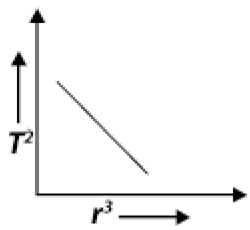
(a) zero (b) sideways (c) upward (d) downward

14. Choose the correct graph between square of time period and cube of distance of the planet from the sun

(a)


(b)


(c)


(d)


15. The time of ascent when measured from the point of projection of a body projected upwards, the

(a) Time of ascent > Time of descent

(b) Time of ascent = Time of descent

(c) Time of ascent < Time of descent

(d) All of these

LEVEL - II

1. Astronomers have discovered a planet orbiting a nearby star. It is estimated that the mass of this planet is 16 times that of our earth and its density is one-fourth of the earth's density. Assume that planets are spheres of uniform density. If your weight on the earth is 1000 N, then your weight on this planet would be

(a) 5000 N (b) 1000 N (c) 2000 N (d) 4000 N

2. Let A and B be the points respectively above and below the earth's surface each at a distance equal to half the radius of the earth. If the acceleration due to gravity at these points be g_A and g_B, then g_A : g_B is:

(a) 1: 1 (b) 9: 8 (c) 8: 9 (d) zero

3. Read the given statements and choose the correct option.

Statement 1: Acceleration due to gravity at a place on earth remains constant for all objects.

Statement 2: Acceleration due to gravity doesn't depend on the mass of the object, but only on mass of earth.

(a) Both statement 1 and statement 2 are true and statement 2 is correct explanation of statement 1

(b) Both statement 1 and statement 2 are true but statement 2 is not correct explanation of statement 1.

(c) Statement 1 is true but statement 2 is false

(d) Statement 1 is false but statement 2 is true.

4. According to the chart, on which planet would a ball fall the fastest?

Planet	Earth	Jupiter	Neptune	Saturn
Acceleration Due to gravity	10	26	14	12

(a) Jupiter (b) Saturn (c) Neptune (d) Earth

5. The gravitational force between two masses kept in air at a certain distance is x N. The same two masses are now kept in water and the distance between them is same. The gravitational force between these masses in water is y N. Then

(a) x > y (b) x < y (c) x = y (d) Cannot say

6. At a certain place, value of g is 1% less than its value on the surface of Earth. If the radius of Earth is given to be 6400 km, then the place is:
   (a) 64 km below the surface of the Earth
   (b) 64 km above the surface of the Earth
   (c) 30 km above the surface of the Earth
   (d) 32 km below the surface of the Earth
7. If suddenly the gravitational force of attraction between the Earth and a satellite revolving around
   (a) Fall onto the Earth
   (b) Move in a direction tangential to its original orbit
   (c) Escape horizontally
   (d) None of these
8. If the sun and the planets carried huge amount of opposite charges,
   (a) all three of Kepler’s laws would still be valid
   (b) only the third law will be valid
   (c) the second law will not change
   (d) the first law will still be valid
9. If the mass of sun were ten times smaller and gravitational constant G were ten times larger in magnitude.
   (a) walking on ground would become more difficult
   (b) the acceleration due to gravity on earth will not change
   (c) rain drops will fall much faster
   (d) air planes will have to travel much faster
10. Satellites orbiting the earth have finite life and sometimes debris of satellites fall to the earth. This is because
   (a) the solar cells and batteries in satellites run out
   (b) the laws of gravitation predict a trajectory spiralling inward
   (c) of vicous forces causing the speed of satellite and hence height to gradually decrease
   (d) of collisions with other satellites.

ADDITIONAL EXERCISE - SECTION - A

1. If the distance between the earth and the moon becomes just half the present value, then the gravitational force become:

   (A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

2. If earth stops rotating, the value of g at the equator

   (A) will increase (B) will decrease

   (C) will remains constant (D) none of these

3. The value of the acceleration due to gravity at the surface of the earth is g. Its value at a depth h below the earth's surface is

   (A) $g\left(1 - \frac{h}{r}\right)$ (B) $g\left(1 + \frac{h}{r}\right)$ (C) $g\left(1 - \frac{h^2}{r}\right)$ (D) zero

4. An artificial satellite is revolving round the earth. A ball is dropped out of the satellite from a sidewall

   (A) it will fall on the earth

   (B) it will move away from the earth

   (C) it will keep revolving round the earth in the same orbit with the same time period as the satellite

   (D) none of these

5. The velocity of the satellite in orbit depends on

   (A) radius of the orbit

   (B) acceleration due to gravity

   (C) product of radius and acceleration due to gravity

   (D) none of these

6. Weightlessness in a space is due to

   (A) inertia (B) zero gravity

   (C) zero acceleration (D) centre of gravity

7. The height at which g will be $\frac{1}{4}$ the of its value at the earth surface is

   (A) h = R (B) h = $\frac{R}{2}$ (C) h = 2R (D) none of these

8. Newton's law of gravitation

   (A) can't be verified but is true (B) can be verified in the laboratory

   (C) is valid only on earth (D) is valid only in the solar system

9. When an object is thrown up the force of gravity is

(A) have the same speed (B) have the same velocity

(C) have the same acceleration (D) none of these

10. In vacuum all freely falling objects

(A) have the same speed (B) have the same velocity

(C) have the same acceleration (D) none of these

11. At which of the following location, the value of g is the largest?

(A) on top of Mount Everest (B) on top of Qutub Minar

(C) at a place on the equator (D) a camp site in Antarctica

12. A stone is dropped from a cliff. Its speed after it has fallen 100mis (in m/s)

(A) 9.8 (B) 44.2 (C) 19.6 (D) 98

13. A ball which is thrown up attains a maximum height of 100m. Its initial speed was

(A) 9.8 m/ s (B) 44.2m/ s (C) 19.6 m/ s (D) zero

14. A stone dropped from the roof of a building takes 4 s to reach the ground. The height of the building is

(A) 19.6 m (B) 938 m (C) 136 m (D) 78.4 m

SECTION - B

1. Gravitational force is

(A) proportional to the product of their masses

(B) proportional to the ratio of their masses

(C) inversely proportional to the square of distance between them

(D) inversely proportional to the distance between them

2. The gravitational force is responsible

(A) for rainfall and snowfall (B) occurrence of tides

(C) for the flow of water in rivers (D) none of these

3. The value of G is independent of

(A) the nature size and mass of interacting bodies

(B) the space where bodies are kept

(C) both are incorrect

(D) None

4. Which is correct

(A) $g = 9.8 \text{ m/ s}^2$ (B) $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

(C) $g = 9.3\ m/s^2$ (D) $G = 6.67 \times 10^{-5}\ N\ m^2\ kg^{-2}$

5. Application of Newton's law of gravitation is
   (A) determination of the masses of planets
   (B) estimating the masses of double stars
   (C) twinkling of stars
   (D) none of these
6. Which of the following is correct?
   (A) g is a vector quantity (B) g is a scalar quantity
   (C) G is a vector quantity (D) G is a scalar quantity
7. Which of the following is correct?
   (A) $v = u + at$ (B) $s = ut + \frac{1}{2}at^2$
   (C) $v^2 = u^2 + 2as$ (D) all are correct
8 Which is correct?
   (A) mass is a scalar quantity (B) mass is a vector quantity
   (C) weight is a vector quantity (D) weight is a scalar quantity
9. Which of the following is correct for mass
   (A) mass remains constant at all places
   (B) mass is a vector quantity
   (C) mass is a scalar quantity
   (D) mass is measured in kg
10. Which of the following is correct for weight?
   (A) weight is a scalar quantity (B) weight is a vector quantity
   (C) for weight W=mg (D) for weight W=m/g
11. Read the given statements and choose the correct option.
   Statement 1: Acceleration due to gravity at a place on earth remains constant for all objects
   Statement 2: Acceleration due to gravity doesn't depend on the mass of the object, but only on mass of earth.
   (A)Both statement 1 and statement 2 are true and statement 2 is correct explanation of statement 1.
   (B) Both statement 1 and statement 2 are true, but statement 2 is not correct explanation of statement 1

(C) Statement 1 is true but statement 2 is false

(D) Statement 1 is false but statement 2 is true

12. According to the chart, on which planet would a ball fall the fastest?

Planet	Earth	Jupiter	Neptune	Saturn
Acceleration Due to Gravity	10	26	14	12

(A) Jupiter (B) Saturn (C) Neptune (D) Earth

13. Two rubber balls of the same size are both dropped on the Earth and on the Moon. One ball is solid, and one is hollow. Approximate gravitational field strength on the Earth is 10 N/kg and on the moon is 1.7N/kg. Which ball has the greatest force acting on it?

Type of ball	**Where dropped**
(A)Hollow	On the Earth
(B)Hollow	On the Moon
(C) Solid	On the Earth
(D)Solid	On the Moon

14. A person sitting on a chair in a satellite feels weight less. It is because:

(A) The normal force by the chair on the person balances the earth's attraction

(B)The normal force is zero

(C) The person in the satellite is not accelerated

(D) Cannot say

15. An object of mass 10 kg is at a point Aon a table. It is moved to a point B by a distance 5m. If the line joining A and B is horizontal, then what is the work done on the object by the gravitational force?

(A) 50 J (B) 100 J (C) 60 J (D) Zero

16. An artificial satellite is moving in a circular orbit of radius 42250 km. Find its speed if it takes 24 hours to revolve around the earth.

(A) 3.07 km s^{-1} (B) 5.67 km s^{-1} (C) 6.14 km s^{-1} (D) 1.57 km s^{-1}

17. Read the given statement and mark the correct option:

Statement 1: If an earth satellite moves to a lower orbit, the speed of satellite increases.

Statement 2: The speed of satellite is a constant quantity for all orbits of earth.

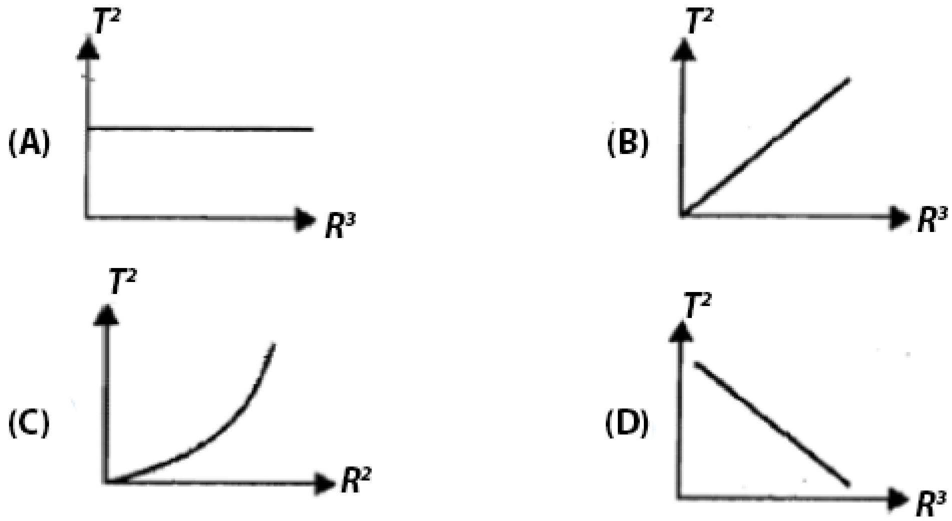
(A) Both statements 1 and 2 are true and statement 2 is the correct explanation of statement 1

(B) Both statements 1 and 2 are true but statement 2 is not the correct explanation of statement 1

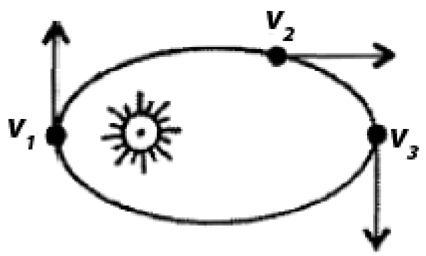
(C) Statement 1 is true but statement 2 is false

(D) Both statement 1 and 2 are false

18. Which of the following graphs between the square of the time period and cube of the distance of planet from the sun is correct?



19. The velocity of a planet revolving around the sun at three different times of a year is shown in the figure. Which among the following alternatives is correct?



(A) $v_2 = \frac{v_1+v_3}{2}$ (B) $v_1 = \frac{v_2+v_3}{2}$

(C) $v_1 > v_2 > v_3$ (D) $v_1 < v_2 < v_3$

20. The gravitational force between two masses kept in air at a certain distance is xN. The same two masses are now kept in water and the distance between the mare same. The gravitational force between these masses in water is yN. Then

(A) x > y (B) x < y (C) x = y (D) Can't say

21 At a certain place, value of g is 1% less than its value on the surface of Earth. If

the radius of Earth is given to be 6400 km, then the place is ________.

(A) 64 km below the surface of the Earth

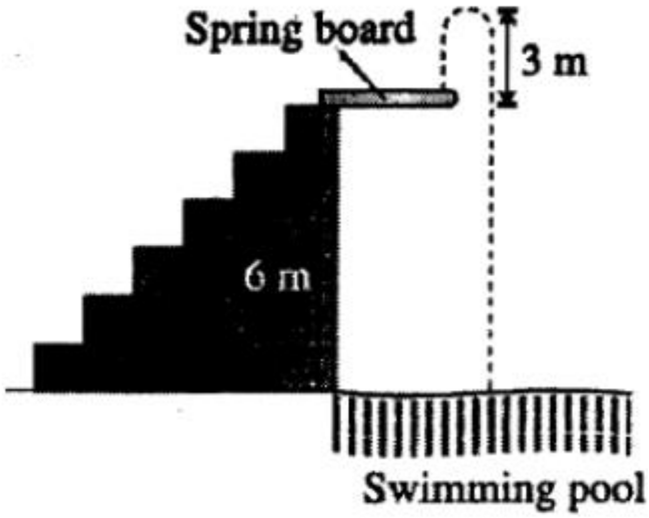
(B) 64 km above the surface of the Earth

(C) 30 km above the surface of the Earth

(D) 32 km below the surface of the Earth

22. If suddenly the gravitational force of attraction between the Earth and a satellite revolving around it becomes zero, then the satellite will ___________.
(A) Fall onto the Earth
(B) Move in a direction tangential to its original orbit
(C) Escape horizontally
(D) None of these

23. A man of mass 55 kg climbs up a flight of steps to reach the spring board. The spring board is 6m above the water surface in a swimming pool as shown in the given figure. He jumps up into air, 3m above the spring board, before falling into water in the swimming pool. If the average resisting force exerted by water on the man is 1500N, then the maximum depth of the man in water will be:



(A) 2.1m (B) 3.3m (C) 4.2 m (D) 5.6m

24. The acceleration due to gravity of the Earth and the Moon are 10 m s^{-2} and 1.7 m s^{-2} respectively. When an astronaut jumps on the Moon, then______________.
(A) He will take a longer time to reach the top and a longer time to come down as compared to jumping on the Earth
(B) He will take a longer time to reach the top and a shorter time to come down as compared to jumping on the Earth
(C) He will take a shorter time to reach the top and a longer time to come down as compared to jumping on the Earth
(D) He will take a shorter time to reach the top and a shorter time to come down as compared to jumping on the Earth

CHAPTER

3

Atoms and Molecules

Matter is made up of small particles which may be atoms or molecules. Different kinds of matter contain different types of atoms or molecules which have different properties, therefore, different kinds of matter have different properties.

An Atom

An Atom is the smallest particle of an element that takes part in chemical reactions. It may or may not have independent existence.

Laws of Chemical Combination

Whenever substances react, they do so according to certain laws. These laws formed the basis of Dalton's atomic theory of matter

1. **Laws of conservation of mass:** This law was stated by French chemist Antoine Laurent Lavoisier. This law states that:

   During any physical or chemical change, the total mass of the products remains equal to the total mass ofthe reactants.

   Lavoisier showed that when mercuric oxide was heated it produced free mercury and oxygen. The sumof masses of mercury and oxygen was found to be equal to the mass of mercuric oxide.

   Mercuric Oxide ⟶ Mercury + Oxygen

   100 g 92.6 g 7.4 g

2. **Law of constant composition or definite proportions:** This law was given by Joseph Proust and it states that :
   "A chemical compound always contains same elements combined together in same proportion by mass".
   This means in a compound, the elements are present in a fixed ratio by mass. e.g., pure water obtained from different sources such as river, well, sea etc. always contains hydrogen and oxygen combined together in the ratio 1 : 8 by mass.
3. **Law of multiple proportions:** This law was discovered by John Dalton and it states that:
   "When 2 elements combine with each other to form two or more than two compounds, the masses of one of the elements which combines with fixed mass of the other, bear a simple whole no. ratio to one another".
   e.g., carbon and oxygen combine with each other to form CO and CO_2.
   In CO, 12 parts by mass of C combines with 16 parts by mass of O.

In CO_2, 12 parts by mass of C combines with 32 parts by mass of O.

Dalton's Atomic Theory

John Dalton put forward a theory in 1808 about the nature of matter. This theory is based on certain postulates called postulates of Dalton's Atomic Theory.

Postulates of Dalton's Atomic theory

The main postulates of Dalton's atomic theory are as follows:

1. All matter, whether an element, a compound or a mixture is made up of extremely small particle called atoms.
2. Atoms of the same element are identical in all respects, i.e., size, shape, mass and properties.
3. Atoms of different elements have different sizes and masses and also possess different properties.
4. Atoms of the same or different elements combine together to form molecules or compounds.
5. When atoms of different elements combine together to form compounds, they do so in a simple whole number ratio such as 1 : 1, 2 : 1, 2 : 3 etc.
6. The number and kind of atoms in a given compound is always fixed.
7. An atom can neither be created nor destroyed, i.e., atom is indestructible.

Symbol

It was Berzelius (1811) who first introduced the use of modern symbol in chemistry.

A symbol is defined as an abbreviation or short-hand sign for the full name of an element. In other words, the

Short-hand representation of an atom of an element is known as symbol.

There are several methods to symbolise an element.

i) By using the initial letter of the English name of an element in capital. e.g., Hydrogen = H, Oxygen = O, Boron = B, Nitrogen = N, etc.

ii) When the names of two or more elements begin with the same letter they are symbolised by using the

First two letters or by using the initial and another which is prominently heard in pronouncing the word.

The first letter is always in capital, e.g., Calcium = Ca, Chromium = Cr, Bromine = Br, Barium = Ba, Argon = Ar, Arsenic = As, etc.

Atomicity

The number of atoms in a molecule of an element is called its atomicity.

i) Thus, the atomicity of inert gases (He, Ne, Ar, Kr, etc.) and metals like Li, Na, K, Cu, etc. is 1.

ii) The molecules of some elements contain 2 atoms, e.g., (H_2, O_2, N_2, Cl_2, etc.). These are diatomic.

iii) The ozone molecule contains 3 atoms and so its atomicity is 3.

iv) Phosphorus molecules (P_4) is tetra atomic.

v) Sulphur molecule (S_8) is polyatomic.

Atomic Mass

Concept of Atomic weight or atomic mass: The weight or the mass of an atom is so negligible that it is not possible to weigh an atom in a chemical balance. Indirect calculations show that the actual weight of one atom of the lightest element hydrogen is 1.67×10^{-24} g. Hence is the necessity of determining the weights of atoms with reference to the weight of an atom of a standard element, or in other words atomic weight or atomic mass refers to the relative atomic weight. It is this relative weight which is known as atomic weight of an element.

Thus, the atomic weight of an element is a number which indicates how many times heavier is the atom of the element, in comparison to an atom of another element taken as a standard.

or,

The atomic weight of an element = $\frac{\text{weight of 1 atom of the element}}{\text{weight of 1 atom of a standard element}}$

Atomic mass unit (amu) may be defined as $\frac{1}{12}$ *th of the mass of an atom of carbon - 12 isotope on the atomic scale, i.e., 1 amu =*$\frac{1}{12}$ *th of mass of C-12 isotope.*

Carbon scale

The atomic weight of an element indicates how many times an atom of the element is heavier than 1/12th part of the weight of an atom of carbon 12 isotope.

Atomic weight = $\frac{\text{weight of an atom of the element}}{\frac{1}{12}\text{th weight of an atom of C}-12\text{ isotope}}$

According to this scale the atomic weight of hydrogen is 1.00796.

Atomic weight of an element does not signify the actual weight of an atom of the element. Since the atomic weight of an element is a ratio, it has no unit - it is only a number.

Gram atomic weight or gram atom

When the atomic weight of an element is expressed in grams, it is known as gram atomic weight or gram atom.

Since gram atom indicates a quantity in grams which is equal to the atomic weight of an element, it has a unit.

Scientifically, gram atom of an element is defined as the weight in grams that contains the same number of atoms as 12 g of carbon-12 isotope.

1 gram atom of oxygen refers to 16 g of oxygen and 1 gram atom of carbon = 12 g of carbon. 0.5 gram atom of carbon = 0.5 × 12 = 6 g of carbon.

Molecule A molecule is a group of two or more atoms which are held together strongly by some kind of attractive forces. Such an attractive force holding the atoms together is called a chemical bond.

A molecule is the smallest particle of an element or a compound which can exist freely under ordinary condition and shows all the properties of that substance.

Molecular Weight

The molecular weight is always the sum of the atomic weights of the atoms in the molecule. Consequently by knowing the composition of the molecule, the molecular weight can be easily calculated.

Element or compound	Molecular formula	Molecular weight
Hydrogen	H_2 – two atoms of hydrogen	2 × 1 =2
Oxygen	O_2 – two atoms of oxygen	2 × 16 = 32
Carbon di-oxide	CO_2 – 1 atom of carbon and 2 atoms of oxygen	12 × 1 + 2 × 16 = 44
Nitric acid	HNO_3 – 1 atom of hydrogen, 1 atom of nitrogen and 3 atoms of oxygen	1 × 1 + 14 × 1 + 3 × 16 =63
Sulphuric acid	H_2SO_4 – 2 atoms of hydrogen, 1 atom of sulphur and 4 atoms of oxygen	2 × 1 + 1 × 32 + 4 × 16=98
Glucose	$C_6H_{12}O_6$ – 6 atoms of carbon,12 atoms of hydrogen and 6atoms of oxygen	6 × 12 + 1 × 12 + 6 × 16 =180

Gram molecule or Gram molecular weight or Gram mole

The gram-molecular weight of a substance (element or compound) is simply the molecular weight of the substance expressed in grams. It is more commonly known as gram molecule or gram mole or mole.

Ions

An atom or a group of atoms which carries positive or negative charge is called an ion. The ion carrying positive charge is called a cation and the ion carrying negative charge is called an anion. The ions consisting of only single atoms are called monoatomic ions whereas an ion consisting of a group of atoms is called polyatomic ions. The compounds consisting of cations and anions are called ionic compounds.

e.g Ca^{2+} ,O^{2-} ,NH^{+},SO_4^{2-}, NO_3^{-}

Valency

The number of atoms of some element which can combine with 1 atom of other element is known as the valency of other element or in general combining capacity of other elements.

Hence, the valency denotes the combining capacity of an atom of an element with the atoms of other elements which is measured by the number of hydrogen atoms, which can combine with or are displaced by one atom of the said element.

Thus, 1 atom of bromine, 1 atom of oxygen, 1 atom of nitrogen and 1 atom of carbon combines with 1, 2, 3 and 4 atoms of hydrogen to form HBr, H_2O, NH_3 and CH_4 , respectively.

Compounds of hydrogen	**Formula**	**No. of H-atoms combined with one atom of the element**	**Valency of the element**
Hydrogen bromide	HBr	1	Br = 1
Water	H_2O	2	O = 2
Ammonia	NH_3	3	N = 3
Methane	CH_4	4	C = 4

There are some species which are positively or negatively charged and some are neutral. These species maybe atom or group of atoms, e.g., K^+, $Zn^{+2,}$ NH_4^+, etc.

Monovalent	**Divalent**	**Trivalent**	**Tetravalent**
Nitrate (NO_3^-), Cyanide (CN^-)	Sulphide(S^{2-})	Phosphide (P^{3-})	Ferrocyanide [$Fe(CN_6)^{4-}$]
Nitrite NO_2^- , Fluoride (F^-)	Sulphite (SO_3^{2-})	Phosphate (PO_4^{3-})	
Hydroxyl (OH^-), Chloride (Cl^-)	Sulphite (SO_4^{2-})	Nitride (N^{3-})	
Bromide (Br^-) , Iodide (I^-)	Carbonate(CO_3^{2-})	Ortho borate (BO_3^{3-})	
Chlorate (ClO_3^-),Cyanate (CNO^-)	Silicate (SiO_3^{2-})	Ferri cyanide $[Fe(CN)_6]^{3-}$	
Bicarbonate (HCO_3^-)	Zincate (ZnO_2^{2-})		
Bisulphate (HSO_4^-)	Chromate (CrO_4^{2-})		
Bisulphite (HSO_3^-)	Dichromate $Cr_2O_7^{2-}$		
Aluminate (AlO_2^-)			
Metaborate (BO_2^-)	Thiosulphate ($S_2O_3^{2-}$)		
Permanganate(MnO_4^-)			

Variable Valency

The valency of an element is not constant. There are some elements which have more than one valency.

Thus, valency may not necessarily be constant as in transition elements,

(Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu etc.)

Iron shows a valency of +2 (Fe^{2+}) and +3 (Fe^{3+}).

Formula

Formula is the symbolic representation of the molecule of a substance. In other words, the short representation of one molecule of an element or compound is known as formula.

It represents the relative weights of the element present in the molecular weight of the compound. Thus the formula of sulphuric acid is H_2SO_4.

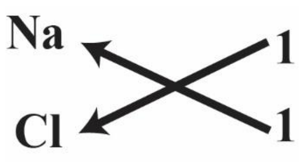
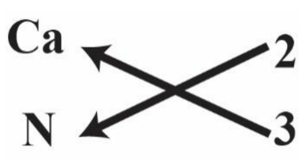
Formula of Compounds

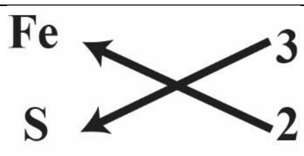
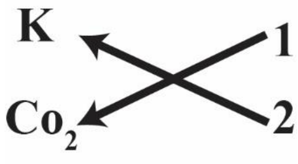
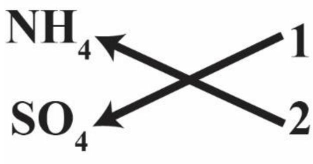
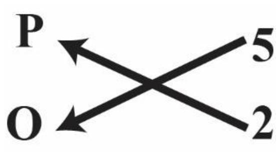
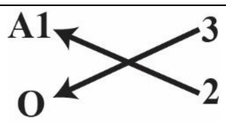
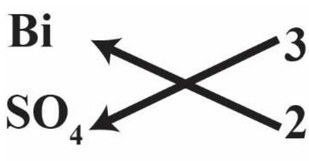
Classification of elements and radicals in terms of valency has made the study of chemistry much easier. Moreover, the formula of a chemical compound can be found out with the knowledge of valency. An ionic compound consists of

3 two parts, a positive part and a negative part. The compound is, as a whole, electrically neutral.

Hence, the total positive charge contained in the metallic part or basic radical must be equal to the negative charge contained in the non-metallic part or acid radical.

The above procedure will be more clarified in the following table:

Compound	Valencies of the atom and radicals in the compound	Formula of the compound
Sodium chloride		NaCl
Calcium nitride		Ca_3N_2
Ferric sulphide		Fe_2S_3

		
Potassium carbonate		K_2CO_3
Ammonium sulphate		$(NH_4)_2SO_4$
Phosphorous pentoxide		P_2O_5
Sodium metaborate		$NaBO_2$
Aluminium oxide		Al_2O_3
Bismuth sulphate		$Bi_2(SO_4)_3$

Avogadro's Number

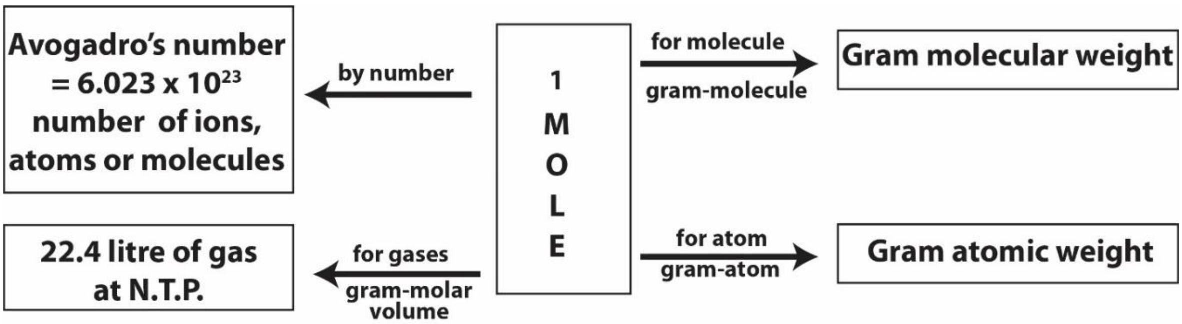
According to Avogadro's hypothesis under the same conditions of temperature and pressure equal volumes of all gases contain the same number of molecules.

The number of particles present in one mole of a substance (element or compound) is known as Avogadro's number. It is equal to 6.023×10^{23}. Avogadro number is represented as 'N'.

Mole Concept

1 gram mole or gram molecule represents the molecular weight of the substance expressed in grams. Thus 1 gram molecule of water is 18 g of water. It is evident from Avogadro's hypothesis that one mole or 1 gram mole of any gas or vapour contains the same number of molecules. The importance of using the quantity known as mole is that it contains the same number of molecules irrespective of the gas or vapour. This number is called Avogadro's number which is 6.023×10^{23}. Again the gram molecular volume of all gases or vapour at N.T.P. is 22.4 litre. Therefore it can be fairly argued that the number of molecules present in 22.4 litre of any gas or vapour at N.T.P. is 6.023×10^{23}.

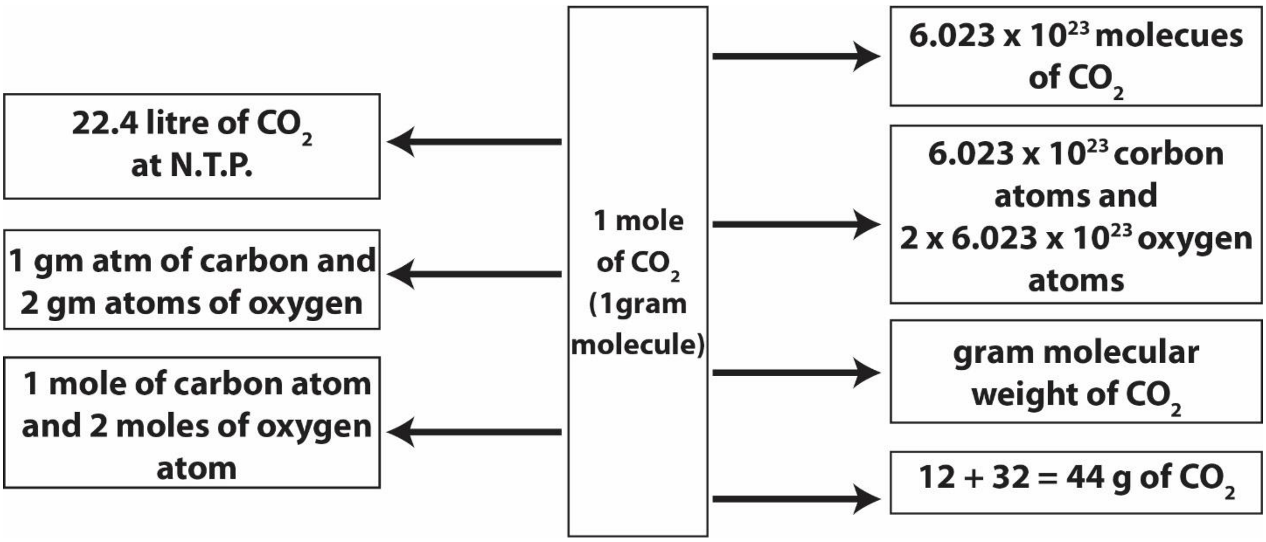
Mole is a unit of counting the number of atoms or molecules present per gram atom or gram molecules of the substance. It also represents the weight of the substance (element or compound) expressed in gram which contains Avogadro number or 6.023 × 1023 particles (atoms or molecules) of that substance.



No. of moles (n) = $\frac{\text{m (given mass)}}{\text{M (atomic / molecular mass)}}$

No. of molecules (N) = $n \times N_A \;\; or \frac{m}{M} \times N_A$

For example: 1 mole of CO_2 are represented as follows.



Molarity: Molarity is the number of moles of solute present per litre solution. Unit of molarity is mol/L it represented as M.

$$M = \frac{\text{no.of moles of solute}}{\text{Volume of solution in L}}$$

Molality: Molality is the number of moles of solute present per kilogram of solvent. Unit of molality is mol/kg it is represented as m.

$$\text{Molality} = \frac{\text{no.of moles of solute}}{\text{Mass of solvent in kg}}$$

Mole fraction: Mole fraction is the ratio of number of moles of the component to the total number of moles of all components present in the solution. It is unitless.

$$\text{Mole fraction of } A(\chi_A) = \frac{n_A}{n_A+n_B}$$

$$\text{Mole fraction of } B(\chi_B) = \frac{n_B}{n_A+n_B}$$

$$A(\chi_A) + B(\chi_B) = 1$$

Percentage Composition and Molecular Formula

Percentage composition of a compound refers to the amount of various constituent elements present per 100 parts by mass of the substance. It can be calculated from the formula of the compound by using the following relation:

$$\text{\% (mass) of an element} = \frac{(\text{No.of atoms of elements})\times(\text{At.mass of element})\times 100}{\text{Molar mass of substance}}$$

For example:

The % of iron (Fe), Sulphur (S) and Oxygen (O) in ferric sulphate can be calculated as follows:

The formula of the compound is $Fe_2(SO_4)_3$.

The molar mass is = 2 × 56 + 3(32 + 64) = 400 u.

$$\text{\% of iron} = \frac{2\times56}{400} \times 100 = 28\%$$

$$\text{\% of sulphur} = \frac{3\times32}{400} \times 100 = 24\%$$

$$\text{\% of oxygen} = \frac{3\times6}{400} 4 \times 100 = 48\%$$

REVISION EXERCISE - LEVEL - I

1. How many moles of calcium carbonate ($CaCO_3$) are present in 10 of the substance? (Ca = 40 u; C = 12 u; O = 16 u)
2. Calculate the mass of 3.011×10^{24} atoms of carbon.
3. How many moles of oxygen atoms are present in one mole of the following compounds?
   (i) Al_2O_3 (ii) CO_2 (iii) Cl_2O_7 (iv) H_2SO_4 (v) $Al_2(SO_4)_3$
4. Calculate the number of moles in 12g of oxygen gas.
5. Calculate the number of moles present in 14g of carbon dioxide.
6. Find the mass of 5 moles of aluminium atoms?
7. Calculate the molar mass of sulphur.
8. Calculate the mass of 0.2 mole of water molecules.
9. Which has greater number of atoms, 100g of sodium or 100g of iron?
10. How many atoms of oxygen are present in 300 g of $CaCO_3$.
11. The mass of one atom of an element 'A' is 2.65×10^{-23}g. Calculate its atomic mass and name the element.
12. Calculate the number of moles of magnesium in 0.478g of magnesium
13. Find the number of aluminium ions present in 0.051g of aluminium oxide (Al_2O_3).
14. What is the molarity of 5.30 g of Na_2CO_3 dissolved in 400.0 mL solution? (Atomic mass of Na = 23, C = 12, O = 16).
15. How many moles of Na_2CO_3 are there in 10.0 L of 2.00 M solution?

LEVEL - II

1. Explain the formation of (i) sodium ion, and (ii) chloride ion, from their respective atoms giving thenumber of protons and number of electrons in each one of them. What is the reason for positive charge on a sodium ion and a negative charge on a chloride ion?
2. In which of the following cases the number of hydrogen atoms is more? Two moles of HCl or onemole of NH_3.
3. Find the number of oxygen atoms in 88 gm CO_2?
4. Calculate the number of water molecules contained in a drop of water weighing 0.6 gm.
5. Find the number of moles present in 24.0088×10^{23} particles of carbon dioxide.
6. 0.100 mole of NaCl is dissolved into 100.0 grams of pure H_2O. What is the mole fraction of NaCl?
7. A solution is prepared by mixing 25.0 g of water (H_2O) and 25.0 g of ethanol (C_2H_5OH). Determinethe mole fractions of each substances.
8. A solution contain 10.0 g pentane (C_5H_{12}), 10.0g hexane (C_6H_{14}) and 10.0g benzene (C_6H_6). Whatis the mole fraction of hexane
9. The molality of an aqueous solution of sugar ($C_{12}H_{22}O_{11}$) is 1.62 m. Calculate the mole fractions ofsugar and water
10. How many grams of water must be used to dissolve 100.0 grams of sucrose ($C_{12}H_{22}O_{11}$) to preparea 0.020 mole fraction of sucrose in the solution? (C =12, H = 1, O = 16)
11. Sea water contains roughly 28.0g of NaCl per litre. What is the molarity of sodium chloride in seawater? (Na = 23, Cl = 35).

12. What is the molarity of 245.0 g of H_2SO_4 dissolved in 1.000 litre solution? (H = 1, S = 32, O = 16)
13. What volume (in mL) of 18.0 M H_2SO_4 is needed to contain 2.45 g
14. What volume (in mL) of 18.0 M H_2SO_4 is needed to contain 2.45 g H_2SO_4?
15. A solution of H_2SO_4 with a molal concentration of 0.010 m has a density of 1.354. Calculate itsmolarity

MULTIPLE CHOICE QUESTIONS

1. Magnesium is two times heavier than C-12 atoms, what shall be the mass of Mg atom in terms of atomic mass units.
   (a) 26u (b) 48u (c) 6u (d) 24u
2. Ratio of masses of atoms of elements present in water molecule will be:
   (a) 1:2 (b) 2:1 (c) 1:8 (d) 1:16
3. Chemical formula of the compound using zinc ion and phosphate ion.
   (a) $Zn_2(PO_4)_3$ (b) $ZnPO_4$ (c) $Zn_3(PO_4)_2$ (d) $Zn(PO_4)_2$
4. If one mole of carbon atoms weighs 12 gram what is the mass (in gram) of 1 atom of carbon?
   (a) 3.66×10^{-23}g (b) 6.022×10^{-23} (c)1.66×10^{-23}g (d) 1.992×10^{-23}g
5. Mass of 4 moles of aluminium atoms will be:
   (a) 206g (b) 27g (c) 108g (d) 208g
6. Number of moles in 12g of oxygen gas will be:
   (a) 0.375 mol (b) 0.835 mol (c) 6.022 mol (d) 1.66mol
7. Number of molecules of sulphur (S8) present in 16g of solid sulphur will be:
   (a) 4.88×10^{22} molecules (b) 3.76×10^{22} molecules
   (c) 6.022×10^{22} molecules (d) 1.22×10^{22} molecules
8. An atom is the smallest unit and is indivisible. This theory was proposed by
   (a) Dalton (b) Rutherford (c) Thomson (d) Bohr
9. Sample of carbon dioxide were collected from different environments. While testing all the samples were found to contain carbon and oxygen in the mass ratio of 3:8. This is in agreement with the law of
   (a) Conservation of mass (b) Multiple proportion
   (c) Constant proportion (d) None of these
10. An ionic compound will be formed by the combination of one of the following pairs of elements. This pair of elements is:
    (a) Chlorine and calcium (b) Calcium and sodium
    (c) Sulphur and oxygen (d) Chlorine and chlorine
11. The number of atoms in 4.25 g of NH_3 is approximately
    (a) 1×10^{23} (b) 2×10^{23} (c) 4×10^{23} (d) 6×10^{23}
12. Which of the following contains maximum number of atoms?
    (a) 6.023×10^{21} molecules of CO_2 (b) 22.4 L of CO_2 at STP
    (c) 0.44 g of CO_2 (d) None of these
13. 7.5 g of a gas occupy 5.6 litres of volume at STP. The gas is
    (a) NO (b) N_2O (c) CO (d) CO_2
14. The smallest particle that can exist independently and constitute all substances is:
    (a) Atom (b) Molecule (c) Electron (d) Proton
15. The molarity of water
    (a) 5.55 (b) 55.55 (c) 45.5 (d) 4.55
16. The molecular formula of potassium permanganate is:

(a) KNO_3 K_2MnO_4 $KMnO_4$ KNO_2

17. What mass of NaF must be mixed with 25 mL of water to create a 3.5% by mass solution?
(a) 0.91g (b) 6.8g (c)8.6g (d) 4.8g
18. An 800g solution of Kool-Aid contain 780 g of water. What is the mass per cent of solute in this solution?
(a) 3.5% (b) 1.2% (c) 2.5% (d) 3.8%
19. The mass percent of a solution created by adding 10g of olive oil to 90g of vegetable oil will be:
(a) 20% (b) 10% (b) 10% (d) 40%
20. If a 4000g solution of salt water contains 40g of salt, what is mass percent of solution?
(a) 9.0% (b) 6.0% (c) 8.0% (d) 1.0%
21. Calculate the volume percent of solution formed by combining 25 mililiters of ethanol with enoughwater to make 200 milliliters of solution.
(a) 3.2% (b) 1.0% (c) 8.9% (d) 7.5%
22. The mass of sodium in 11.7 g of sodium chloride is
(a) 2.3 g (b) 1.0% (c) 1.3% (d) 2.1%
23. The mass of sodium in 11.7 g of sodium chloride is
(a) 2.3 g (b) 1.0% (c) 1.3% (d) 2.1%
24. A compound consists of 47.8% zinc and 52.2% chlorine by mass. The empirical formula is $Zn_x\ Cl_y$where *x* and *y* can have the values
(a) 1 and 1 respectively (b) 1 and 2 respectively
(c) 2 and 1 respectively (d) 2 and 3 respectively
25. Number of gram-atoms in 8 g of He are
(a) 2 (b) $1.204 \times 10_{24}$ (c) 1.204×10^{23} None of these
26. What is the total number of atoms present in 25.0 mg of camphor, $C_{10}H_{16}O$?
(a) 9.89×10^{19} (b) 2.67×10^{21} (c) 6.02×10^{20} (d) 2.57×10^{21}
27. Volume at STP of 0.42 g of CO_2 is the same as that of
(a) 0.01 g of hydrogen (b) 0.085 g of NH_3
(c) 320 mg of gaseous SO_2 (d) 3 g atoms of CO_2
28. One mole of CO_2 contains
(a) 6.02×10^{23} atoms of C (b) 6.02×10^{23} atoms of O
(c) 18.1×10^{23} molecules of CO_2 (d) 3 g atoms of CO_2
29. The number of moles of H_2 in 0.224 litres of hydrogen gas at STP (273 K, 1 atm) (assuming ideal gas behaviour) is
(a)1 (b) 0.1 (c) 0.01 (d) 0.001
30. The number of molecules in 16 g of methane are
(a) 3.0×10^{23} (b) 6.02×10^{23} (c) $\frac{16}{6.02}$ x 10^{23} (d) $\frac{16}{3.0}$ x 10^{23}

ADDITIONAL EXERCISE

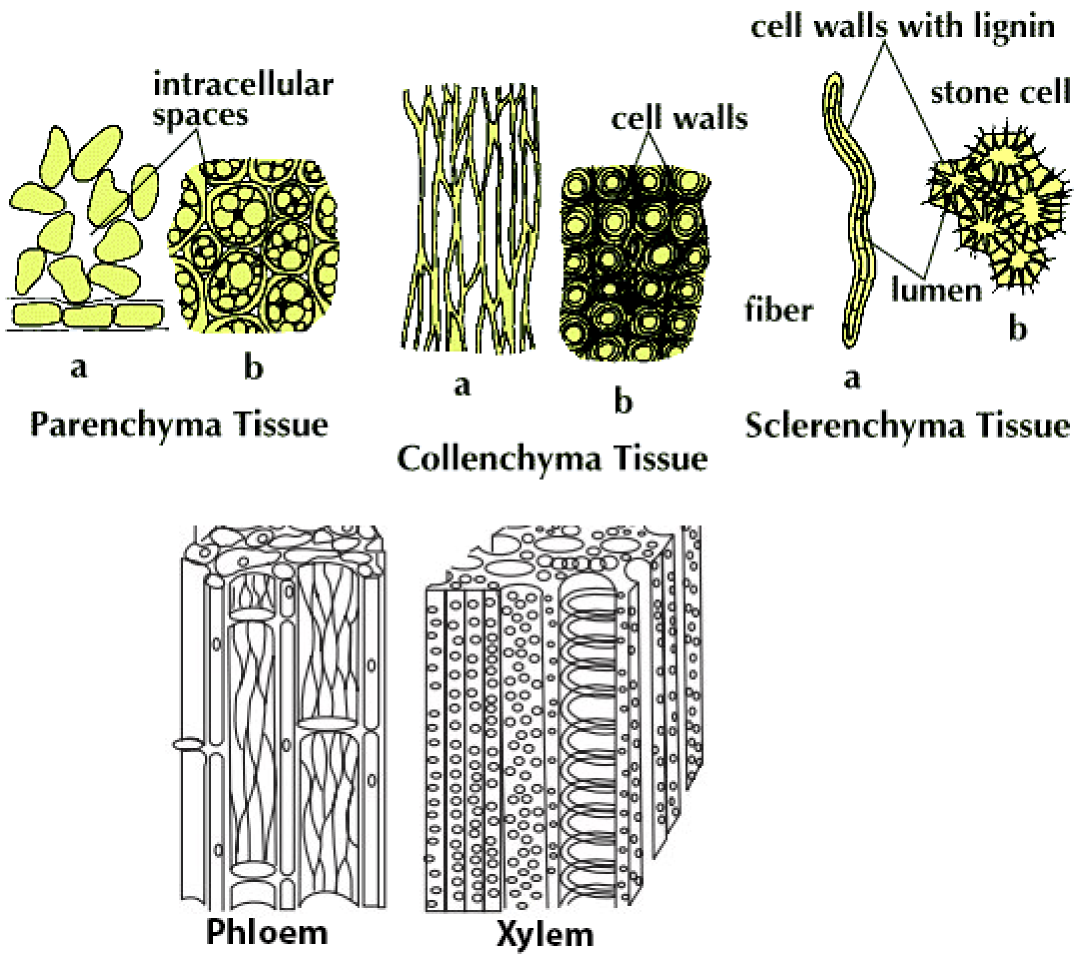
1. Mass of one molecule of Phosphorous (P-31) is
(A) 31 g (B) 2.059×10^{-22} g (C) 5.147×10^{-23} g (D) 124 g
2. What is the formula for the compound formed by Calcium and Nitrogen
(A) CaN (B) Ca_2N (c) Ca_2N_3 (D) Ca_3N_2
3. Which sample at STP has the same number of molecules as 5 liters of NO_2 (g) at STP?
(A) 5 grams of H_2 (g) (B) 5 litre of CH_4 (g)
(C) 5moles of O_2 (g) (D) 5×10^{23}molecules of CO_2(g)
4. Calculate the mass of Lithium that contains same number of atoms as present in 8 g of Magnesium.
(A) 8g (B) 3g (C) 7 g (D) 2.3 g

5. What will be the volume of Cl_2 at STP produced during electrolysis of $MgCl^2$ which produce 6.5 gMg
   (A) 5.099 litre (B) 5.99 litre (C) 12.02 litre (D) 3.099 litre
6. Which is not a molecular formula?
   (A) $C_6H_{12}O_6$ (B) $Ca(NO_3)_2$ (C) $C_2H_4O_2$ (D) N_2O
7. How many years it would take to spend Avogadro's number of Rupee at the rate of 1million rupees in
   (A) 19.098×10^{19} years (B) 19.098 year
   (C) 19.098×10^{9} year (D) None of these
8. Which is heaviest :
   (A) 25 g of Hg (B) 2 mole of H_2O (C) 2 mole of CO_2 (D) 4 g-atom of O
9. The percentage by mole of NO_2 in a mixture of NO_2(g) and NO(g)having average molecular mass 34 is :
   (A) 25% (B) 20% (C) 40% (D) 75%
10. A balanced chemical equation is in accordance with
    (A)Avogadro's law (B) law of constant proportions
    (C) law of conservation of mass (D) law of gaseous volumes
11. Which one of the following pairs of substances illustrates the law of multiple proportions?
    (A) Carbon monoxide and Carbon dioxide
    (B) Sodium chloride and Sodium bromide
    (C) Water and Heavy water
    (D) Magnesium oxide and Magnesium hydroxide.
12. Which is the smallest possible unit of a chemical compound?
    (A) Atom (B) Electron (C) Proton (D) Molecule
13. The hydrogen phosphate of a metal has the formula$MHPO_4$. The formula of its chloride would be
    (A) MCl (B) MCl_2 (C) MCl_3 (D) M_2Cl_3
14. A compound having the empirical formula (C_3H_4O) has a molecular weight of 168. The molecular formula of this compound is -
    (A) $C_9H_{12}O_3$ (B) C_3H_4O (C) $C_6H_8O_2$ (D) $C_9H_{12}O_2$
15. X and Y atoms have 2 and 6 valence electrons in their outermost shells respectively. The compound
    (A) XY_2 (B) Y_2X_6 (C) YX_2 (D) XY
16. The haemoglobin from the red blood corpuscles of most mammals contains approximately 0.33% of
    (A) 2 (B) 3 (C) 4 (D) 5
17. Insulin contains 3.4% sulphur. The minimum molecular weight of insulin is -
    (A) 941.176 u (B) 944 u (C) 945.27 u None
18. Rakesh needs 1.71 g of sugar ($C_{12}H_{22}O_{11}$) to sweeten his tea. What would be the number of
    (A) 3.6×10^{22} (B) 7.2×10^{21} (C) 0.05×10^{23} (D) 6.6×10^{22}
19. The total number of AlF_3 molecule in a sample ofAlF_3 containing 3.01×10^{23} ions of F is
    (A) 9.0×10^{24} (B) 3.0×10^{24} (C) 7.5×10^{23} (D)10^{23}
20. The largest number of molecules among the following is -
    (A) 28 g of CO (B) 46 g of C_2H_5OH
    (C) 36 g of H_2O (D) 54 g of N_2O_5

CHAPTER 2 Tissues

Plant Tissues

Based on the capacity to divide, the plant tissues have been classified into two fundamental types, **meristematic** and **permanent.**



Meristematic Tissues

A meristem or meristematic tissue **(Gk. *meristos* – divided) is a simple tissue composed of a group of** similar **and** immature cells **(*meristematic cells*) which can divide and form new cells.**

Characteristics of Meristematic Cells

- Ability to grow and divide

- Small immature cells
- Isodiametric, rounded, oval or polygonal
- Absence of intercellular spaces
- Walls are thin, elastic and made of cellulose
- Nucleus conspicuous
- Cytoplasm dense
- Vacuoles absent or very small
- Crystals absent
- Endoplasmic reticulum small.
- Pro plastids are present instead of plastids.
- Mitochondria have simple structure.
- Rate of respiration is very high.

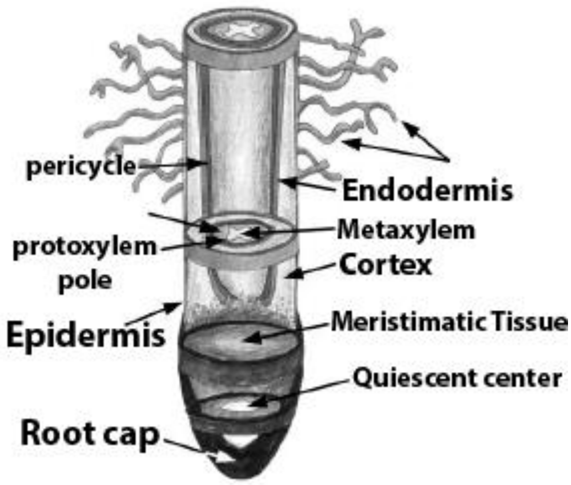


Figure 2.2 section of root showing Meristimatic Tissue

Types of Meristems

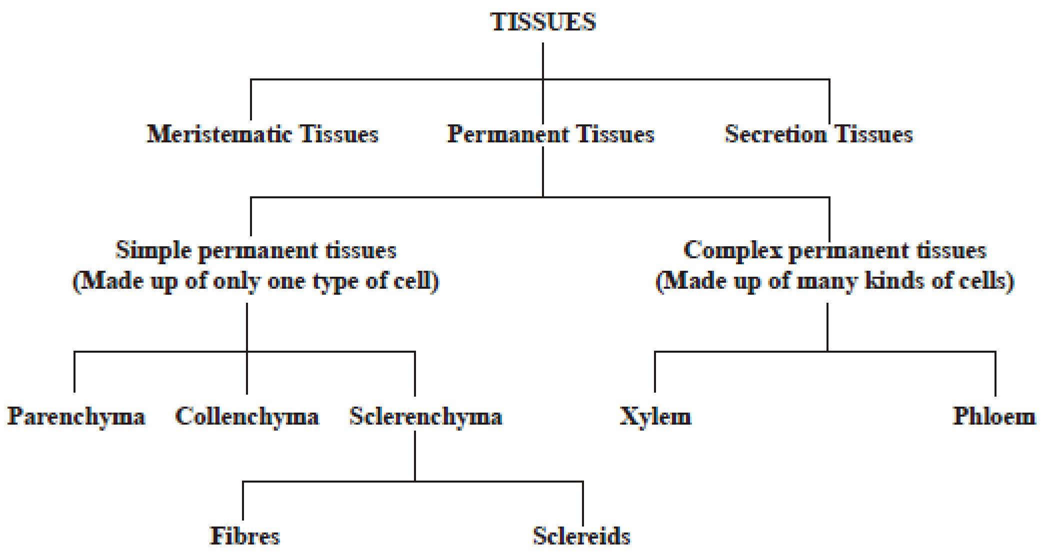
The meristems are classified variously on the basis of origin and position in the plant body, function, plane of division and stage or method of development.

[A] Depending upon the position in the plant body, meristems are of the following three types. (figure)

1. **Apical meristem:** The meristem in located at the growing apices of main and lateral shoots and roots. These cells are responsible for linear growth of an organ.

2. **Intercalary meristem:** This meristem is located in between the regions of permanent tissues. It is believed that some portions of apical meristems are left behind due to tissue differentiation of an organ. These cells remain meristematic and

continue to add new cells to the organ. These meristems are usually responsible for growth in length and present mostly at the base of node (e.g. *Mentha viridis* - Mint), base of inter mode (e.g., stem of many monocots, viz. Wheat and Grasses) or at the base of the leaf (e.g., *Pinus*). The intercalary meristems ultimately disappear and give rise to permanent tissues.



3. **Lateral meristem:** This meristem consists of initials which divide mainly in one plane (periclinal and result increase in the diameter of an organ. Examples, cambium, cork cambium and marginal meristem of some leaves.

Differences between apical and lateral meristems are given in table and positions of leaf primordia, and differentiating vascular tissues are given in figure.

[B] Based on the **origin and method of development**, meristems are of the following three types:

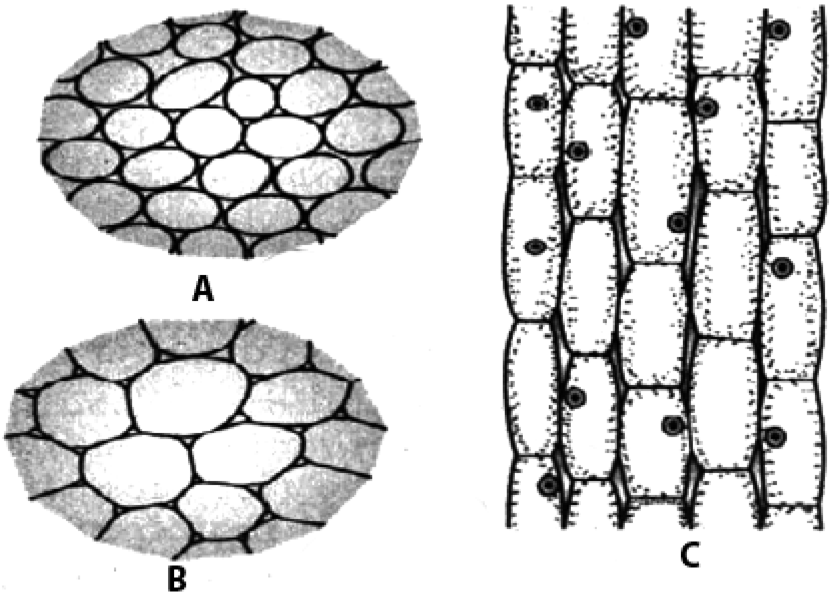
1. **Promeristem.** The promeristem originates from embryo and, therefore, called **primordial** or **embryonic meristem.** It is present in the regions where an organ or a part of plant body initiated. A group of initial cells that lay drawn the foundation of an organ or a plant part is called **promeristem.** This group consists of a limited amount of cells which divide repeatedly to give rise primary meristem. In other words, the promeristem no more exists in the products of promeristem and it is restricted to a limited amount in a particular organ of a particular plant. For example, the promeristem is present in the extreme tip of a young stem consisting of only few cells. These cells divide and give rise a next zone of primary meristem, which by their activity give rise to different tissues of young stem.

Apical Meristem	Lateral Meristem
1. This meristem is located at the growing apices of main and lateral shoots and roots.	1. The meristem is located on the sides of plant organs.
2. They are mostly primary meristems.	2. They are primary as well as secondary in origin.
3. These meristems give rise to growth in length	3. These meristems give rise to growth in girth.
4. Examples of apical meristem are shoot apex and root apex.	4. Examples of lateral meristem are vascular cambium, cork cambium and marginal meristem of some leaves.

2. **Primary meristem:** According to the definition, a primary meristem originates from embryonic state (i.e., from promeristem or embryonic meristem) of the plant which is concerned with the formation of primary permanent tissues of the primary plant body. In early stages of growth, the different organs of a primary body are initiated by the activity of promeristems. They give rise to primary meristem and further development of primary body, differentiation of various tissues and organisation takes place by the activity of primary meristem. For example, a procambium at the apex of shoot give rice — (a) **protoderm**, which produces epidermal tissue system (b) **procambium,** which produce primary vascular elements, and (c) **ground meristem,** which produces cortex and pith. The protoderm, procambium and the ground meristems are the kinds of primary meristem.
3. **Secondary meristem:** They originate as new meristems from the permanent tissues which have already undergone differentiation. They do not have their own promeristem. The secondary meristems arise in plant organs whenever and wherever they are needed. For example, they originate as **vascular cambium** from interfascicular regions in dicots when secondary growth is needed, arise as cork cambium when formation of periderm and heading of wounds is needed. The cambium of root and the accessory cambium rings are best examples of secondary meristem.

Permanent Tissues: They are those tissues, the cells of which have lost the capacity to divide and have attained a permanent shape, size and function due to morphological, biochemical and physiological differentiation. Depending upon their origin, permanent tissues are of two types, **primary** (derived from apical and intercalary meristem) and

secondary (derived from a lateral meristem). On the basis of composition, permanent tissues can be simple, complex and special (*e.g.,* seretory).



Parenchyma cells as seen in (A-B) cross section and (c) longitudinal sections

Simple Permanent Tissues: A simple permanent tissue is that tissue which is made up of similar permanent cells that carry out the same function or similar set of functions. Simple permanent tissues are of three types – parenchyma, collenchyma and sclerenchyma.

Parenchyma

(Gk. *para* - beside, *engchyma* – tissue)

Parenchyma is a simple permanent living tissue which is made up of thin-walled similar isodiametric cells. It is the **most abundant** and common tissue of plants. Typically the cells are isodiametric (all sides equal). They may be oval, rounded or polygonal in outline. The cell wall is made up of **cellulose.** Cells may be closely packed or have small intercellular spaces for exchange of gases. Internally each cell encloses a large central vacuole and a peripheral cytoplasm containing nucleus. The adjacent parenchyma cells are connected with one another by plasmodesmata. They, therefore, form symplasm or living continuum.

Functions

- Storage of food
- Providing turgidity to softer parts

- Checking water loss in the form of epidermis
- Formation of water absorbing epiblema in root
- Photosynthesis in the form of chlorenchyma
- Secretion

Collenchyma

(Gk. *kola* – glue, *enchyma* – tissue)

Collenchyma is a simple permanent tissue of refractile non-lignified living cells which possess pectocellulose thickenings in specific areas of their walls. The cells appear conspicuous under the microscope due to their higher refractive index. The cells are often enlongated, but appear circular, oval or angular in transverse section. Internally, each cell possesses a large central vacuole and a peripheral cytoplasm. Wall possesses uneven longitudinal thickenings in specific areas. Collenchyma is found below the epidermis in the **petiole, leaves** and **stems** of herbaceous dicots**,** especially in the region of ridges (*e.g.,* Gourd).

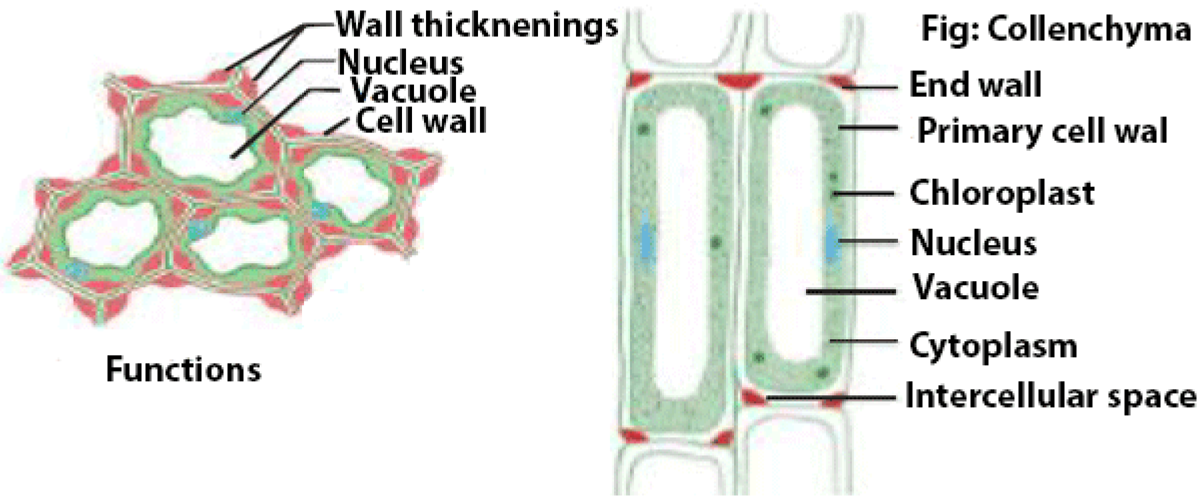


Fig: Collenchyma

Functions

- It provides mechanical strength to young dicot stems, petioles and leaves.
- While providing mechanical strength, collenchyma also provides flexibility to the organs and allows their bending, *e.g.., Cucurbita* stems.
- It prevents tearing of leaves.
- Collenchyma allows growth and elongation of organs.
- Being living, its cells store food.

Sclerenchyma

The cells of sclerenchyma are long, narrow, thick and lignified. These are closely packed without intercellular spaces. They are usually pointed at both ends. Often oblique thin areas are found in their walls. These are called pits. The middle lamella that is the wall between adjacent cells is conspicuous. Sclerenchyma cells are dead cells devoid of protoplasm. Sclerenchymatous cells are found abundantly in plants. The length of sclerenchyma cells varies form 1 mm to 550 mm in different plants. Their main function is to give mechanical support to plants. Sclerenchyma is of two types, **sclerenchyma fibres** and **sclereids**.

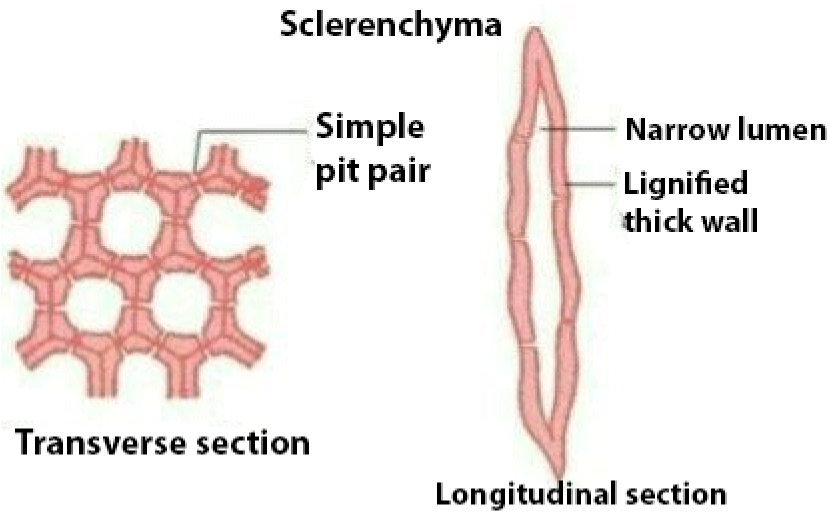


Fig: Sclerenchyma

Protective Tissues

These tissues are usually present in the outermost layer of the plant body such as leaves, flowers, stem and roots. This layer is one cell thick and may be covered with cutin. These tissues protect the inner tissues present in the plant body.

As roots and stem grow older with time, tissues at the periphery become cork cells. Cork cells are dead and do not have any intercellular spaces. The walls of these cells are heavily thickened by the deposition of suberin. They prevent loss of water.

Examples: *Epidermis* and *Periderm* (usually many cell layered, replaces old epidermis as plant grows. It is produced by cork cambium).

Vascular Tissue System

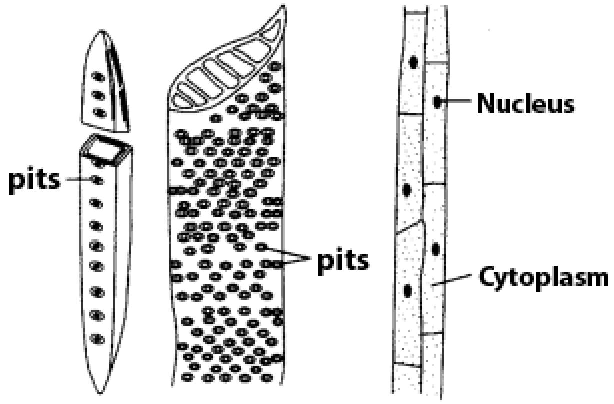
Vascular plants have a specialized tissue called vascular tissue. Vascular tissue carries water and nutrients throughout the plant and helps support the plant.

There are two kinds of vascular tissue. Both kinds of vascular tissue contain specialized conducting cells

Xylem

- Moves water and minerals upward from roots to leaves.

- When water and minerals are absorbed by the roots of a plant, these substances must be transported up to the plant's stem and leaves. Xylem is the tissue that carries water and dissolved substances upward in the plant.
- Xylem consists of tracheids vessels, parenchyma and xylem fibres.
- **Tracheids** are long, thick walled, sclerenchyma form, narrow cells of xylem with thin separations between them. Water move from one tracheid to another through pits, which are thin, porous areas of the cell wall.
- **Vessel elements** are short, sclerenchyma form, wide cells of xylem with no end walls. Vessel elements do not have separations between them, they are arranged end to end liked barrels staked on top of each other. The vessels are wider than tracheids, and more water moves through them.



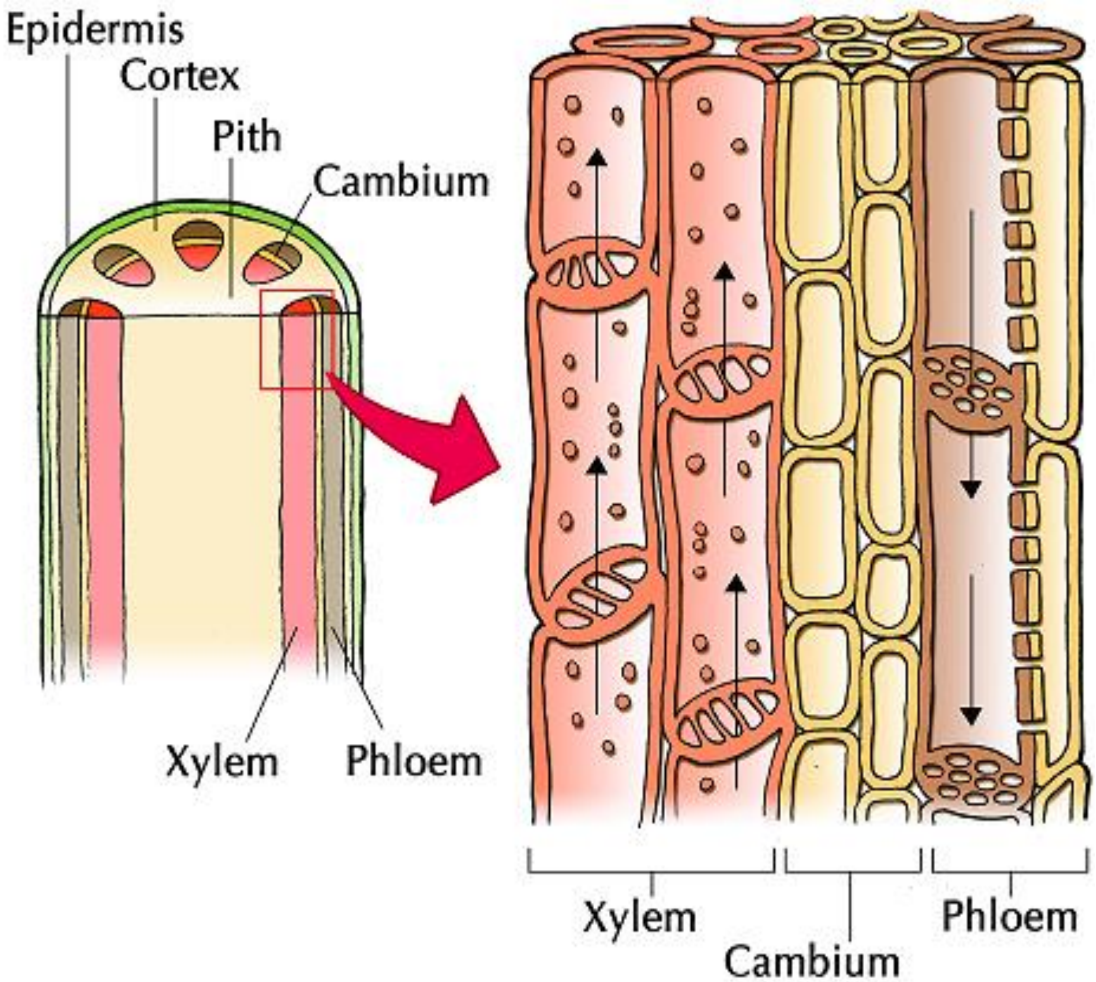
Section of xylem

- The function of xylem parenchyma is storage of food and sideways conduction of water.
- Fibres are mainly supportive in function.
- Angiosperms, or flowering plants, contain tracheids and vessel elements.
- Gymnosperms, or cone bearing seed plants, contain only tracheids.

Phloem

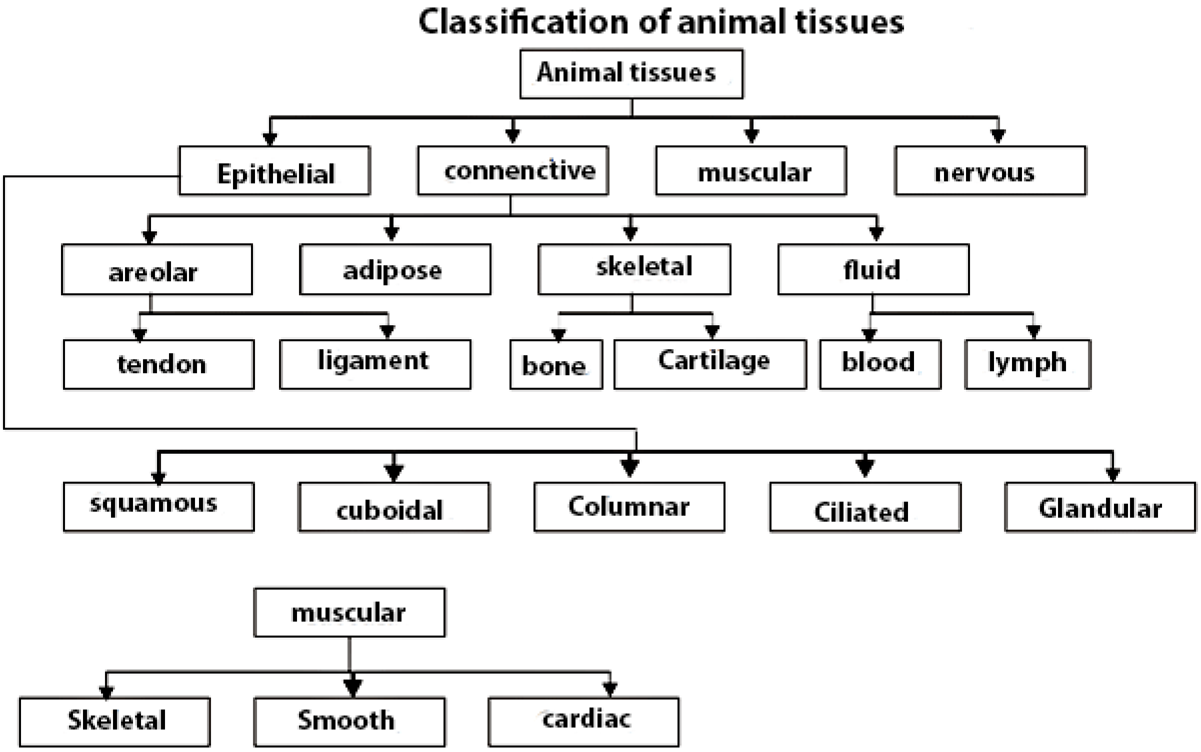
- Moves sugars or saps in both directions throughout the plant originating in the leaves.
- Sugars made in the leaves of a plant by photosynthesis must be transported throughout the plant.
- Phloem tissue conducts sugars upward and downward in a plant.
- The sugars move as sugary sap.

- Phloem consists of sieve tubes, companion cells, phloem fibres and phloem parenchyma.
- **Sieve tube members** are cells of phloem that conduct sap. Sieve tube members are stacked to form long **sieve tubes**. Compounds move from cell to cell through end walls called **sieve plates**.
- **Companion cells** are parenchyma to us cells of phloem that enable (assist) the sieve tube elements to function.



- Phloem parenchyma are living cells found in between sieve tubes. Their function is storage food.
- Phloem fibres are the only non-living components of phloem and their function is providing mechanical supper.
- Each sieve tube element has a companion cell. Companion cells control the movement of substances through the sieve tubes.
- The partnership between these two cells is vital; neither cell can live without the other.

Animal Tissue



Introduction

The term "tissue" was given by a French scientist, Bichat (1771-1802). It is the "group of cells of similar origin, structure and function". The term histology, (study of tissue) was given by Mayer (1819), a German scientist. Marcello Malpighi (1828-1694) was the first to make this kind of study hence he is known as founder of histology.

Histology is the branch of science which deals with the structures and functions of tissues. All multicellular organisms are made up of different types of tissues.

Animal tissues are divided into four major classes on the basis of their functions.

Epithelial tissue: Covers or lines the free surfaces of other tissues. It serves several functions such as protection, secretion, excretion and also forms receptors.

Connective tissue: Supports and connects various tissues.

Muscular tissue: Causes movement of the skeleton and other internal organs. Enables movement by and contraction and relaxation.

Nervous tissue: Transmits messages in the form of impulses, thus coordinating the activities of body.

- The tissues are classified on the basis of their origin and mode of formation. The basic types of tissues and their origin are as follows:
- Epithelial tissue – From all 3 germ layers (ectoderm, mesoderm, and endoderm)

- Connective tissue – only Mesoderm
- Muscular tissue – only Mesoderm
- Nervous tissue – only Ectoderm

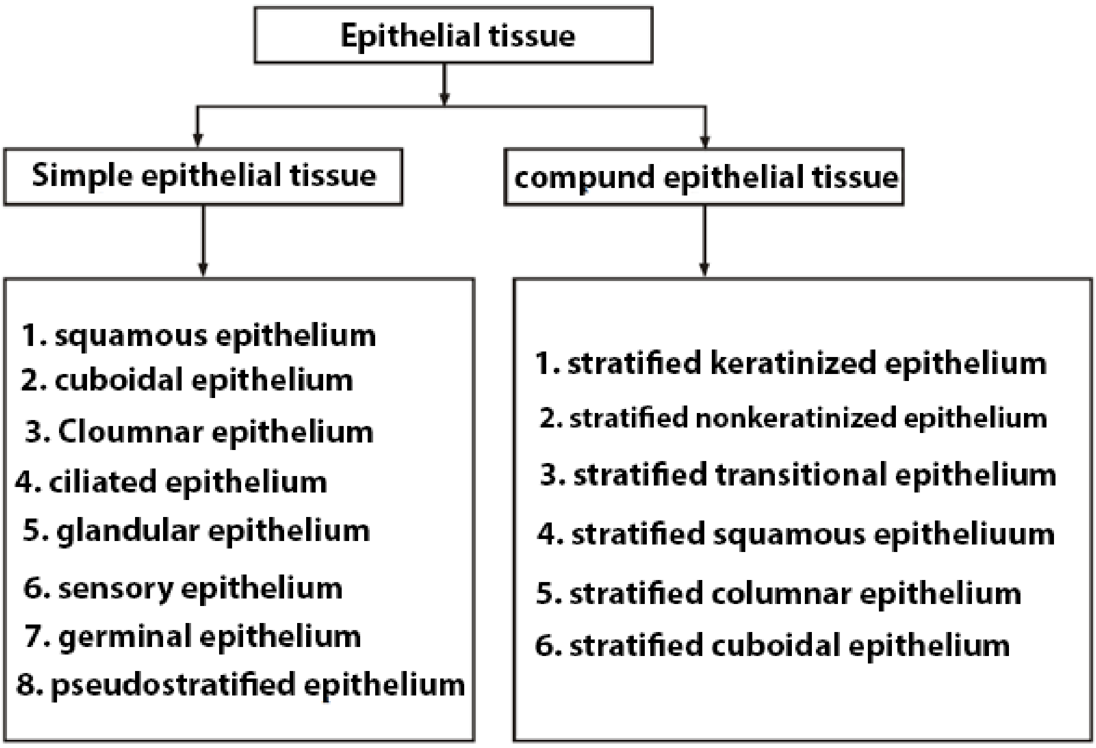
Epithelial Tissue

One of the simplest animal tissues is epithelium. Epithelium is a lining tissue. In its simplest form it consists of a single layer of cells covering the surface of the animals and the organs, cavities and tubes within it. The epithelium lining the inside of heart, blood vessels and lymph vessels is referred to as endothelium.

Structure: Typically, all individual cells of the epithelium are firmly attached with each other. These cells rest on a non-cellular basement membrane.

Functions: Functions are varied, some of which involve protection of underlying tissues, production of motion (ciliated epithelium), absorption of digested material, secretory and also sensory function (olfactory region of nose, taste buds of tongue, retina of eye are all example of sensory functions of epithelium).

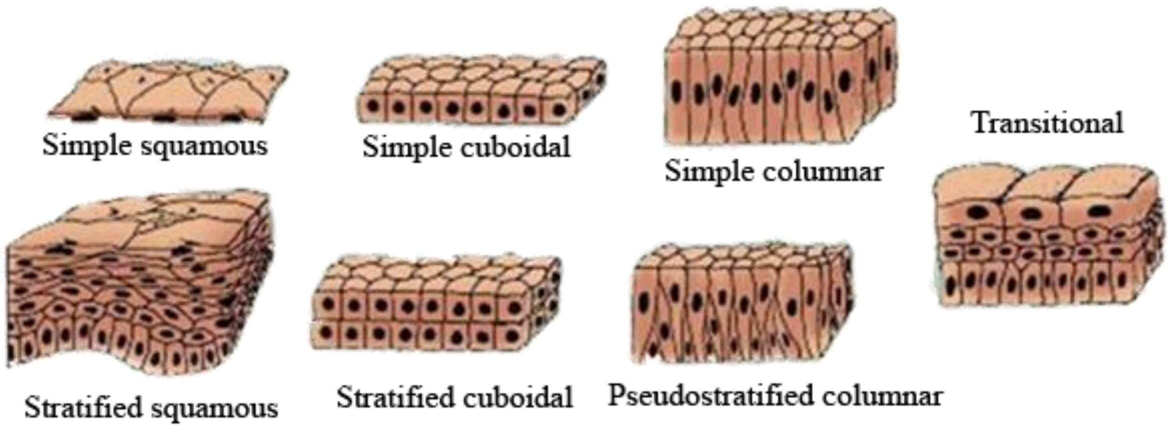
Epithelia are classified on the basis of shape, function and number of layers it is made up of:



(a) Simple squamous epithelium: In such a type of epithelium, the cells are flattened. This gives a sheet like appearance in surface view. It is found in places where the protective covering also needs to be readily permeable to molecules in solution for e.g., the lining of capillaries, alveoli in lungs.

It is also called *as pavement epithelium or tessellated epithelium* as cells arranged like tiles.

Examples: Lining of blood vessel's wall (endothelium), Mesothelium, Bowman's capsule, Loop of Henle, Alveoli of lungs, Terminal bronchiole, etc.



(b) Columnar Epithelium

- Cells are tall, pillar-like with subcentric nucleus which lie in the same line in all cells.
- No intercellular space at apical surface but may be little at basal surface.
- Main functions include absorption, storage, synthesis and secretion.
  Ex. Lining (mucosa) of stomach and small intestine.
- Within the intestine, gives a typical brush border look due to microvilli at a spored surface.

(c) Cuboidal epithelium

Cells are cubical in shape. Normally this epithelium forms the lining of glands or their ducts. A classical example is the epithelial lining of the follicles of thyroid.

(d) Ciliated epithelium

A specialized form of lining tissue is ciliated epithelium. Usually columnar in shape, the free surfaces of each cell bears numerous cilia capable of beating rapidly and rhythmically. This helps in causing motion producing currents. In mammals ciliated epithelium lines tubes and cavities in which materials have to be moved. e.g., ciliated epithelium is found in the lining of respiratory tract for expelling dust particles; in oviduct ciliated epithelium causes movement of ova.

(e) Glandular epithelium

- Cells are secretory in function.

- The cells of this tissue are of any shape like cuboidal, columnar, oval, spherical or irregular.
- Cells together may take different shape or remain single to act as gland, hence classified as two main types.

Simple glandular epithelium (single cell individually acts as gland, e.g., Goblet cells).

Compound glandular epithelium (many cells together form a unit gland, e.g., all common glands).

- Tubulo-alveolar or racemose gland: This type of gland comprises of both alveolar (as secretory) and tubular (as duct) parts. Examples: All common exocrine glands (salivary gland, mammary gland, pancreas, Cowper's gland).

Exocrine and Endocrine Glands

- Both are formed by the invagination of epithelial lining into the connective tissue part.
- Exocrine glands retain the connection with upper epithelial layer that forms duct; secretions released outside, i.e. at the site of action through its own duct.
- Endocrine glands later loose connection from parent (upper) epithelium and becomes ductless, secretion product (hormones) are released inside, i.e., into the blood, which carries it to the site of action.

(f) Sensory epithelium

- Cells of cuboidal or columnar shape; apical surface sensitive to stimulus.
- Basal surface with nerve connection to convey information to brain; highly specialized; form lining of all sensory organs. Examples: Retina of eyes, organ of corti, cristae and maculae in internal ear; Schneider's epithelium in olfactory chamber, Paccinian corpuscles in skin, taste buds on tongue, etc.

(g) Germinal epithelium

- Cells undergo meiotic division; for the formation of gametes only. Examples: Lining of ovary and seminiferous tubules of testis.

(h) Pseudostratified epithelium

- This type of epithelium is ciliated.
- Cells are arranged in a single layer, but due to excess and early growth of neighbouring cells some cells remain subdued and cannot reach the surface hence appear to be in another layer.

Examples: Patches in the lining of pharynx, nasal chamber, trachea, covering of epiglottis, some part of vasa deferentia and oviduct funnel, etc.

Compound Epithelial Tissue

(i) Stratified Epithelium: The cells of stratified epithelium are arranged in many layers. The deepest layer, resting on a basement membrane, consists of columnar cells. The intermediate cells are polyhedral or cubical and the superficial layers are made up of flattened cells.

(j) Stratified Squamous: This is found on surfaces which may be subjected to friction, mechanical injury or desiccation. The layering may vary from a few layers (e.g., corneal epithelium) to many layers (e.g., epidermis). Stratified squamous epithelium is of 2 types:

Non-*keratinized* and *keratinized*.

Non-keratinized epithelium is found on covering surfaces, which are not subjected to desiccation while the keratinized variety is present on exposed skin surfaces.

- Non-*keratinized*
  Lining of mouth, pharynx, oesophagus and anus.
  Urethra (near outlet).
  Vagina
- *Keratinized*
  Epidermis covering the whole surface of the body.

(k) Stratified Cuboidal

- In this epithelium, there is a superposition of polyhedral cells, with the superficial cells taking a more cuboidal shape. They are found in seminiferous tubules of testis, Graafian follicles of ovary, ducts of sweat glands, sebaceous glands, intermediate zones of urethra and conjunctiva.

(l) Stratified Columnar

- The deeper cells are irregular polyhedrons but the surface cells are columnar, found in excretory ducts of salivary, mammary glands, pharynx and Larynx.

(m) Transitional Epithelium

It is a variety of stratified epithelium found in the urinary tract. All the cells are living, and the surface layer is not squamous; in this respect it differs from stratified epithelium. The deepest cells are usually columnar or cubical, the more superficial cells are irregularly polyhedral or “pear-shaped”. The surface layer consists of large cells, often binucleate, of "umbrella" shaped, the concavity fitting over the cells in the next deeper layer. On stretching, the epithelium becomes markedly thinner found in pelvis of the kidney, ureters, bladder, first part of the urethra.

(n) Stratified compound epithelium

- Cells undergo both morphological and positional changes but reversible. When not required the cells resume original shape and position. Examples: Lining of urinary bladder, upper part of ureter and pelvis, etc.

Connective Tissue

The connective tissues of the body connect and anchor various parts to each other. They are often referred to as supporting tissues because they give support to the body and its organs. All of the supporting tissues possess two characteristics in common:

- They are all developed from the embryonic mesenchyme, which is itself derived from the primitive mesoderm.
- They all possess a relatively large amount of intercellular matrix and fibres of various types. The cells are relatively less important than the matrix and fibres. The matrix is permeated by tissue fluid which contains albumin and a variable content of salts and glucose.

Cells of various types remain scattered within the matrix which is more than cellular component. Due to wide variety classified initially into three main types:

(i) Connective tissue proper

(ii) Skeletal connective tissue

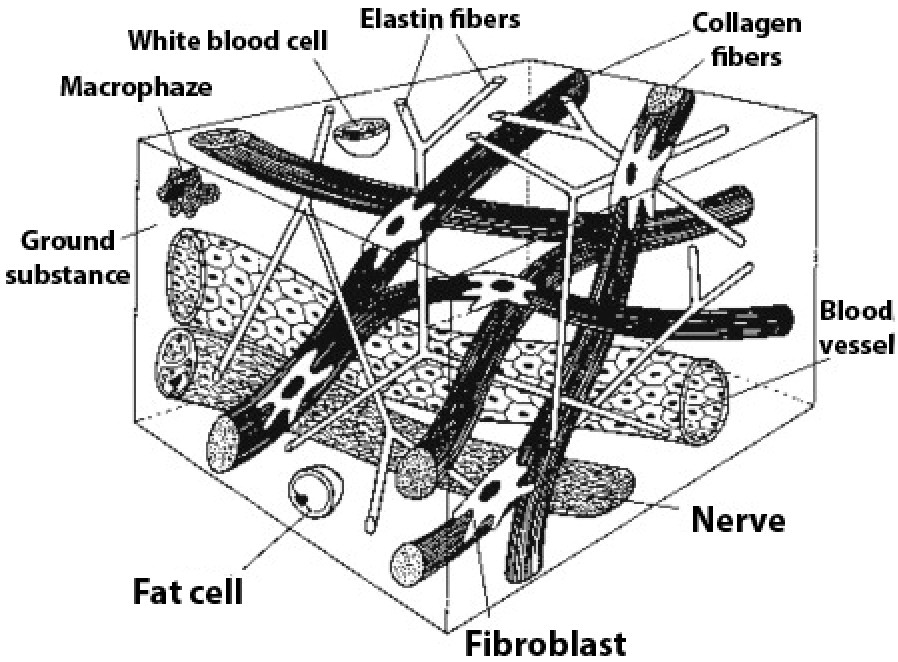
(iii) Fluid connective tissue

I. Connective Tissue Proper

The connective tissue as such refers to connective tissue proper. Features of typical connective tissue are as follows:

- Owing to many types of cells and large amount of matrix the whole structure is described in three main parts:
  – Cellular component – all cell types.
  – Fibrous component – fibres in the matrix.
  – Amorphous substance – non-fibrous (liquid) part of matrix.
- **Fibroblasts:** These are large, flat and of irregular shape, mostly seen with fibres. Main cell type; most numerous and produces most part of the matrix

- **Mast cells**: These are second most numerous cell types; small, spherical or oval and with granulated cytoplasm. Secretes various substances like:
  – Heparin (glycoprotein) is an anticoagulant of blood.
  – Histamine (derivative of amino acid histidine) is an activator, vasodilator, causes all body reactions (allergies) and inflammatory actions.
- **Adipocytes (fat cells):** Large, spherical with accumulated fat, which occupies central part in the cell pushing the cytoplasm and nucleus to periphery, hence looks like *signet-ring*. Found mainly as the component of adipose tissue.
- **Fibrous component :** Fibres of connective tissues are of following types:
- **White collagen fibres:** These are thick and unbranched bundles of fibres. Fibres are made up of collagen proteins. These are inelastic in nature and less durable.
- **Yellow elastic fibres:** These are thin and branched bundles of fibres. Fibres are made up of elastin proteins. These are of elastic nature and highly durable.
- **Reticular fibres**: These are thin and network like fibres found only in special parts, made of reticulin protein. Its contact provides *positional signal* to the epithelial cells for division
- **Amorphous substance**: These are non-fibrous, jelly-like and colloidal in nature. Macromolecules are mainly in the form of protein or its derivative (as building material). Collagen is the most abundant protein.



(a) Areolar Tissue

Areolar tissue is the most typical kind and can be regarded as the general form from which the other varieties are specialized. It is a loose; irregular connective tissue which has a very widespread distribution. Areolar tissue connects the skin to the underlying structures and fills any unoccupied spaces between organs. It penetrates with the blood vessels and nerves into the various tissues and organs. In the fresh conditions it is soft and transparent and contains numerous potential cavities.

The presence of these spaces is responsible for the name (areolar – "little areas" or spaces).

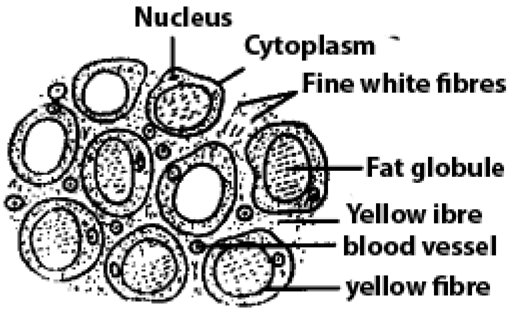
Pathologically, these spaces may become filled with fluid resulting in oedema (swelling).

Structure: Areolar tissue consists of a matrix that contains various kinds of cells and fibres namely m collagen and elastic fibres.

Intercellular Matrix: In a fresh spread preparation of areolar connective tissue, the matrix is optically homogenous and transparent. The intercellular matrix contains several protein polysaccharide complex.

(b) Adipose connective tissue

- Fat (food) storing tissue, mainly in subcutaneous layer and termed as *panniculus adiposus*.
- Made of adipocytes, interstitial cells give rise to adipocytes and some elastic fibres which provide strength.
- Increase in the amount of this tissue is called obesity



Structure of a adipose tissue

- Fat is deposited in three forms: olein, stearin and palmitin, which is more influenced by female sex hormone (estrogen).
- Acts as thermal insulator hence specially important for homoeotherms.
- As shock absorber, it forms the padding around internal organs. Examples: Mammary gland, Camel's hump, Blubber of Whale, Dolphin, etc. These are the type of common white fat.
- Brown fat is specially found in mammalian baby and the hibernating mammals. Looks brown due to high number of blood vessels and mitochondria.
- Protects the baby from temperature-shock after birth, and provides extra energy and heat.

White fibrous connective tissue

- White fibrous connective tissue is cord like structure, mainly made of collagenous fibres.

  Examples: Tendon which connects muscle to bone.

Yellow elastic connective tissue

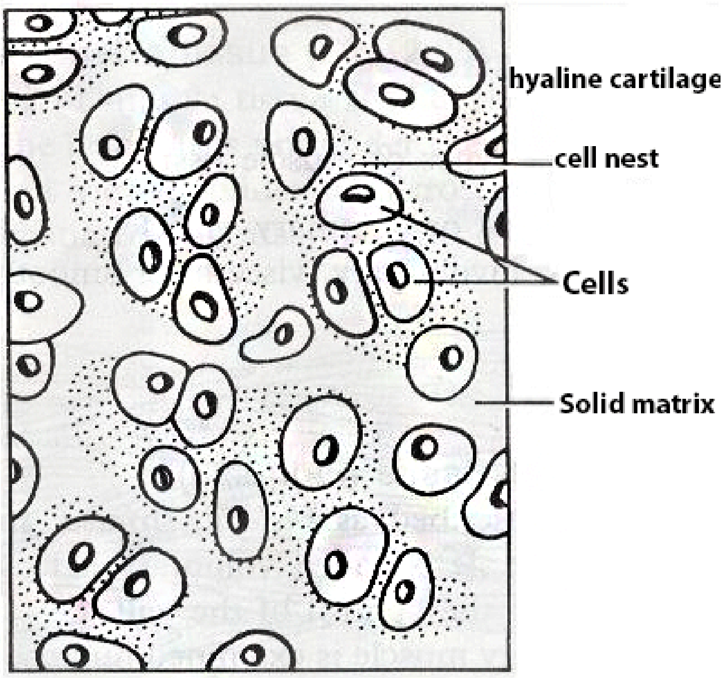
- Yellow elastic connective tissue is cord like and made of mainly elastic fibres. Examples: Ligament which connects bone to bone.

II. Skeletal Connective Tissue

- The skeletal connective tissue forms the framework of the body and provides surface for the attachment of muscle.
- It is mainly of two types: cartilage and bone.

Cartilage

- Skeleton in vertebrates initially formed as cartilage hence is primitive to bone.
- The mesenchyme differentiates into *chondrioblasts* which give rise to *chondriocytes*; the fibrous covering is *perichondrium*.
- Chondriocytes exist as singlet, doublet and *cell nests* within the lacunae
- The cartilaginous tissues are classified as follows:
- Hyaline cartilage is transparent, smooth, glass-like; with less or no fibres. It is major type of cartilage in the body. Examples: Cartilages of larynx, tracheal rings, epiphysis and hyoid apparatus of frog, etc.
- Fibrous cartilage is with abundant collagenous fibres hence, appears opaque; strongest cartilage present between two bones. Examples:
  Intervertebral disc and pubic symphysis of mammals, Ischium, pubis and mentomeckelian cartilage of frog.
- Elastic cartilage consists mainly of elastic fibres, hence forms thin and elastic parts like. Examples: Pinna, nose tips, epiglottis in mammals, eustachian tube, etc.

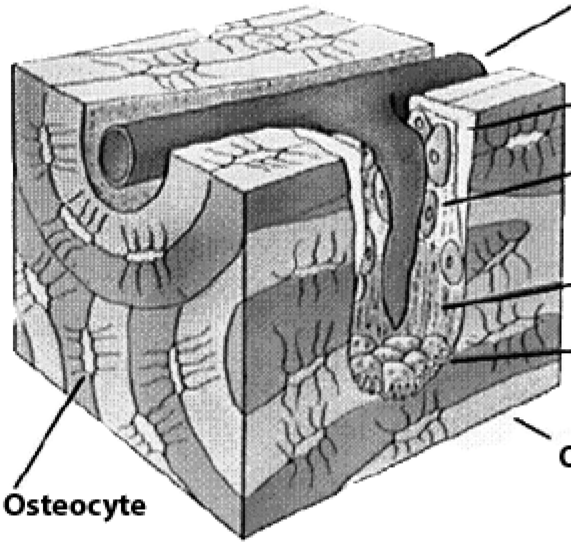


- Calcareous cartilage has extra deposition of $CaCO_3$, hard like bone. Examples: Supra scapula in pectoral girdle of frog.

Bone

- The outermost fibrous layer of bone is periosteum. Osteoblasts give rise to osteocytes or bone cells. These cells are arranged in concentric layers around the central cavity, through this cavity also pass nerves and blood vessels.
- Endosteum forms inner boundary of this cavity

Fig. Bone



- Fibres of Sharpey are the radial fibres traversing inside, its number increases with age.
- Other characteristics are comparable to cartilage are as follows:

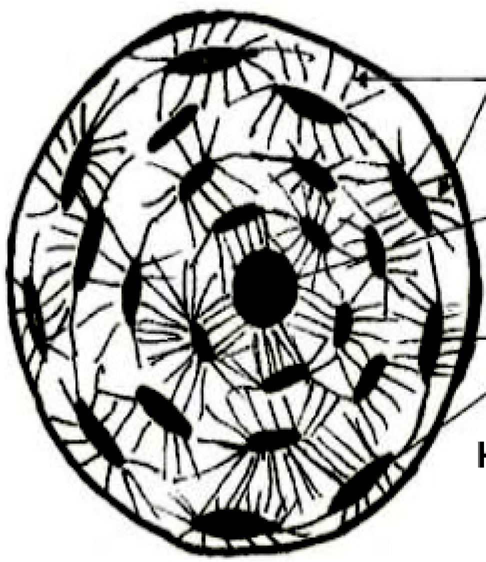


Table: Comparision between cartilage and bones

Characters	Cartilage	Bone
1. Content of matrix	Mainly (90%) organic substance	Mainly inorganic substance (65-70%) and rest organic substance (30-35%)
2. Main organic substance	Chondroitan sulphate	Collagen substance
3. Special protein	Chondrin	Ossein
4. Inorganic substance	Mainly $CaCO_3$	Mainly $Ca(PO_4)_2$
5. Cavity	Absent	Central cavity called as *bone marrow*
6. Position of basic components	Chondriocytes are scattered in matrix	Osteocytes are arranged in concentric layers
7. Layers of matrix	Absent	Layers form lamellae
8. Lacunae	Without canaliculi	With canaliculi
9. Coverings	Only perichondrium	Both periosteum and endosteum
10. Blood supply & Nerve supply	Absent	Both types of supplies takes place through a special cavity and canals
11. Mode of nutrition	Received by simple diffusion	Received directly through blood supply

- Birds bones are pneumatic, sustain air cavities to reduce the weight of bone as a flight adaptation.

Mammalian bone

- Consists of many longitudinal Haversian canals parallel to the central cavity and transverse Volkmann's canals connecting them. Through these, pass blood vessels and nerves.
- Osteocytes get arranged around Haversian canals instead of central cavity.
- All these together form ***Haversian system*** or ***osteon***, a structural unit of mammalian bone, each consists of osteocyte lamella.
- Mammalian bones are classified on the basis of structure into following types:

Spongy bone

- In the limb bones the two ends have many small cavities giving a spongy look, then contain red bone marrow where only RBC's are formed; thus absent in frog.

Compact bone

- The shaft region has only central cavity and the rest part is compact; contains yellow bone marrow where all blood cells are formed.
- The yellowish color is also due to presence of adipose cells in it.

Cartilage bone

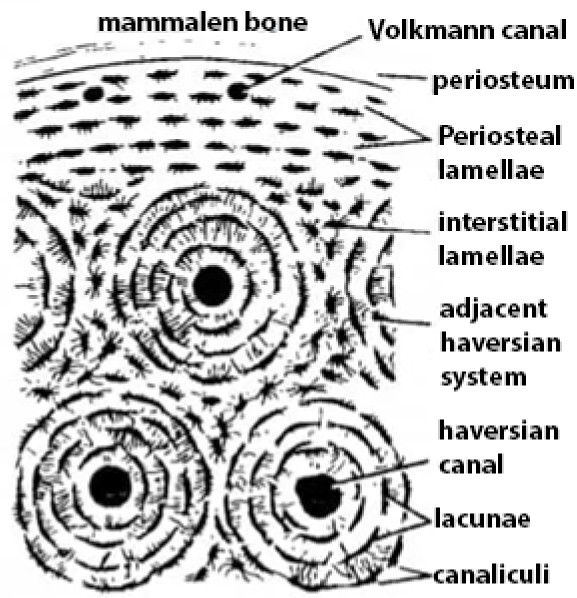
- Initially formed as cartilage but later replaced by bone. Examples: All long and thick bones of the body, *e.g.,* limbs, girdles, vertebral column, etc.

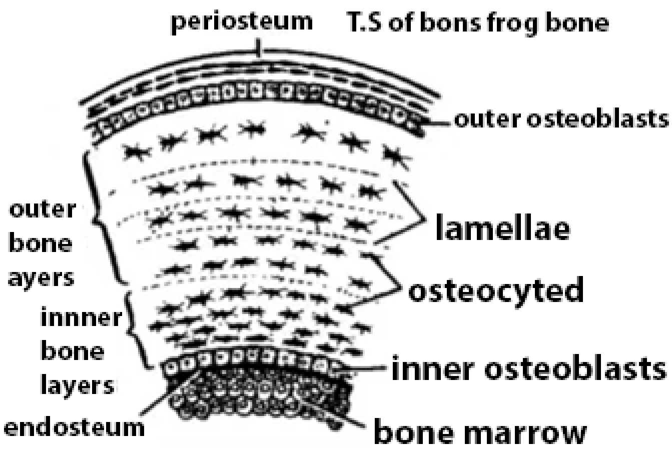
Investing bone

- Initially formed as bone from the dermis of skin. Examples: all thin and flat bones like all skull bones except occipital segment.

Sesmoid bone

- Formed by the ossification of fibrous connective tissue, i.e., tendon, ligament. Examples: Patella (knee cap)





Additional information about bones

- It is the hardest tissue; It is the Homeostatic reservoir of calcium in the body.
- Formation and deformation keep taking place hence it need regular maintenance.
- Bone is eaten away by special cells known as *osteoclasts*.
- Calcium and Vitamin D are essential components for its normal form and function.
- Calcitonin, parathormone and also sex hormones play important role in bone metabolism.

- Treatment with acid leads to decalcification, but alkali has no effect.
- Osteomalacia or adult rickets is the deformation or weakening of bone due to deficiency of calcium or Vitamin D in adult.
- Rickets is retarded growth and malformation of bone due to deficiency of Vitamin D, in childhood.
- Osteoporosis is the loss of bone weight due to old age; caused by lower level of sex hormone; more prominent in females.
- Osteomyelitis is the defect of bone due to bacteria or other germs.

III. Fluid Connective Tissue or Blood Vascular Tissue

- In most animals, blood collects excretory and other wastes from the body and flows within closed vessels except few invertebrates.
- Transport, entire materials like nutritive substance, respiratory gases, hormones, etc., throughout the body.
- Constitutes internal environment of the body and defense system.
- In human body, the amount is 5 to 6 litre, i.e., about 8% of total body weight (90 ml/kg). pH is slightly alkaline, strictly maintained; 7.2 for venous and 7.4 in arterial blood.
- Color-Venous blood bluish; Arterial blood bright red.
- Increasing bluishness due to accumulation of venous blood in any part is called as *cyanosis*.
- Blood is described by separating its two components: (i) Intercellular substance or matrix, called *plasma* (54-60%), (ii) Cellular component or formed element includes RBC’s, WBC’s and Platelets (40-46%).

Plasma

- A viscous fluid medium in which blood cells remain suspended, hence also called as *ground substance* of blood.
- Pale yellow but transparent fluid, constituting 4.5 to 5% of the total body weight.
- Plasma consists of mainly water (90 – 92%), Proteins (6 – 7%), minerals (0.9 – 1%), and glucose (0.1%) in almost fixed ratio.
- Other substances whose ratio vary from place to place and time to time include hormones, enzymes, lipids, amino acids, antitoxins, vitamins, urea, uric acid, creatinine, and gases like NH_3, CO_2 and O_2.

- Inorganic salts (minerals) are mainly as bicarbonates and chlorides of sodium, other salts like - phosphates, carbonates, sulphates or iodides of Ca, Mg, K and Fe are present in traces.

Plasma proteins

- Total concentration is 74.83 gm/litre. Most of these are synthesized in liver and of three types:
- (i) Albumin (48 gm/litre), (ii) Globulin (23 gm/litre), and (iii) Clotting protein (3 gm/litre). All these combine with iron, thyroxine and steroid hormones to form *transportable complex*.
- Proteins and minerals maintain *osmotic colloidal* pressure and *p*H of blood (as buffer).
- Globulins, formed in lymphoid organs, help in osmoregulation and transportation. Immunoglobulins act as anti-infective substances (*antibodies*).
- Clotting proteins are fibrinogen and prothrombin, essential for blood clotting.

Cellular Components

Erythrocytes (RBC)

- Erythropoiesis is the process by which RBC's are produced.
- As completely differentiated cell does divide and has no centrosome
- These are also called as *terminated cell* for the loss of its structural and functional organisation and short life span.
- A reservoir of haemoglobin this cell is present in only vertebrate blood and has evolved for the transport of gases.
- Mammalian RBC is without the nucleus, mitochondria, ribosomes, ER and Golgi bodies.
- Viability and source of energy depends upon its membrane and glycolysis.
- Very elastic and flexible, it undergoes deformation to pass through narrow capillaries.
- Contains antigen of blood group and an enzyme, *carbonic anhydrase*, which helps in CO_2 transport.
- Number of RBC's primarily depends upon physical activity of the organism.

Important features of mammalian RBC's

- Vitamin B_{12} vitamin C and folic acid are essential for erythropoiesis.
- The deficiency of vitamin B_{12}, folic acid and other factors lead to improper formation and functioning of RBC. This condition is called as *anemia*. The anemia are of following types:
- *Pernicious anemia* (defective maturation) is due to the deficiency of vitamin B_{12}.
- The largest RBC is in *Amphiuma* and *Proteus*, the tailed amphibian (diameter, 50–50 μ)
- The smallest RBC is in musk deer (diameter 3 – 4 μ)
- Spleen stores extra RBC's thus called as blood bank; destroys dead RBC hence called as *graveyard of RBC*.

Leucocyte (WBC)

- Leucocytes are regarded as policemen of the body, constitutes the defence system.
- Active and nucleated cells of dividing nature; can change shape; they are mobile and phagocytotic.
- They can cross through blood capillaries wall (diapedesis) since most of their functions are outside blood.
- Number (Total Count) is 5000 to 9000/ mm^3, depicts body's state of infection, it is in higher range in sick people.
- Leucocytosis – Number above 12000/mm^3.
- Leucopenia – Number below 4000/mm^3.
- Leukemia (*Blood cancer*) – Number rises abnormally beyond control.
- Life span – of these cells varies from few hrs to years depending upon the type.
- Leucocytes are classified into following two broad types:

Agranulocytes: No granules in cytoplasm and nucleus and large, it does not have any definite shape.

These are: (1) Monocyte, (2) Lymphocyte

1. **Monocytes:** These are the largest WBC (diameter 10 μ to 18 μ); macrophages (phagocytotic) in tissue fluid, they function to mop up unwanted material. Number (differential count) is approximately 4 to 11% i.e., about 200 to 700/mm^3. Nucleus is large, kidney or bean-shaped, may also be horse-shoe shaped.

2 Lymphocyte: These are both small and large sized (diameter ~ 6 μ to 15 μ) They are of two types, *B-cell* and *T-cell,* number (D.C.) is 20 to 35% i.e., ~ 1500 – 2500/mm^3. Nucleus enormously large, spherical and occupies most area of cell. Cytoplasm is peripheral, rim-like. Its main role is in maintaining the *immune system*; forming antibodies. The T–lymphocytes form various cell types in the immunity system and according to their functions called as *T-helper cell*, *T-killer cell*, *T-suppressor cell* and *T-memory cell*.

- Null cells are the third type of lymphocytes of unknown property.

Granulocytes - presence of granules in the cytoplasm and are of 3 types:

1. **Neutrophils**: They take color with neutral stains (dyes). These are the main WBC type; with maximum number and variety of functions. The number is 60–65% of the total WBC i.e., 4000 to 5000/mm^3, large sized and phagocytotic in nature. Neutrophils secrete chemicals like pyrogens, toxins (killer substance), antitoxins, inflammatory and anti-inflammatory substance, lysozymes etc. Nucleus is bi-lobed and the number of lobe increase with age.

   In females, drumstick body (modified Barr bodies) is present in the nucleus for its action, also called as *shock troops*.

2. **Eosinophil or acidophil:** They take color up with acidic dyes, hence cytoplasm is alkaline. Number is 2 – 4% i.e. 70 – 300/mm^3, diameter is approximately 7.5–9 μ. The number increases i.e., *eosinophilia* (in helminthic and the respiratory tract infections). The nucleus is bi-lobed, also phagocytic, but less motile than neutrophils, granules lysosomal with high peroxidase content.

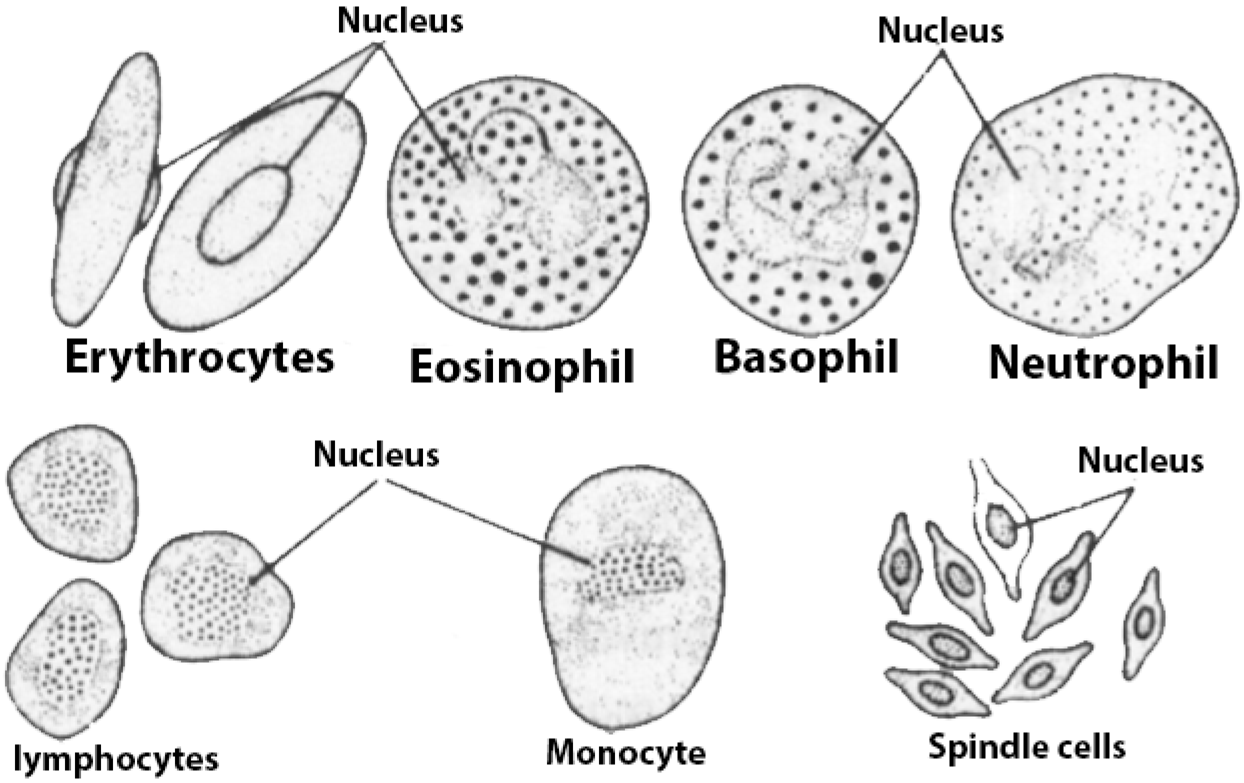
   - Hanker (1980) discovered a new type of eosinophil named as *medusa cell* of large size and phagocytotic nature.

3. **Basophils:** They take up color with basic dye, thus cytoplasm is acidic. Number (D.C) is 0.5 to 1% i.e., 35 -150/mm^3, most difficult to locate in a blood smear. Nucleus is twisted, S-shaped. Cytoplasm contains heparin, histamine and serotonin like mast cell.

Blood platelets

- These are only found in mammalian blood and protoplasmic fragments are without nucleus, these are formed from *megakaryocytes* in the bone marrow.
- They contain blood clotting factors, decrease in number is called *thrombocytopenia***,** and it hampers blood clotting.

- In other vertebrates (frog) and invertebrates, instead of platelets, these are nucleated cells called as *thrombocytes* or *spindle cells*.



Different types of blood corpuscles
Fig. formed elements of blood

Muscular Tissue

- Most specialized tissue, only function is to generate movement or force.
- Converts chemical energy into mechanical energy like automobile machine.
- Machinery for movement is the set of *actin* and *myosin* proteins (both filamentous) as main component of the cytoplasm.
- The cell (completely differentiated) doesn't divide, has no golgi bodies and centrosome and other parts called as: cytoplasm is called as *sacroplasm,* plasmalemma as *sarcolemma,* endoplasmic reticulum is *sarcoplasmic reticulum* (reduced as vesicles).
- Mitochondria (Sarcosomes) are the main organelle, largest in size and number. In vertebrate there are three types of muscles; (1) Skeletal muscle, (2) Cardiac muscle and (3) Smooth muscles

Skeletal muscle

- Associated with skeleton, generates external movement in the body.

- Cells are coenocytic long and cylindrical called as *muscle fibre* which is formed by the fusion of many myoblasts**.**
- Only parallel fibres are present in a muscle.

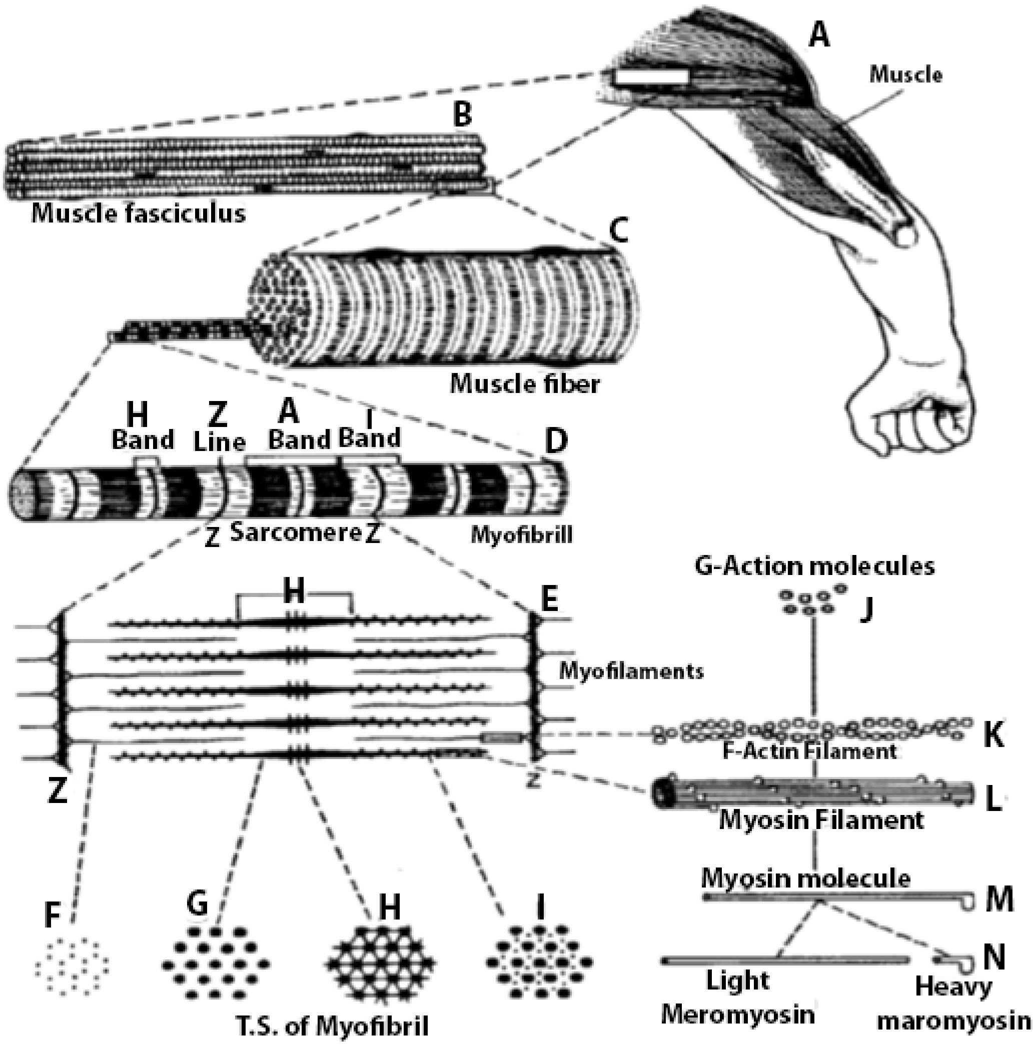


Fig. *Structure of skeletal muscles*

- Sarcolemma form T-tubules by transverse invagination at intervals.
- Striations (alternate *dark* and *light* bands) are formed due to particular arrangement pattern of actin and myosin filaments – as revealed in electron microscopic structure of a *myofibril*.
- Lengthwise, each myofibril consists of many sarcomeres, which are plate like zigzag structures through which thin filaments pass, they are the functional units of muscles and constitute the area between two Z-lines.
- Well defined Z-line, (Z-disc or Krause's membrane or Dobies membrane) present.
- Dark band (**Anisotropic band or A-band**) consists of both actin (thin) and myosin (thick) filaments.
- Light band (**Isotropic band or I-band**) consists of only actin filaments.
- **H–Zone or Hensen's disc** within dark band consists of only myosin part.

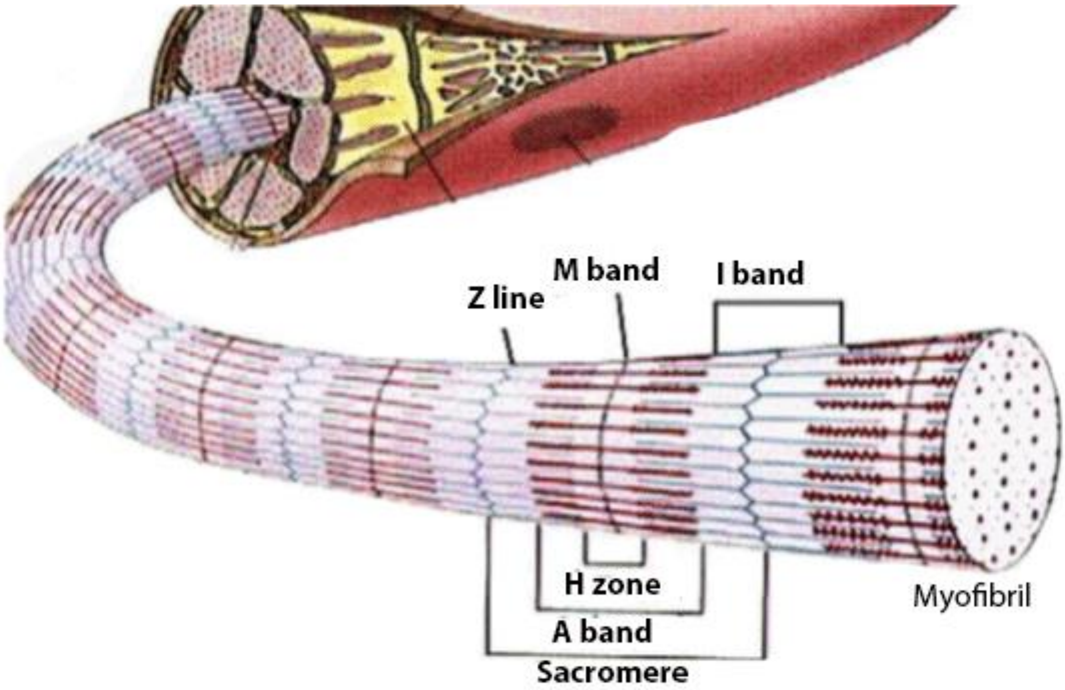
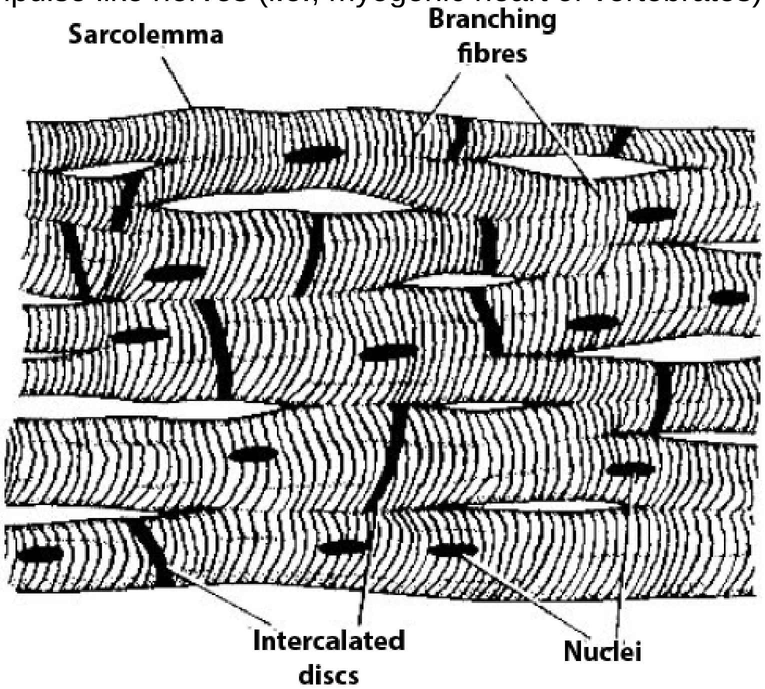


Fig.

Cardiac muscle

- The most versatile, muscle is exclusively found in heart.
- Being striated, it is structurally similar to skeletal muscle except the presence of –
- Cross fibres, besides parallel fibres
- Intercalated discs which represent the joints of two myoblasts.
- Contraction pattern is rhythmic and also fast, controlled by autonomous nervous system (ANS).
- Untiring muscle (no fatigue), works ceaselessly throughout the life.
- Generates impulse like nerves (i.e., myogenic heart of vertebrates).



Structure of cardiac muscle

Smooth Muscle

- Unstriped; hence structurally different from striated and cardiac muscles.
- Muscle cells are spindle-shaped (not cylindrical), uninucleated each formed of single myoblast.
- Actin filaments remain attached, to dense bodies in the cytoplasm.
- There is no myofibril, no sarcomere and no Z-line.
- Actin and Myosin filaments not arranged in any pattern hence no striations.
- Involuntary muscle, under ANS, control, found in visceral organs (visceral muscle). Contraction in such muscles is termed as *peristalsis*, is *rhythmic*, *slow* and *prolonged*.

Nervous Tissue

- This is coordinating or controlling tissue of the entire of the body's structural or functional organization.
- Exhibits highest degree of irritability and conductivity.
- It is ectodermal in origin and completely differentiated tissue, cells do not divide at all (except some glial cells).
- Receiving, integrating, transforming and transmitting the coded information of stimuli to evoke response in the body are its specialized functions. This tissue consists of the following two types of cells:
- Neurons - the actual nerve cell.
- Neuroglia - the supporting cells.

Neuroglia (glial cells)

- Non-excitable, supporting cells of both ectodermal and mesodermal origin.
- Constitutes 80% part of nervous system to provide neurons with nutrition, mechanical support, protection, electrical insulation and packing material.
- For above functions these cells (Astrocytes) make *blood-brain barrier*.
- The neuroglia are of following two types representing the peripheral nervous system (PNS):
  a) Schwann cells – These cells constitute the myelinated covering of axon or nerve in vertebrates.
  b) Sheath cells – These are generally found in invertebrates.

Neurons

These are structural and functional units of the nervous tissue and highly excitable as well as specialized for generation and conduction of impulse.

- The longest (up to 1.5 m. or more), the most sophisticated and the most complicated in structure and function.
- Number is higher in large sized animals 100 billion in human body.
- Neurons consist of three parts; *dendron*, *cyton* and *axon*. Cyton is the cell body, while dendron and axon are the branches of the cell.
- Nissl's granules remain only in the cyton part and they are the sites for protein synthesis.
- Neurons possess network of microtubules throughout the cell and constitute a transporting system.
- Centrosome is absent so neurons are unable to regenerate.

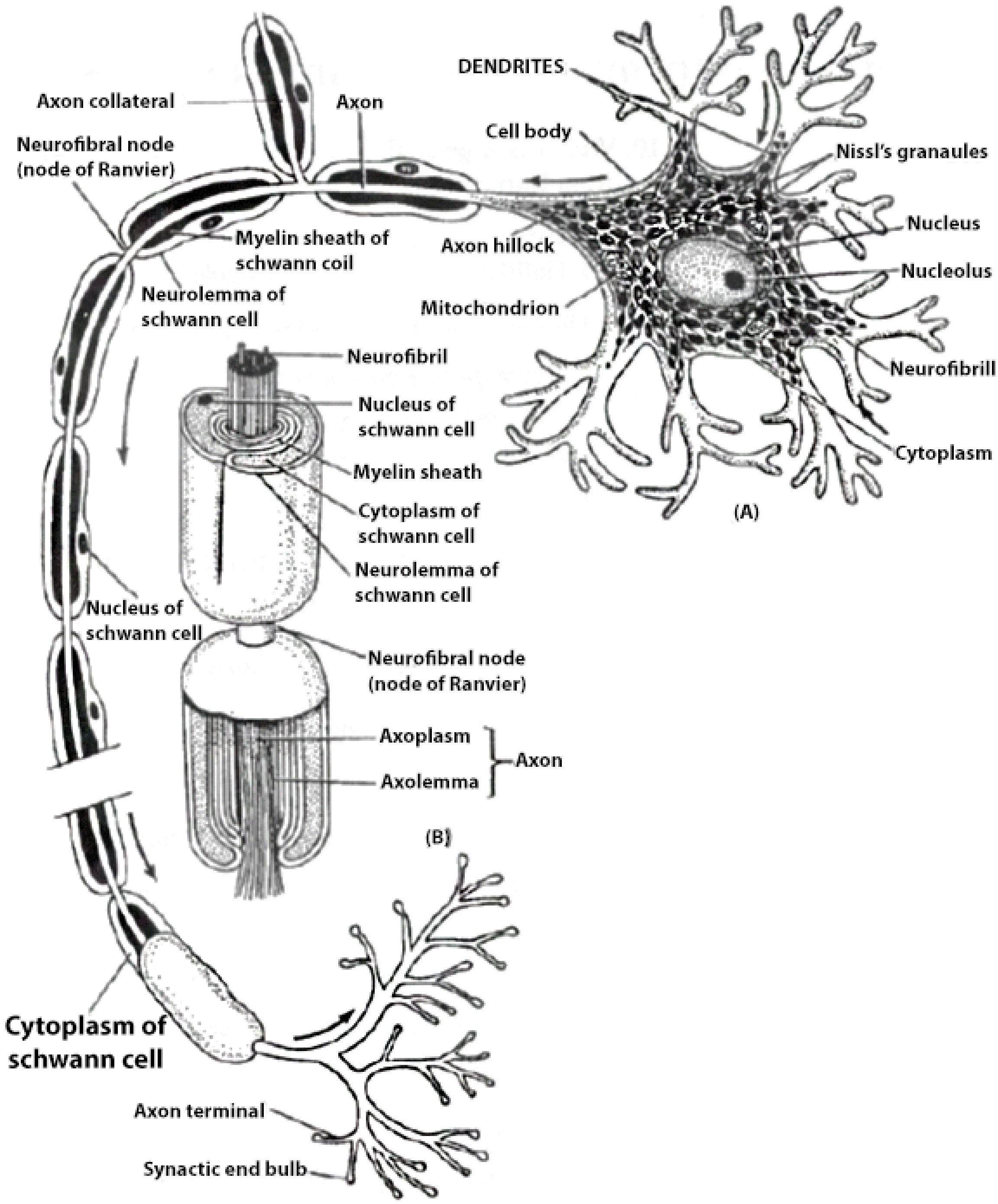
Dendron

- These are also called as *afferent branches* and are the receiving branch, impulse enters the nerve cells through it. At the terminal end each dendron gives rise to fine branches called *dendrites*.

Axon

- It is also referred to as *afferent branch*, directly attached with cyton. The point of attachment with cyton is called as *axonic hillock*. Axon is regarded as the main branch of neuron and it is uniform in thickness. The number of axon always remains one. Impulse passes out of neuron through this…
- Terminal branches (arborization) are telodendria, sometimes they terminate with a prominent knob like structure which are called as *terminal buttons* or *end bulbs* or *secretory vesicles*.

  Certain axon of neurons also gives out branches at right angle and they are called as *collateral fibres*.
- The cytoplasm of axon is *axoplasm* with abundant neuro-fibril and mitochondria. The plasma membrane of axon is called as *axolemma*. Axons are main components of peripheral nervous system (PNS).
- In myelinated nerve it is covered with myelinated sheath made of Schwann cells. Cut gap at certain intervals in this sheath is called as *node of Ranvier*.



(A) Structure of a nerve cell (B) L.S of axon

Fig,: *A. Structure of a neuron, B. L.S. of axon*

Cyton (perikaryon or soma or cell body)

- This part of neuron constitutes the grey matter of CNS, outside CNS in clusters form ganglion.
- Abundant cytoplasm with almost all organelles and Nissl's granules as colored body made of RNA, Nissl's granules are also called as *trigoid bodies* which are

secretory in nature. Ribosomes and Golgi bodies are also present in the cytonic region.

Synapse

- It is the joint of two neurons - between the axon of one neuron and the dendron of another neuron.
- Axon is termed as *pre synaptic terminal* while, dendron as *post synaptic terminal*.
- Synapse are classified into following two types and their classification is based on following distinguishing features:
  a) Creminal synapse
  b) Electrical synapse

Neurotransmitters

- There are more than 30 types of substances discovered so far which are directly or indirectly involve in the conduction of nerve transmission and they are collectively called as *neurotransmitters*.
- Acetylcholine, this is *excitatory* for nerve and general muscle, while, inhibitory for cardiac muscle.
- Sympathin, Histamine, Noradrenaline (adrenalin) are excitatory for all muscles and nerves.
- Serotonin, dopamine and GABA (γ-amino-butyric acid) are inhibitory for all.

Types of neurons

- The neurons are classified on the basis of number of dendrons and functionality.
- On the basis of number of dendrons, neurons are classified into following types:
  – **Unipolar neurons:** Without dendron, only one branch present is axon.
  Examples: Embryonic brain cells.
  – **Bipolar neurons:** Only one dendron and one axon hence with two branches.
  Examples: Neurons present in the retina of eyes.
  – **Multipolar neurons:** More than one dendron, hence, has many branches.
  Examples: All common neurons.
- On the basis of functional characteristics, neurons are of following three types:
  – **Sensory neuron:** Carries impulse from sensory organ to brain, also called as *afferent neuron*.
  – **Motor neuron:** Carries impulse from brain to effector organ (i.e. muscles) hence, also called *efferent neuron*.
  – **Interneuron:** Acts as adaptor between sensory and motor neurons in the CNS.

REVISION EXERCISE - LEVEL - I

1. Find out incorrect sentence:

   (a) Parenchymatous tissues have intercellular spaces

   (b) Collenchymatous tissues are irregularly thickened at corners

   (c) Apical and intercalary meristems are permanent tissues

   (d) Meristematic tissues, in its early state, lack vacuoles

2. Girth of stem increases due to:

   (a) apical meristem (b) lateral meristem

   (c) intercularly meristem (d) vertical meristem

3. Which cell does not have perforated cell wall?

   (a) Tracheids (b) Companion cells

   (c) Sieve tubes (d) Vessels

4. Intestine absorb the digested food materials. What type of epithelial cells are responsible for that?

   (a) Stratified squamous epithelium (b) Columnar epithelium

   (c) Spindle fibres (d) Cuboidal epithelium

5. A person met with an accident in which two lung bones of hand were dislocated. Which among the following may be the possible reason?

   (a) Tendon break (b) Break of skeletal muscle

   (c) Ligament break (d) Areolar tissue break

6. While doing work and running, you move your organs like hands, legs etc. Which among the following is correct?

   (a) Smooth muscles contract and pull the ligament to move the bones

   (b) Smooth muscles contract and pull the tendon to move the bones

   (c) Skeletal muscles contract and pull the ligament to move the bones

   (d) Skeletal muscles contract and pull the tendon to move the bones

7. Which muscles act involuntarily?

   (i) Striated muscles (ii) Smooth muscles

   (iii) Cardiac muscles (iv) Skeletal muscles

   (a) (i) and (ii) (b) (ii) and (iii) (c) (iii) and (iv) (d) (i) and (iv)

8. Meristematic tissues in plants are:

   (a) localised and permanent (b) not limited to certain regions

   (c) localised and dividing cells (d) growing in volume

9. Cartilage is not found in:

(a) nose (b) ear (c) kidney (d) larynx

10. Bone matrix is rich in:

(a) fluoride and calcium (b) calcium and phosphorus

(c) calcium and potassium (d) phosphorus and potassium

11. Contractile proteins are found in

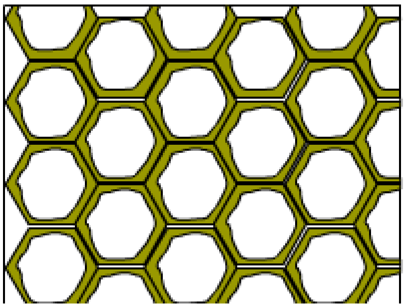
(a) bones (b) blood (c) muscles (d) cartilage

12. Nervous tissue is not found in:

(a) brain (b) spinal cord (c) tendons (d) nerves

LEVEL – II

1. Consider the given diagram carefully and select the true statement regarding this tissue.



Sclerenchyma

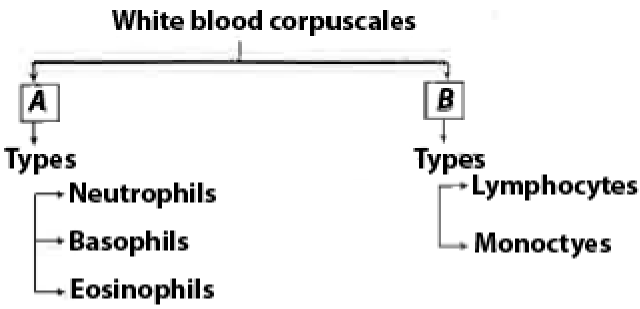
(a) This is a complex and dead xylem tissue found in plants

(b) This is a simple tissue, i.e. sclerenchyma which are basically dead cells

(c) This is a complex tissue, i.e. phloem and helps in food transportation

(d) This is a simple tissue, i.e. epithelial tissue, found on the upper surface of plant cells and give mechanical support to the plant

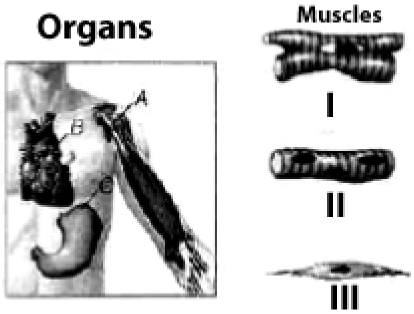
2. Complete the following chart.



Identify A and B from the options given below:

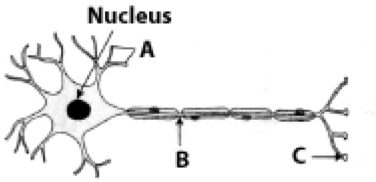
A	B
(a) Erythrocytes	Leucocytes
(b) Granulocytes	Agranulocytes
(c) Granulocytes	Agranulocytes
(d) Agranulophils	Granulocytes

3. Consider the given diagram carefully and observe that which of the given matches correctly.



(a) A-II, B-I, C-III (b) A-I, B-II, C-III (c) A-II, B-III, C-I (d) A-III, B-II, C-I

4. A nerve tissue is composed of various nerve cells as given below. Identify the parts of this cell, termed as *A*, *B* and *C*.



(a) A-Axon, B-Dendrites, C-Nerve endings
(b) A-Dendrites, B-Axon, C-Nerve endings
(c) A-Nerve endings, B-Axon, C-Dendrites
(d) A-Axon, B-Dendrites, C-Nerve endings

5. **Assertion (A):** Non-striated muscles are said to be voluntary in nature.
Reason (R): Non-striated muscles can be moved according to will.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A)
(c) (A) is true, but (R) is false
(d) Both (A) and (R) are false

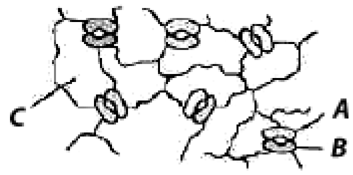
6. In given below crossword consists of names of different types of tissue

E	C	Q	X	Y	B	D	F	G	H	J	B
P	A	R	E	N	C	H	Y	M	A	K	C
I	R	T	S	R	U	Q	P	N	M	L	L
T	D	W	X	Z	B	G	B	C	D	F	E
H	I	G	H	C	O	L	U	M	N	A	R
E	A	J	K	L	I	A	M	P	N	Q	E
L	C	P	N	M	D	N	W	T	R	S	N
I	Q	W	X	K	A	D	V	V	W	B	C
A	Z	Y	B	J	L	U	H	G	F	D	H
L	D	F	G	V	O	L	U	N	T	R	Y
W	K	L	M	P	N	A	X	Y	L	E	M
M	Y	Z	X	W	Q	R	X	Y	B	D	A

How many of them are animal tissues?

(a) 2 (b) 5 (c) 8 (d) 6

7. A microscopic view of plant tissue is given below, observe it carefully and name *A*, *B* and *C* correctly.



A	B	C
(a) Stomata	Gurad cell	Epidermal cell
(b)Guard cell	Stomata	Epidermal cell
(c) Epidermal cell	Guard cell	Stomata
(d)Stomata	Epidermal cell	Guard cell

8. Tissue responsible to increase the length of the root and stem is:

(a) lateral meristem (b) apical meristem

(c) intercalary meristem (d) both (a) and (c)

9. Strain used to differentiate the plant tissues for cells for clear demarcation is:

(a) safranin (b) glycerine (c) acetocarmin (d) crystal violet

10. Chlorenchyma contains maximum:

(a) collenchyma (b) chloroplast (c) vacuoles (d) all of these

11. A chemical substance which acts as cement and hardens the sclerenchyma is:

(a) suberin (b) lignin (c) cutin (d) cellulose

12. Usually the number of guard cell in a stoma is:

(a) one (b) two (c) three (d) four

13. **Match the columns**

(A) Eosinophil (i) Erythrocyte

(B) Chondrocyte (ii) Adipose tissue

(C) Mast cell (iii) Areolar tissue

(D) Fat droplet (iv) Leucocyte

(E) Haemoglobin (v) Hyaline cartilage

(a) (A) → (iv), (B) → (iii), (C) → (v), (D) → (ii), E) → (i)

(b) (A) → (iv), (B) → (v), (C) → (ii), (D) → (iii), (E) → (i)

(c) (A) → (iv), (B) → (v), (C) → (iii), (D) → (ii), (E) → (i)

(d) (A) → (v), (B) → (iv), (C) → (iii), (D) → (ii), (E) → (i)

14. (A) Tracheid (i) Lateral meristem

(B) Companion cell (ii) Epidermis

(C) Sclerenchyma (iii) Simple permanent tissue

(D) Guard cell (iv) Xylem

(E) Cambium (v) Phloem

(a) (A) → (i), (B) → (ii), (C) → (v), (D) → (iv), E) → (iii)

(b) (A) → (ii), (B) → (i), (C) → (v), (D) → (iv), (E) →(iii)

(c) (A) → (iv), (B) → (v), (C) → (iii), (D) → (ii), (E) →(i)

(d) (A) → (i), (B) → (ii), (C) → (iii), (D) → (v), (E) →(iv)

15. **Fill in the blanks:**

i. In human beings, ... contract and relax to cause movement.

ii. Most of the plant tissues are ....

iii. Animals consume .... energy as compared to plants.

iv. ....is present at the growing tips of roots and stems.

v. Parenchyma consists of relatively ....cells with thin cell walls.

vi. ....of a coconut is made of sclerenchyma.

vii. Stomata are enclosed by..............kidney-shaped cells called guard cells.

viii. Epithelial tissue cells are .... packed.

ix. Skin epithelial cells are arranged in many layers to prevent wear and ....

x. The blood plasma contains proteins, ... and hormones.

xi. .... smoothens bone surfaces at joints and also present in nose and ear.

xii. Muscular tissue consists of elongated cells called ....

xiii. .... cells are long with pointed end and uninucleared.

xiv. An individual nerve cell may be up to a .... long.

xv. .... is made of neurons that receive and conduct ....

16. **Write True or False**

i. The girth of the stem increases due to lateral meristem.

ii. Lymphocyte and basophil are types of erythrocytese.

iii. Haversian canal contains blood vessels and nerve fibres.

iv. The alternate light and dark bands are found in smooth muscles.

v. Heart muscle cells are cylindrical, branched and multinucleated.

vi. Canaliculus contains and slender process of bone cell or osterocyte.

vii. Adipose tissue acts as an insulator.

viii. Striated muscles are long, cylindrical, unbranched and multinucleated.

ADDITIONAL EXERCISE – SECTION - A

1. The meristematic cells have :
   (A) Thin walls (B) prominent nuclei
   (C) Absence of vacuoles (D) All of the above
2. Meristems helps in :
   (A) Absorption of water (B) Absorption of minerals
   (C) Transport of food (D) Growth of plants
3. The division in meristematic cells is :
   (A) Mitotic (B) Amitotic
   (C) Meiotic (D) All of the above
4. The cells having the ability to divide are :
   (A) Specialised (B) Glandular (C) Meristematic (D) Permanent
5. A nail inserted some years back at 1.5mtr height on a tree trunk shall :
   (A) remain where it was (B) move up wards
   (C) move down wards (D) move laterally
6. Most metabolism of the plants is carried in tissue :
   (A) Phloem (B) Meristem (C) Collenchyma (D) Parenchyma
7. Which tissue provides maximum mechanical strength to the plants :
   (A) Parenchyma (B) Xylem (C) Phloem (D) Collenchyma
8. Xylem & Phloem belong to the group of :
   (A) Simple tissue (B) Latex tissue
   (C) Complex tissue (D) None of these
9. Which constitutes the thickening in collenchyma :

(A) Suberin (B) Cutin (C) Pectin (D) Lignin

10. Vessles and Companion cells occur in :

(A) Angiosperm (B) Gymnosperm (C) Pteridophytes (D) Bryophytes

11. Which one is not a plant fibre :

(A) Coir (B) Flex (C) Hemp (D) Silk

12. Which of the following acts a middle man?

(A) W.B.C. (B) Plasma (C) Blood (D) Lymph

13. The main difference between bone and cartilage is of :

(A) Mineral distribution (B) Cell structure

(C) Lymph vessels (D) Haversian system

14. Tendon is made up of:

(A) Yellow fibrous connective tissue (B) Adipose tissue

(C) Modified white fibrous tissue (D) Areolar tissue

15. Muscles get fatigued due to accumutation of :

(A) ATP (B) CO_2 (C) Lactic acid (D) Poly Molecule

16. Chemical ions responsible for muscle contraction are :

(A) Ca^{2+} and K^+ (B) Na^+ and K^+

(C) Na+ and Ca^{2+} (D) Ca^{2+} and Mg^{2+} ion

17. Collagen is :

(A) Protein (B) Fat (C) Sugar (D) Starch

18. Sprain is due to pulling of :

(A) Muscles (B) Ligaments (C) Tendens (D) Nerves

19. Power of regeneration is poor in:

(A) Brain cells (B) Bone cell

(C) Muscle cell (D) All of the above

20. Which one contain voluntary muscles?

(A) Heart (B) Hind limb (C) Liver (D) Lung

SECTION - B

1. Fibres of striped muscles are :

(A) Oval (B) Spindle-Shaped

(C) Cylindrical (D) Oblong

2. Striped muscle fibres are :

(A) Mutinucleate (B) Uninucleate (C) Binucleate (D) Anucleate

3. In a striped muscle fibre, the nuclei lie :
(A) Centrally (B) Peripharally (C) Polar (D) Diffused

4. Striped muscle fibres possess :
(A) Longitudnal Striations (B) Oblique striations
(C) Transverse striations (D) No striations

5. Striations appear in striped muscles due to :
(A) Presence of alternate light and dark bands
(B) Dispersion of pigments
(C) Presence of intercalated discs
(D) Occurrence of actin strands

6. A characteristics of striped strands :
(A) Rapid contraction (B) Forceful contraction
(C) Fatigued after some time (D) All the above

7. Striped muscle is specialised to :
(A) Elongate (B) Contract (C) Relax (D) Both B and C

8. Striped muscle works :
(A) Automatically or involuntarily (B) Under control of will or voluntarily
(C) Under control of reflex system (D) All the above

9. Nucleated part of nerve cell is called :
(A) Axon (B) Dendrites (C) Cyton (D) None of above

10. Cell body of a nerve cell contains :
(A) Nucleus (B) Cytoplasm
(C) Nissl granules and neuro fibrils (D) All the above

11. Dendrites occur over :
(A) Muscle fibre (B) Nerve cell (C) Spinal cord (D) Brain

12. Nerve cells are also called :
(A) Neurons (B) Neurites (C) Neurines (D) Nuerofibrils

13. Dendrites are :
(A) Long unbranched processes (B) Long branched processes
(C) Short branched processes (D) Short unbranched processes

14. Along fibre like process coming out of the cyton of a nerve cell is :
(A) Axon (B) Dendron (C) Neurolemma (D) Neurofibrils

15. Buttons are :

(A) Terminal sharp ends of dendrons

(B) Terminal swollen ends of dendrons

(C) Terminal pointed ends of axon branches

(D) Terminal knob-like swollen ends of axon branches

16. Nerve fibre is :

(A) A thin nerve

(B) Fibre formed by many axons

(C) Fibre formed by ensheathing of axon

(D) Fibre formed by ensheathing of a dendrite

17. Medullated nerve fibre is axon covered by :

(A) Neurolemma

(B) Meduallary sheath

(C) Both neurolemma andmedullary sheath

(D) None of the above

18. Nodes of Ranvier are areas where :

(A) Joints occur between adjacent axons

(B) Axon terminals forms ynapses with dendrite tips

(C) Dendrites of one nerve cell are attached to dendrites of an adjacent one

(D) Non medullated areas of a medullated nerve fibre

19. Impulse is brought to a nerve cell through a :

(A) Dendrite (B) Neurofibril (C) Axon (D) Nissl granules

20. A nerve cell transmits its impulse to another through its :

(A) Dendrite (B) Axon (C) Cyton (D) All the above